Homework 6
due at 14:00 on October 13, 2009

1. Suppose there are two fair dice. We assume that one dice is red and the other is green (that is, distinguishable). Let us record the numbers that are up in this order as \((n, m)\) \((n, m \in \{1, 2, \cdots, 6\})\).
   (1) To know a particular pair of numbers \((a, b)\) unambiguously what information (in bits) do you need?
   (2) You are told that the sum \(a+b\) is not less than 5 (5 not included). What is the information you gain from this message?
   (3) Next, you are told, one of the dice shows the face less than 3. What is the information you gain? (You must know the info obtained from (2) already.)
   (4) Now, you are told that actually, the one of the dice in (3) is the red one. What is information in this message?
   (5) Finally, you are told that the face pair is actually \((2, 5)\). What is the information in this final statement?

As you guess, in whatever order the information is given, the total information you gain does not depend on the actual ‘path,’ because the extent of your ignorance is a ‘state function.’

2. There is a classical ideal gas of volume \(V\) consisting of \(N\) molecules whose internal degrees of freedom are expressed by a single harmonic oscillator with a frequency \(\nu\). The frequency depends on the volume of the system as

   \[
   \frac{\partial \log \nu}{\partial \log V} = \gamma.
   \]

(1) Obtain the pressure of the system.
(2) Obtain the constant pressure specific heat \(C_p\).

3. Inside the F\(_1\)ATPase is a rotator \(\gamma\) to which a long actin filament (it is a straight stiff bar of length 30 nm) is perpendicularly attached. Thus, the filament swings back and forth when the ATPase is waiting for an ATP molecule.
   (1) The root\(^1\) mean square angle fluctuation of the stiff filament was 30 degrees at 290K. If the temperature is raised by 10\%, by what percentage will the angular fluctuation change? Assume that the molecular structure is not affected by this temperature change.
   (2) What is the torsional spring constant of this rotator captured by the surrounding ring?
   (3) Now, adding an appropriate polymers to the ambient solution, the effective viscosity of the solution is doubled. What is the mean square angle fluctuation of the filament? You may assume that the polymers do not affect the ATPase itself.

4. Write down the magnetic susceptibility under constant volume of a magnet in terms of the magnetization fluctuation.

\(^{1}\)In the original version, this crucial word was missing.