Homework 3
due at 14:00 on September 22, 2009

1. We have two ideal gases with the same volume $V$, pressure $P$, and temperature $T$. These two gases consist of different chemical species. Assume the whole system is thermally isolated during the following processes.
(1) Two boxes containing the above gases are connected. That is, now the total volume of the mixture is $2V$. Find the entropy change due to this procedure of joining two boxes.
(2) Find the entropy change if two gases are mixed into a single volume of $V$.
(3) How can you actually measure the entropy change in (1) experimentally?

2. The internal motion of some ring puckering molecules (e.g., cyclobutanone) can be described by the following Hamiltonian:

$$H = \frac{p^2}{2m} + \frac{a}{4}x^4,$$

where $m$ is the effective mass of the oscillator and $a$ is a positive constant. Obtain the constant volume specific heat of this gas around the room temperature.

3. Let $\rho$ be the density operator of a single $1/2$ quantum spin system whose Hamiltonian is given by $H = -\gamma \sigma \cdot B$, where $\sigma$ is $(\sigma_x, \sigma_y, \sigma_z)$ in terms of the Pauli spin operators.
(1) Obtain the matrix representation of $\rho$ that diagonalizes $\sigma_z$.
(2) Find the average of $\sigma_y$.
(3) Obtain the matrix representation of $\rho$ that diagonalizes $\sigma_x$.

4. The potential energy of a permanent electric dipole has a potential energy $U = -p \cdot E$ in the electric field $E$. Obtain the electric susceptibility of the system.