

Last Name: \_\_\_\_\_ First Name \_\_\_\_\_ Network-ID \_\_\_\_\_  
Discussion Section: \_\_\_\_\_ Discussion TA Name: \_\_\_\_\_

*Instructions—*

**Turn off your cell phone and put it away.**

**This is a closed book exam. You have ninety (90) minutes to complete it.**

1. Use a #2 pencil; do **not** use a mechanical pencil or a pen. Fill in completely (until there is no white space visible) the circle for each intended input – both on the identification side of your answer sheet and on the side on which you mark your answers. If you decide to change an answer, erase vigorously; the scanner sometimes registers incompletely erased marks as intended answers; this can adversely affect your grade. Light marks or marks extending outside the circle may be read improperly by the scanner.
2. Print your last name in the **YOUR LAST NAME** boxes on your answer sheet and print the first letter of your first name in the **FIRST NAME INI** box. Mark (as described above) the corresponding circle below each of these letters.
3. Print your NetID in the **NETWORK ID** boxes, and then mark the corresponding circle below each of the letters or numerals. Note that there are different circles for the letter “I” and the numeral “1” and for the letter “O” and the numeral “0”. **Do not** mark the hyphen circle at the bottom of any of these columns.
4. **This Exam Booklet is Version A.** Mark the **A** circle in the **TEST FORM** box at the bottom of the front side of your answer sheet.
5. Stop **now** and double-check that you have bubbled-in all the information requested in 2 through 4 above and that your marks meet the criteria in 1 above. Check that you do not have more than one circle marked in any of the columns.
6. Do **not** write in or mark any of the circles in the STUDENT NUMBER or SECTION boxes.
7. On the **SECTION line**, print your **DISCUSSION SECTION**. (You need not fill in the COURSE or INSTRUCTOR lines.)
8. Sign (**DO NOT PRINT**) your name on the **STUDENT SIGNATURE line**.

*Before starting work, check to make sure that your test booklet is complete. You should have 11 **numbered pages** plus two Formula Sheets.*

*Academic Integrity—***Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including dismissal from the University.**

*Exam Grading Policy—*

The exam is worth a total of 114 points, and is composed of three types of questions:

**MC5:** *multiple-choice-five-answer questions, each worth 6 points.*

**Partial credit will be granted as follows.**

- (a) If you mark only one answer and it is the correct answer, you earn **6** points.
- (b) If you mark *two* answers, one of which is the correct answer, you earn **3** points.
- (c) If you mark *three* answers, one of which is the correct answer, you earn **2** points.
- (d) If you mark no answers, or more than *three*, you earn **0** points.

**MC3:** *multiple-choice-three-answer questions, each worth 3 points.*

**No partial credit.**

- (a) If you mark only one answer and it is the correct answer, you earn **3** points.
- (b) If you mark a wrong answer or no answers, you earn **0** points.

**TF:** *true-false questions, each worth 2 points.*

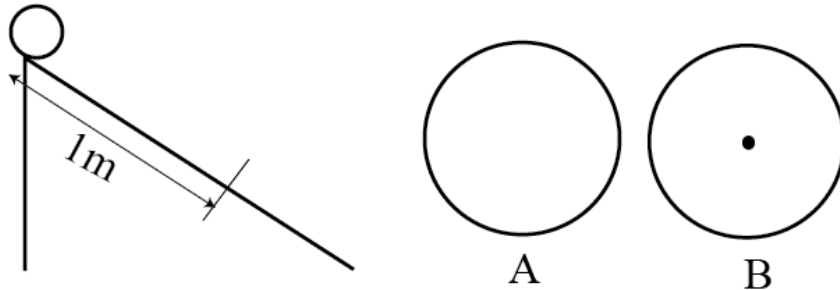
**No partial credit.**

- (a) If you mark only one answer and it is the correct answer, you earn **2** points.
- (b) If you mark the wrong answer or neither answer, you earn **0** points.

*Unless told otherwise, you should assume that the acceleration of gravity near the surface of the earth is  $9.8 \text{ m/s}^2$  downward and ignore any effects due to air resistance.*

*The following 3 questions concern related physical situations:*

In the figure below, A is a circular cylinder of radius  $R$  with moment of inertia  $I$  ( $> 0$ ). B is an identical cylinder except that a point mass of mass  $m_p$  ( $> 0$ ) is attached at the center.



We release the cylinders gently at the top of the slope in the figure without motion.

1. Suppose the slope is frictionless. After sliding down the slope for 1 m (see the figure), which has a faster translational motion?

- a. A
- b. B
- c. A and B have the same speed.

$I_A = I_B$ , but since there is no friction, rotational motion does not matter. It is just the sliding block problem, so masses do not matter.

mechanical energy conserved

2. Now, suppose the cylinders roll down the slope without slipping. After rolling down the slope for 1 m (see the figure), which has a faster translational motion?

Now,  $I_A = I_B$ ,  $M_A < M_B$ . Look at the following general relation:

- a. A
- b. B

$$(1/2)[I/R^2 + M]V^2 = Mgh$$

- c. A and B have the same speed.
- Thus, the effect of rotation is less for B; B is faster.

3. Suppose the cylinder in B has a total mass  $m = 2.3$  kg (= mass of the cylinder plus mass of the point-mass), a moment of inertia  $I = 0.1$  kgm<sup>2</sup>, and  $R = 0.2$  m. The slope is 30 degrees. Given the angular acceleration is 11.7 rad/s<sup>2</sup>, what is the magnitude of the static frictional force between the cylinder and the slope?

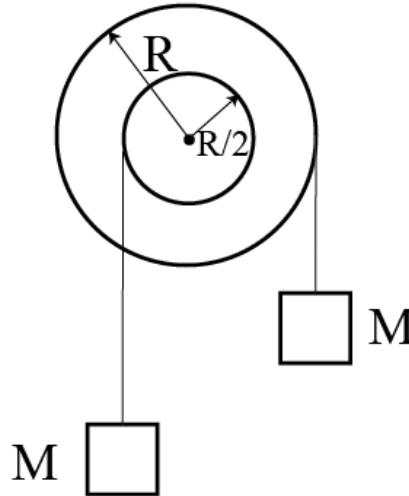
- a. 5.85 N
- b. 8.90 N
- c. 11.7 N
- d. 15.8 N
- e. 21.3 N

$$I\alpha = \tau = FR. \quad \text{Therefore,}$$

$$F = I\alpha/R = 11.7/2 = 5.85 \text{ N.}$$

*The following 2 questions concern related physical situations:*

Consider a pulley made of two connected cylinders of radius  $R$  and  $R/2$  as shown in the figure. The moment of inertia of the pulley is  $I$ . Two blocks with the same mass  $M$  are hung with weightless strings that do not slip against the pulley.



4. The masses start to move and the pulley starts to rotate from the initial stationary condition.  $M = 1.2$  kg and  $R = 1.5$  m. When the right mass goes down by 1m, what is the total kinetic energy of the system? Notice that the distance the left mass moves is just half of the right mass.

- a. 3.7 J
- b. 4.8 J
- c. 5.9 J
- d. 7.0 J
- e. 8.1 J

The left mass goes up by 0.5 m, so the total potential energy loss by the mass system is  $Mg - Mg/2 = Mg/2$ . This must have turned into the kinetic energy =  $0.6g = 5.9$  J

going up = gain

5. Suppose the speed of the right mass after descending by 1m in the preceding problem is  $v_0$  m/s. Now, let us double the masses  $M$  to  $2M$  and the moment of inertia  $I$  to  $2I$  while keeping  $R$  as before. Then, what is the speed of the right mass after descending by 1m as in the preceding problem in terms of  $v_0$ ?

- a.  $0.5 v_0$
- b.  $1.0 v_0$
- c.  $1.1 v_0$
- d.  $1.4 v_0$
- e. none of the above

The total kinetic energy is

$$(1/2)Mv_0^2 + (1/2)M(v_0/2)^2 + (1/2)I(v_0/R)^2 = Mgh/2$$

Therefore, even if  $M$  and  $I$  are doubled,  $v_0$  remains the same.

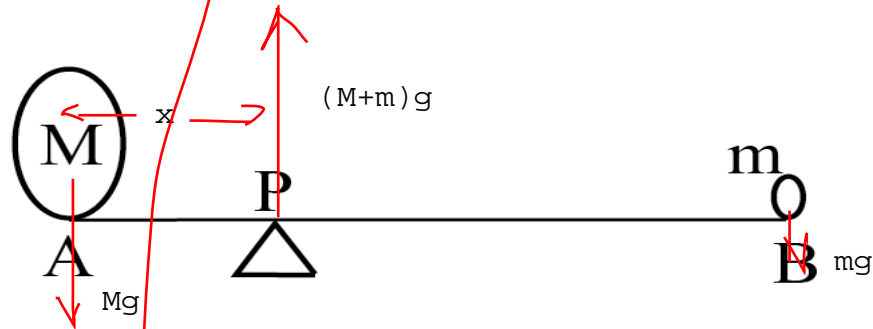
Right mass

Left mass

pulley

*The following 2 questions concern the same physical situation:*

There is a weightless bar of length 1m. At one end A is a mass  $M = 30$  kg and at the other end B is a mass  $m = 5$  kg. The bar is at rest on the pivot P. (Note: Assume + is for counterclockwise rotation.)



6. What is the torque around B due to mass M?

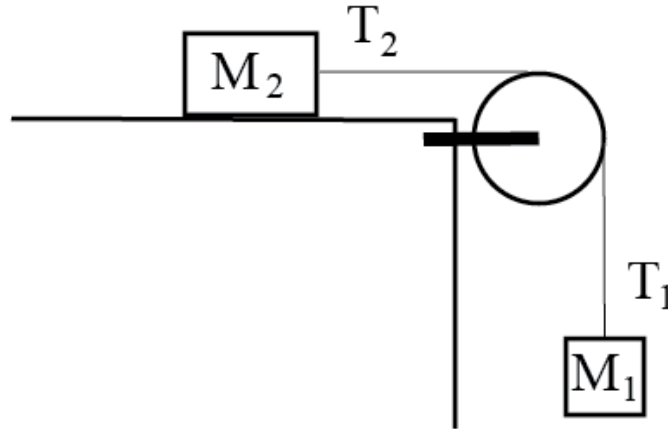
- a. +294 Nm       $\tau_B = Mg = 30g = 294 \text{ Nm}$   
 b. +187 Nm  
 c. 0 Nm  
 d. -187 Nm  
 e. -294 Nm

7. What is the torque around A due to the force exerted by the pivot?

- a. +98 Nm       $Mgx = mg(1-x)$  or  
 b. +49 Nm       $30x = 5(1-x)$  so  $x = 1/7$ .  
 c. 0 Nm  
 d. -49 Nm      Therefore,  $+(M+m)gx = 35g$  times  $1/7$   
 e. -98Nm       $= 5g = 49 \text{ Nm}$ .

*The following 2 questions concern the same physical situation:*

There is a table whose top surface is frictionless and horizontal. A massive pulley is attached to its edge and two masses  $M_1$  and  $M_2$  are connected with a weightless string passing through the pulley as illustrated below. The block  $M_1$  has mass 1.3 kg and  $M_2 = 5$  kg. The string does not slip against the pulley whose radius  $R$  is 30 cm and whose moment of inertia is  $I = 1.2 \text{ kgm}^2$ .



8. The tension  $T_1$  is 11.9 N. Find the angular acceleration of the pulley.

- a.  $1.1 \text{ rad/s}^2$
- b.  $2.2 \text{ rad/s}^2$
- c.  $3.3 \text{ rad/s}^2$
- d.  $4.4 \text{ rad/s}^2$
- e.  $5.5 \text{ rad/s}^2$

If we know the acceleration  $a$  of  $M_1$ , then  $a/R$  must be the angular acceleration of the disk.

The acceleration of  $M_1$  is

$$M_1 a = Mg - T_1$$

[The downward direction is assumed to be positive]

$$1.3 a = 1.3 g - 11.9. \text{ Therefore, } a = 0.646 \text{ m/s}^2.$$

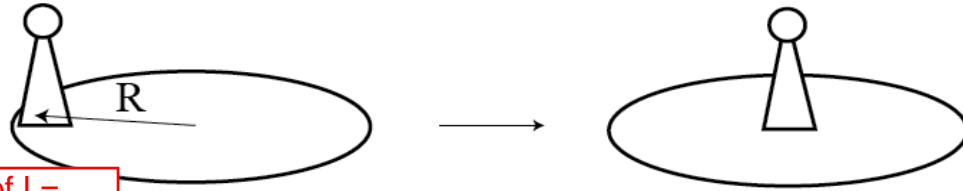
$$\text{Hence, } \alpha = a/R = 0.646 / 0.3 = 2.154 \text{ /s}^2.$$

9. Would the angular acceleration be larger, the same, or smaller, if the mass of the pulley were zero, but the radius was the same?

- a. Larger. This should be intuitively clear;
- b. Smaller. Smaller  $I$  means easier to move, so  $a$  must be larger.
- c. Not enough information. Consequently,  $\alpha$  must also be larger.

*The following 2 questions concern the same physical situation:*

A round horizontal table is turning freely. It has a moment of inertia  $I = 60 \text{ kgm}^2$  with a radius  $R = 2\text{m}$ . On the table stands a child of mass  $15 \text{ kg}$ . In the following you may ignore the size (cross section) of the child (i.e., you may regard her as a point mass).



Recall the definition of  $I = \sum mr^2$

10. From the initial position at the edge of the table the child moves to the center. What happens to the angular speed of the table?

- a. It speeds up.
- b. It slows down.
- c. It stays the same speed.

$L$  is conserved, and  $L = I\omega$   
 $I$  decreases, so  $\omega$  increases.

$$I_{\text{initial}} = 60 + 15(2)^2 = 120 \text{ kgm}^2$$

$$I_{\text{final}} = 60 \text{ kgm}^2$$

The contribution of the child.

11. To perform the motion in the preceding problem the child has to do  $W = +120 \text{ J}$  of work. Find the angular momentum of the system.

- a.  $170 \text{ kg m}^2/\text{s}$
- b.  $245 \text{ kg m}^2/\text{s}$
- c.  $420 \text{ kg m}^2/\text{s}$
- d.  $560 \text{ kg m}^2/\text{s}$
- e.  $720 \text{ kg m}^2/\text{s}$

The kinetic energy  $= L^2/2I$ . This implies  
 $\Delta E = (L^2/2)[1/I_{\text{final}} - 1/I_{\text{initial}}]$ .  
 Therefore,  
 $120 = (L^2/2)[1/60 - 1/120] = L^2/240$ .  
 That is,  $L = 169.7 \text{ kgm}^2/\text{s}$ .

**The following 4 questions concern the same physical situation:**

Two blocks moving in a straight line, along the x-axis, on a frictionless, horizontal surface collide in a *completely elastic* collision (no energy is lost in the collision).

Block 1 has  $m_1 = 2 \text{ kg}$ ,  $v_1 = 3.3 \text{ m/s}$ , Block 2 has  $m_2 = 0.1 \text{ kg}$ ,  $v_2 = -3.3 \text{ m/s}$ .

12. What is the magnitude of the total momentum of the blocks after the collision?

- a. Less than it was before the collision.
- b. **The same as it was before the collision.** The total momentum is conserved.
- c. More than it was before the collision.

13. The energy of block 2 will

- a. **Increase.** Suppose Block 2 has the same speed after the collision. To keep the total momentum Block 1 must keep moving in the same direction, but the speed must be smaller than the initial speed. This means the total energy is not conserved. Therefore, Block 2 must be faster after collision.
- b. Decrease.
- c. Stay the same.

14. Suppose that the blocks had a partially inelastic collision instead. Would it be possible for block 1 to have a speed of 4 m/s after the collision?

- a. Yes
  - b. **No**
  - c. It is impossible to tell from the information given.
- The initial total kinetic energy is about 11.43 J. If 4m/s were the speed of Block 1, then the total kinetic energy could not be less than 16 J. Impossible.

15. Block 2 then undergoes another collision so that its velocity is now  $v_x = -18 \text{ m/s}$ ,  $v_y = 0 \text{ m/s}$ . Then it explodes into two pieces 2a and 2b of mass  $m_{2a} = 0.03$  and  $m_{2b} = 0.07 \text{ kg}$ ; block  $m_{2a}$  moves with velocity  $v_x = -20 \text{ m/s}$ ,  $v_y = 3 \text{ m/s}$ . What is the velocity of block 2b?

- a.  $v_x = +16 \text{ m/s}$ ,  $v_y = -2 \text{ m/s}$
  - b.  $v_x = -17 \text{ m/s}$ ,  $v_y = +1.3 \text{ m/s}$
  - c.  $v_x = -24 \text{ m/s}$ ,  $v_y = -4 \text{ m/s}$
  - d.  **$v_x = -17 \text{ m/s}$ ,  $v_y = -1.3 \text{ m/s}$**
  - e.  $v_x = -25 \text{ m/s}$ ,  $v_y = +2 \text{ m/s}$
- The total momentum must be conserved:  
 $0.03(-20, 3) + 0.07(v_x, v_y) = 0.1(-18, 0)$ .  
 Therefore,  $-0.6 + 0.07v_x = -1.8$ , so  $v_x = -17.1 \text{ m/s}$ .  
 $0.09 + 0.07v_y = 0$ , so  $v_y = -1.29 \text{ m/s}$ .

*The following 3 questions concern the same physical situation:*

16. A car of mass  $M = 1000 \text{ kg}$  is moving in a straight line on level ground at  $70 \text{ m/s}$ . It applies its brakes and slides (skids) to a halt. Which of the following forces does nonzero work?

- a. Normal force
  - b. Gravitational force
  - c. Kinetic friction
  - d. Static friction
  - e. Centripetal force
- The force in the vertical direction cannot do any work: a, b are out. e is irrelevant. d is also not working.

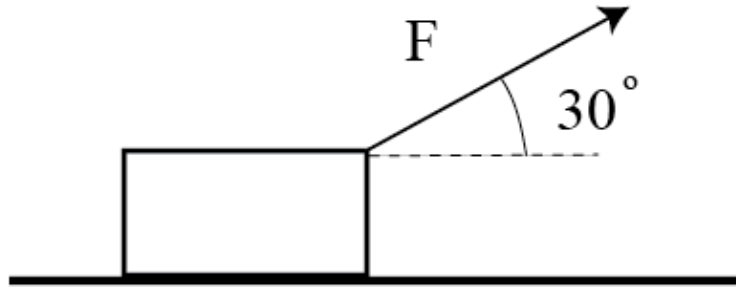
17. Suppose that the car starts up again and drives up a hill of height  $h = 300 \text{ m}$  (obviously this is not in Champaign-Urbana) at constant speed  $v = 30 \text{ m/s}$ , and that the wheels turn without slipping. What is the net work done (sum of the work done by all external forces), on the car, as it climbs the hill?

- a.  $Mgh$
  - b. 0
  - c.  $(1/2) M v^2$
- Since the speed is constant, the sum of the external forces must be zero. Therefore, the work done by the external forces must be zero.

18. At the top of the hill the car turns around, stops, and rolls freely, without air resistance or any other kind of friction, back to the bottom of the hill. What is its kinetic energy at the bottom of the hill?

- a. 300,000 J
  - b. 2,940,000 J
  - c. 4,400,000 J
  - d. 10,100,000 J
  - e. 39,000,000 J
- $Mgh = 1000 \times 9.8 \times 300 = 2940000 \text{ J}$

*The following 3 questions concern the same physical situation:*



19. A box sits on the floor. A force  $F = 200 \text{ N}$  is then applied to the right but slightly upward at an angle of  $30^\circ$  from the horizontal (see diagram) such that the box moves 5 meters horizontally to the right. How much work is done by the force?

- a. 50 J
  - b. 866 J
  - c. 1900 J
  - d. 2430 J
  - e. 3230 J
- $W = F D \cos(30^\circ) = 1000 \sqrt{3}/2 = 866.0 \text{ J}$

20. Suppose that the force (of magnitude 200 N) is applied horizontally instead. How much work is done by the force?

- a. Less than when the force was applied at an angle.
- b. The same as when the force was applied at an angle.
- c. More than when the force was applied at an angle.

21. Suppose that the force is applied horizontally still, and the box moves with a speed of  $0.2 \text{ m/s}$ . What power is supplied by the force?

- a. 10 W
  - b. 18 W
  - c. 30 W
  - d. 40 W
  - e. 44 W
- $\text{Power} = F \text{ times speed} = 200 \times 0.2 = 40 \text{ W.}$

This problem is out of the HE2 range this time.

22. A Hooke's law spring with spring constant  $2 \text{ kg/s}^2 (= 2 \text{ N/m})$  is initially at its resting position ( $x = 0$ ). A time-variable force  $F$  is applied so that the spring is compressed by  $0.1 \text{ m}$  ( $x = -0.1 \text{ m}$ ), then it expands past its resting position to  $0.1 \text{ m}$  ( $x = +0.1 \text{ m}$ ). How much work is done by the force  $F$ ?

- a. 0
- b. 0.01 J
- c. It is impossible to tell from the information given.

This is a work-energy theorem problem. The final total energy is  $(1/2)kx^2 = (1/2) 2 \times 0.1^2 = 0.01 \text{ J}$  more, which must have been done by the force.

23. A ball of mass  $0.1 \text{ kg}$  is falling straight down with speed  $10 \text{ m/s}$  just before it bounces off the floor. It then rises (straight up) to a height of  $4 \text{ m}$ . What is the impulse delivered to the ball by the floor?

- a.  $0.98 \text{ kg m/s}$
- b.  $1.2 \text{ kg m/s}$
- c.  $1.9 \text{ kg m/s}$
- d.  $2.0 \text{ kg m/s}$
- e.  $4.7 \text{ kg m/s}$

The momentum just before bouncing is  $-1 \text{ kg m/s}$

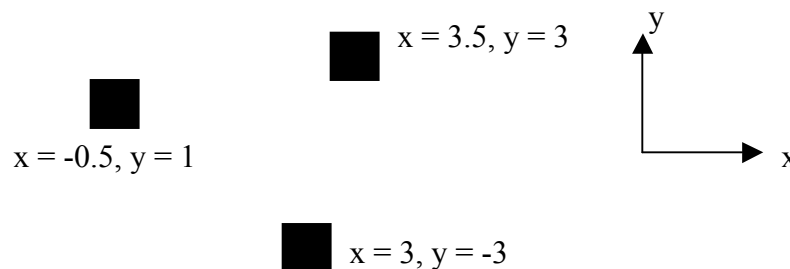
From the max height after bouncing,  $Mgh = p^2/2M$

tells us the momentum just after bouncing:

$$p = M\sqrt{2gh} = 0.1\sqrt{8 \times 9.8} = 0.8854 \text{ kg m/s}.$$

Therefore,  $\Delta p = 0.8854 - (-1) = 1.885 \text{ kg m/s}$ .

24. A system of 3 identical masses is arranged as shown in the figure. What is the x-coordinate of the center of mass?



- a. 0
- b. 2
- c. 3
- d. 3.5
- e. 4

We have only to compute the average of the x coordinates:  
 $(-0.5 + 3 + 3.5)/3 = 2.$

Check to make sure you bubbled in all your answers.  
 Did you bubble in your name, network-ID, and exam version?