HW 1 due at 11 am on Sept 10 (W), 2014.

If you submit your solutions written in (some version of) TeX\textsuperscript{1} by 21:00 on Sunday (before the deadline) (clearly declare it is a draft), you can get my response within a day. If you submit later, I will respond but cannot guarantee that I will reply quickly.

I believe if you take more than 15 minutes to answer any question, you’d better review the relevant portion of the lecture notes.

No solution without your justification will get any credit.

1. (1) Two fair coins are thrown but you cannot see them. You are told at least one coin exhibits a Head (H) and that if there is a coin exhibiting a Tail (T), you will be awarded $1,000. However, to participate in this game, you must pay a participation fee of $500. Will you still play the game, expecting some monetary gain?

(2) Now, it has turned out that the coins are not fair, but H is more likely with probability \( p \). What is the largest \( p \) that your expected monetary gain is not negative with the same reward and the participation fee?

2. There are 4 boxes and only one box contains a prize of $1,000. First, you are told that you can choose two boxes but you are not allowed to open them. Then, someone who knows where the prize is opens one of the two boxes you have not chosen and shows you that it is empty. Finally, you are asked to choose one box from the remaining still closed three boxes (that is, you can select one box anew). What is your expected monetary gain under your wisest choice?

3. (1) In an equilibrium\textsuperscript{2} mixture ideal gas maintained at temperature \( T \) are two molecules, 1 and 2, with mass \( m \) and \( M \), respectively. Suppose \( m/M = 0.31 \). What is the ratio between the mean square relative velocity of these two molecules and the mean square velocity of molecule 1?

(2) What is \( \langle (1/2)m(v_1 - v_2)^2 \rangle \) in the \( M/m \rightarrow \infty \) limit? The answer should be obvious, so state your answer first with your supporting argument and then confirm it, using the formulas (you should have used to answer (1)).

4. There is a gas of mass 15 g in a container of 18 liter. In equilibrium, its pressure is 1.0 atm. What is the root mean-square velocity of the molecules in the gas?

\textsuperscript{1}An example of HW1 (2012) explanation is posted.

\textsuperscript{2}We have not clearly defined what ‘equilibrium’ is, but here you may understand that the system is isolated and left alone for a sufficiently long time. The molecules move in a mutually unrelated manner, and, in particular, the equipartition of energy holds.