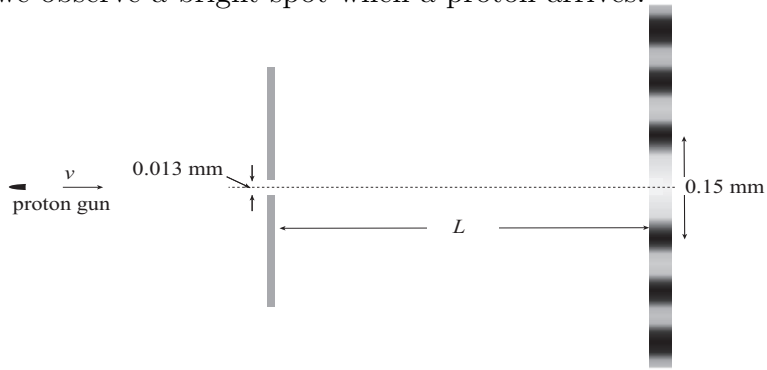


Name: \_\_\_\_\_ Section: \_\_\_\_\_ Score: \_\_\_\_\_/20

1. Protons are sent one by one with the same speed  $v = 2.5 \text{ km/s}$  from far behind a narrow slit of width  $13 \mu\text{m}$  (see the figure). Distance  $L$  away from the slit is a detecting screen on which we observe a bright spot when a proton arrives.

de Broglie wave  
 $\lambda = h/p$   
 $p = mv$



diffraction  
 dark fringe positions  
 $n \lambda = a \sin \theta$   
 small angle approximation  
 $n \lambda = a(d/L)$

(1) Collecting all the results of numerous protons, we can observe a diffraction pattern with the spacing between the two first dark fringes being  $0.15 \text{ mm}$ . What is the distance  $L$ ? [5]

$p = mv = 1.673 \times 10^{-27} \times 2.5 \times 10^3$ , so  
 de Broglie wavelength =  $6,626 \times 10^{-34} / 2.5 \times 1.673 \times 10^{-24} = 1.58 \times 10^{-10} \text{ m}$ .  
 We can determine  $L$  from  $\lambda = a(d/L)$  as  
 $L = a(d/\lambda)$   
 where  $a = 13 \times 10^{-6}$ ,  $d = (0.13/2) \times 10^{-3}$ ,  $n = 1$ , so  
 $L = 13 \times 10^{-6} \times 0.13 \times 10^{-3} / 2 \times 1.58 \times 10^{-10} = 6.17 \text{ m}$ .

(2) If the kinetic energy of the protons is halved, what is the spacing between the two first dark fringes? [5]

waves with smaller  $\lambda$  go more straight. See the figure on p 141.  
 Large  $p$  gives smaller  $\lambda$ ; more ballistic.

$K = p^2/2m$

$K \rightarrow K/2$  implies  $p \rightarrow p/\sqrt{2}$ , which implies  $\lambda \rightarrow \sqrt{2}\lambda$ , so the pattern expands:  $0.15 \times \sqrt{2} = 0.212 \text{ mm}$

2. When the surface of a metal is illuminated with photons of wavelength 486 nm, we find that the speed of the fastest photoelectron ejected from the surface has a speed of 337 km/s.

(1) What is the work function  $W$  in eV of the metal? [5]

$$1240/\lambda = \text{energy in eV}$$

$$\max K = hf - W$$

$$hf = 1240/486 = 2.55 \text{ eV (incident light)}$$

$$v = 337 \text{ km/s} \rightarrow K_{\max} = (1/2)mv^2 = (1/2) \times 9.11 \times 10^{-31} (337 \times 10^3)^2 \text{ (in J)}$$
$$= (9.11 \times 337^2 / 2 \times 1.6) \times 10^{-31+6+19} = 0.323 \text{ eV}$$

$$\text{Therefore, } W = 2.55 - 0.323 = 2.23 \text{ eV.}$$

(2) The photon is actually produced by a deexcitation of a hydrogen atom to the state with principal quantum number  $n = 2$ . What is the principal quantum number of the initial excited state? [5]

$$E_n = -13.6Z^2/n^2$$

$$\text{so } hf = 13.6Z^2(1/n_{\text{final}}^2 - 1/n_{\text{initial}}^2).$$

$$Z = 1 \text{ for our case.}$$

$$2.55 = 13.6 (1/4 - 1/x^2).$$

Therefore,

$$1/x^2 = 1/4 - 0.1875 = 1/16 \rightarrow n = 4 \text{ initial.}$$