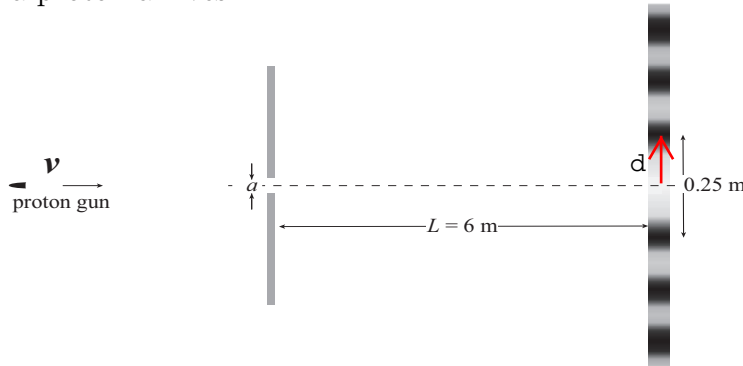


Name: \_\_\_\_\_ Section: \_\_\_\_\_ Score: \_\_\_\_\_/20

1. Protons are sent one by one with the same speed  $v$  through a narrow slit of width  $a = 9 \mu\text{m}$  ( $= 9 \times 10^{-6} \text{ m}$ ). 6 m away from the slit is a detector bank on which we observe a bright spot when a proton arrives.

de Broglie wave  
 $\lambda = h/p$   
 $p = mv$



diffraction  
 dark fringe positions  
 $n \lambda = a \sin \theta$   
 small angle approximation  
 $n \lambda = a(d/L)$

(1) Collecting all the results of numerous protons, we can observe a diffraction pattern whose central peak has width 0.25 mm. What is the speed  $v$  of the protons? [5]

We can determine the wavelength of the proton as  $\lambda = a(d/L)$ , where  $a = 9 \times 10^{-6}$ ,  $d = (0.25/2) \times 10^{-3}$ ,  $L = 6$ ;  $n = 1$ . Therefore,

$$\lambda = 9 \times 10^{-6} \times (0.25/12) \times 10^{-3} = 0.1875 \times 10^{-9} \text{ m}.$$

This means  $p = h/\lambda = 6,626 \times 10^{-34} / 0.1875 \times 10^{-9} = 35.2 \times 10^{-25}$ , so  $v = p/m = 35.2 \times 10^{-25} / 1.6 \times 10^{-27} = 21 \times 10^2 = 2.1 \text{ km/s}$

(2) If the kinetic energy of the protons is doubled, what is the width of the central diffraction peak now? [5]

waves with smaller  $\lambda$  go more straight. See the figure on p 141.  
 Large  $p$  gives smaller  $\lambda$ ; more ballistic.

$$K = p^2/2m$$

$K \rightarrow 2K$  implies  $p \rightarrow \sqrt{2}p$ , which implies  $\lambda \rightarrow \lambda/\sqrt{2}$ , so the pattern shrinks:  $0.25/\sqrt{2} = 0.178 \text{ mm}$

2. When the surface of a metal is illuminated with photons of wavelength 293 nm. The speed of the fastest photoelectron ejected from the surface has a speed of 391 km/s.

(1) What is the work function  $W$  in eV of the metal? [5]

$$1240/\lambda = \text{energy in eV}$$

$$\max K = hf - W$$

$$hf = 1240/293 = 4.23 \text{ eV (incident light)}$$

$$\begin{aligned} v = 391 \text{ km/s} \rightarrow K_{\max} &= (1/2)mv^2 = (1/2) \times 9.11 \times 10^{-31} \text{ kg} \times (391 \times 10^3 \text{ m/s})^2 \text{ (in J)} \\ &= (9.11 \times 391^2 / 2 \times 1.6) \times 10^{-31+6+19} = 0.453 \text{ eV} \end{aligned}$$

$$\text{Therefore, } W = 4.23 - 0.453 = 3.8 \text{ eV.}$$

(2) The photon is actually produced by a deexcitation of an excited  $\text{Li}^{2+}$  ion to the state with principal quantum number  $n = 4$ . What is the principal quantum number of the initial excited state of the ion? [5]

$$E_n = -13.6Z^2/n^2$$

$$\text{so } hf = 13.6Z^2(1/n_{\text{final}}^2 - 1/n_{\text{initial}}^2).$$

$$Z = 3 \text{ for our case.}$$

$$4.23 = 13.6 \times 9 (1/16 - 1/x^2).$$

Therefore,

$$1/x^2 = 1/16 - 0.0345 = 1/35.7 \rightarrow n = 6 \text{ initial.}$$