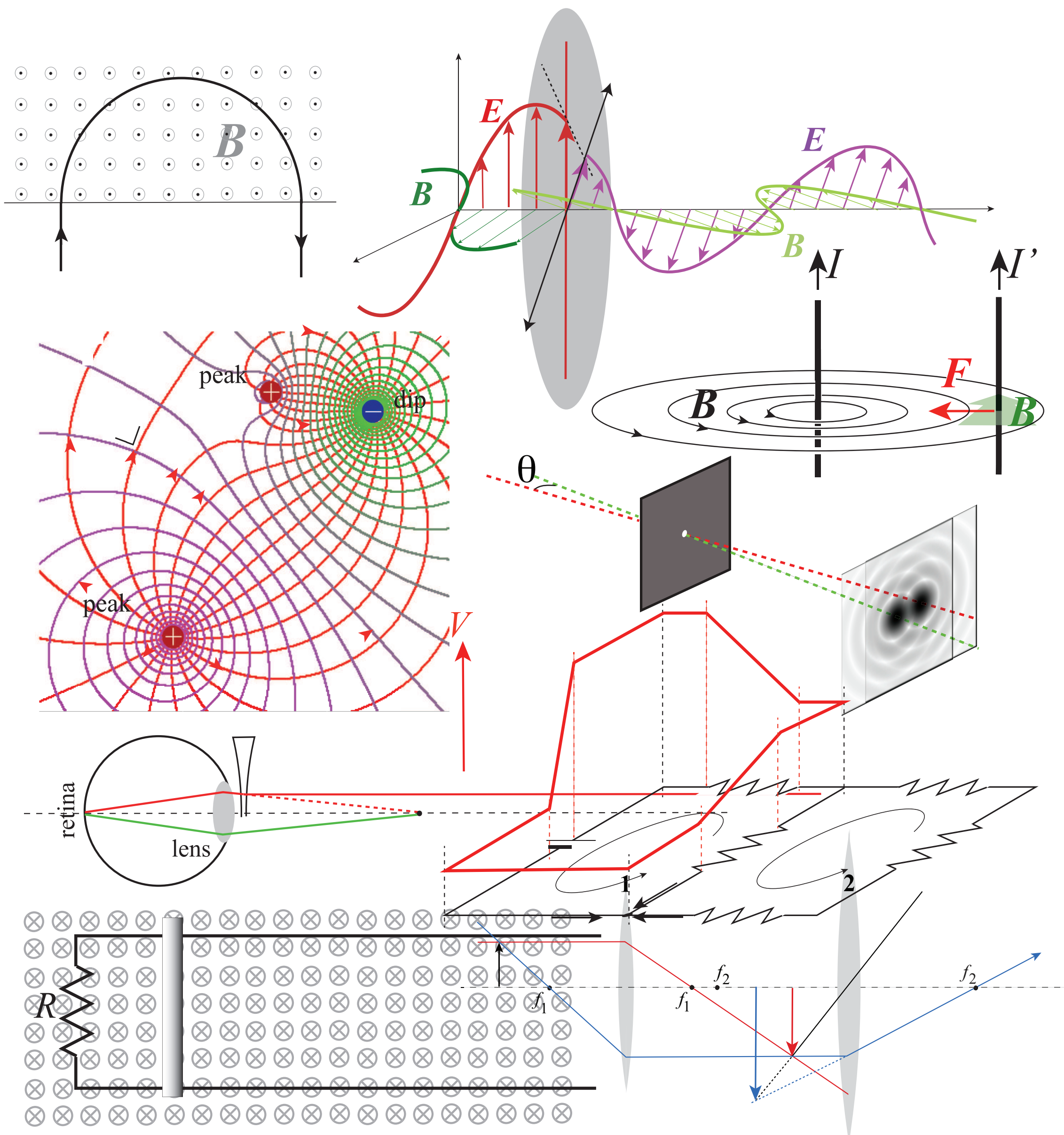
PHYSICS 102 Discussion Booklet—Fall 2016

with sample quizzes

University of Illinois at Urbana-Champaign



Everyone should be able to explain the figures above by December.

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The Solution/Explanation Manual for each weekly discussion problem set will be posted on the course homepage *before* the actual discussion meetings. **You must not abuse the Manual as is clearly explained at the beginning of the Discussion Booklet**.

Physics 102 Discussion
Introduction

Welcome to your Physics 102 Discussion

My Discussion Section is: ______________________________

My TA's name and e-address are: ______________________________

Possible Office Hours I can use are: ______________________________

Discussion is the opportunity to finalize your understanding of the materials covered in the preceding week.

The course is one of the courses you are supposed to improve your quantitative and logical skills.

* No quiz answer will be credited unless your justification is given.
* Symbolic calculations are strongly recommended before entering any numerical values.

Overall Structure of a Discussion Session:

In each Discussion Session, we start with a very brief summary of the preceding week. It is outlined in **Key Points**. Then, hopefully at least occasionally, we wish to have some interesting discussion about topics related to the weekly themes (e.g., why hawks can have sharp eyesight). We go on to solve **Discussion Problems** designed to illustrate key concepts and laws/rules. These problems are similar to the homework questions, but Discussion Session is really the last opportunity to make your understanding sure. The final 20-30 minutes will be devoted to **Quiz** that is directly related to Discussion Problems of the week to check your understanding of the Key Points. Sample quizzes will give you a preview.

Structure of the Discussion Booklet:

The Discussion Booklet is a collection of weekly Discussions.
Each weekly Discussion consists of Key Points, Discussion Problems and a Sample Quiz.

Any participant should read the Key Points before coming to the session. It is strongly recommended that you should read Key Points before doing Homeworks.
Key Points summarize the important concepts and relations (laws and rules). Important concepts are in bold and always with some defining statements. You might think physics is quite different from biology and chemistry and there is nothing to memorize, but it is strongly recommended that important concepts should be memorized to the level that you can explain them to your lay friends. Key formulas that appear in the formula sheet (attached at the end of this booklet) are highlighted with vermillion. You must understand the

contexts in which they show up and know how to use them.

Discussion Problems illustrate key concepts and formulas (especially those in the Formula Sheet).
(i) You must first clearly recognize what the problem asks.
(ii) Then, you must be able to identify (and explain at least to yourself) relevant concepts and laws.
(iii) Try to solve the questions as symbolically as possible. At least, do not jump to use the calculator.
(iv) After getting your answer check whether it is 'reasonable.'

Solution Manual: Unfortunately, we do not have enough time to solve problems together interactively in an orderly fashion as (i)-(iv) above step by step, discussing with all of you and answering all the questions you may have. Therefore, an extensive solution manual will be made available *before* each Discussion Session.

Most professors are against this practice, because they believe students always follow the easiest path and lose learning opportunities. This may be true, but it is also true that obtaining the correct answers in a timely fashion as reinforcing stimuli is always the most effective learning experience as every biologist knows. Therefore, the editor of this Discussion Booklet sticks to the practice that makes the solution manual available.

To use the solution manual properly **you must have some will power**: you must NOT read the solutions BEFORE attempting to solve the problems by yourself. Since you can acquire practically usable knowledge only through active learning process, you must not rely on this solution manual before you try your own ideas; you must use the manual sparingly. If you cannot tell how to proceed, read the first line which is often a basic question, hiding the rest. Its solution is in pale gray fine letters lest it jump into your eyes immediately.

If you think having the solution manual useful, do not abuse it so this practice will not be forbidden by the majority professors.

Quiz: As can be seen from the samples, the quiz problems are directly related to the Discussion Problems. It is a closed book quiz, but you can consult **Formula Sheet** attached at the end of this booklet. The same formula sheet will be used throughout the course; hour and final exams will also use the same formula sheet. You should better become able to explain all the formulas on the sheet by the final exam.

When graded, answers without justification will never be counted, even if the answers are (numerically) correct. However, if the grader can clearly recognize that you understand the key points of the problems, you could even get full credits without the numerical answers. Therefore, try to show your understanding when you compose your answers; it is a good practice to write your answer that you will be able to understand in ten years. Accordingly, careless errors may not be penalize unless the resultant answer is outrageous.

Physics 102 Discussion Week 1

Rudimentary Math Review – simple algebra and vectors

This week we will review **elementary algebra** and **vectors** we need throughout Physics 102.

Key Points:

Crux of algebraic approach:

In this course it is absolutely important to use symbols to solve problems.

∗ You must be able to describe problems in terms of symbolic formulas.

Pretend that you know everything you need.

If there are quantities you want but not given, call them x, etc., and write down the relations and formulas as if you know everything, including x, etc.

The following topics are very elementary. Use the corresponding Discussion Problems to check/consolidate your elementary math knowledge and skill.

Elementary algebra: See Discussions 1-1 and 1-2.

∗ You must be able to solve simultaneous linear equations for x and y:

$$\begin{aligned} Ax + By &= S, \\ Cx + Dy &= T. \end{aligned}$$

∗ You must know the graph of $ax + by = c$ (and its relation to the solutions for simultaneous equations).

∗ You must be able to solve quadratic equations: $ax^2 + bx + c = 0$.

Exponential functions and logarithms: See Discussion 1-3.

∗ $y = 10^x \iff x = \log_{10} y$.

∗ $y = e^x \iff x = \log y$ (natural log; which is occasionally written as $\ln y$).

(You should be able to sketch these functions.)

∗ You must know the following elementary relations:

$\log(xy) = \log x + \log y$,

$\log(1/x) = -\log x$, in particular, $\log 1 = 0$.

The volume/area formulas:

∗ The area of a disk of radius r: $A = \pi r^2$.

∗ The surface area of a sphere of radius r: $S = 4\pi r^2 = 4A$.

∗ The volume of a solid circular cylinder of radius r and height L: $V = \pi r^2 L$.

∗ The volume of a solid circular cone of radius r and height L: $V = \pi r^2 L/3$.

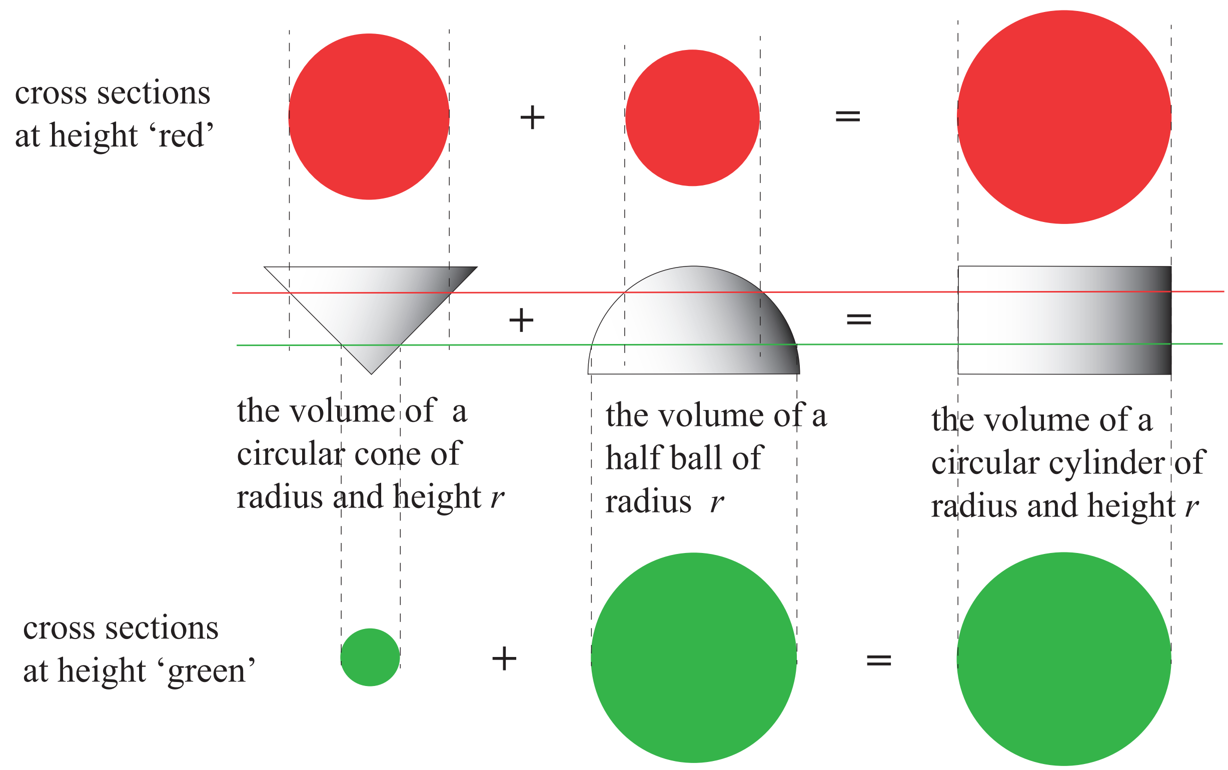
∗ The volume of a ball of radius r: $V = 4\pi r^3/3 = rS/3$.

The volume of a sphere was obtained by Archimedes:
∗ The area of a disk of radius r: $A = \pi r^2$. (What is π?)
∗ The volume of a solid circular cylinder of radius r and height r: $V_{\text{cy}} = \pi r^3$.
∗ The volume of a solid circular cone of radius r and height r: $V_{\text{co}} = \pi r^3/3$. (Can you explain the factor '1/3'?)
∗ The volume of a half ball of radius r: $V_{\text{hs}} = 2\pi r^3/3$.
Notice that $V_{\text{hs}} + V_{\text{co}} = V_{\text{cy}}$.

Archimedes (287 BCE - 212 BCE) was the first to demonstrate that the volume of a ball of radius r is given by $4\pi r^3/3$.

He proved the equality $V_{\text{hs}} + V_{\text{co}} = V_{\text{cy}}$ mentioned above. How did Archimedes do it?

Cicero (107 BCE - 44 BCE) found the headstone of Archimedes' tomb, on which was described the way Archimedes obtained the 'volume of the half ball of radius r' $= 2\pi r^3/3$. The following figure illustrates how he accomplished this.

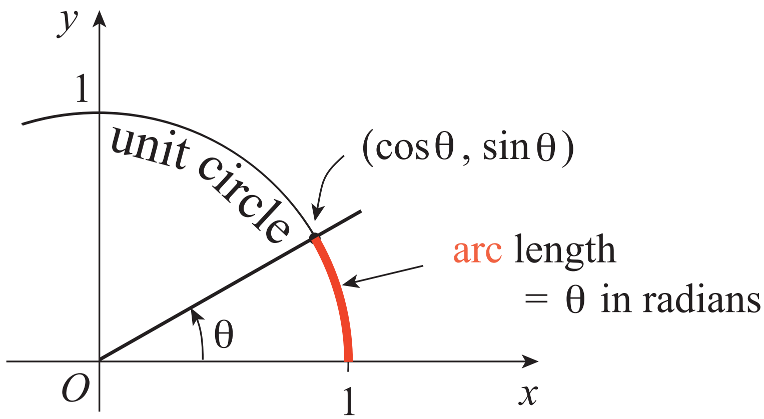


Archimedes realized that if he added the cross sectional areas of the (inverted) cone and of a half ball at the same height, then the sum was always πr^2 (it is a good exercise for you to show this). Thus, he sliced the volumes into parallel thin slices and computed the total volume by stacking these thin slices (basically he invented **integration**). Since the volumes of the cylinder and of the cone were known, he got what he wanted.

Trigonometric functions

∗ $\cos\theta$ and $\sin\theta$ are defined as the x and y coordinates, respectively, of a point on the unit circle as illustrated below. Needless to say,

$$\cos^2\theta + \sin^2\theta = 1.$$



∗ $\tan\theta = \sin\theta/\cos\theta$.

∗ You must be able to explain what **radian** means (see Figure above). What is 38° in radians?

You should remember what 30°, 45°, 60°, 90°, 120°, 180° and 360° are in radians.

Also, you should learn by heart the values of sin, cos and tan at these angles (e.g., $\cos(\pi/6) = \sqrt{3}/2$). Can you sketch the graphs of these functions?

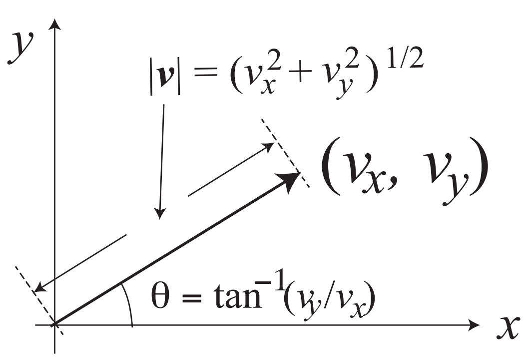
Vectors: See Discussion 1-4

∗ What is a vector? You must be able to give examples and to explain this concept to your friends.

∗ Clearly understand vector addition and subtraction and their graphic representations. If you are not very sure, do all the examples with solutions in Discussion 1-4.

∗ How to describe a vector in terms of components: $\boldsymbol{v} = (v_x, v_y)$.

Magnitude: $|\boldsymbol{v}| = \sqrt{v_x^2 + v_y^2}$.



Discussion 1-1 Simultaneous Equations

This review contains some explanations of simultaneous linear equations.
You must be able to solve simultaneous equations for x and y as unknown of the following form:[1]

$$\begin{aligned} Ax + By &= S, && (1) \\ Cx + Dy &= T. && (2) \end{aligned}$$

Instead of reviewing the general formula, let us review practical procedures through examples.

Example 1.

$$\begin{aligned} x + 2y &= 3, && (3) \\ x + 3y &= 4. && (4) \end{aligned}$$

Look at the structure of the equations BEFORE jumping into any actual calculation and choose the least laborious way. In this case, the easiest way is to subtract (3) from (4):

$$\begin{array}{rrcll} & x+3y & = & 4 & \cdots\ (4) \\ -) & x+2y & = & 3 & \cdots\ (3) \\ \hline & y & = & 1 & \end{array} \qquad (5)$$

That is, $y = 1$. Consequently, $x = 3 - 2y = 1$. After obtaining the solution, you should check it by substituting the obtained values of x and y into (one of) the original equations (not used in getting x): $x + 3y = 1 + 3 = 4$. Correct.

Notice that **writing equations neatly and aligning them properly almost solve the problem**.

Alternatively, you could solve x from (3) as $x = 3 - 2y$ and substitute this into (4) to obtain $(3 - 2y) + 3y = 4$, that is, $y = 1$. The rest is the same as before.

Example 2.

$$\begin{aligned} 2x + 3y &= 4, && (6) \\ x - y &= 1. && (7) \end{aligned}$$

A wise way may be to subtract $2\times(7)$ from (6):

$$\begin{array}{rrlrrclrrcl} & (6), & \text{that is,} & & 2x+3y & = & 4 & \Rightarrow & & 2x+3y & = & 4 \\ -) & 2\times(7), & \text{that is,} & 2\times & x-y & = & 1 & \Rightarrow & -) & 2x-2y & = & 2 \\ \hline & & & & & & & & & 5y & = & 2 \end{array} \qquad (8)$$

That is, $y = 2/5$. Consequently, $x = 1 + y = 7/5$. Check: $2x + 3y = 14/5 + 6/5 = 4$, correct.

Alternatively, you could get x from (7) as $x = 1 + y$ and substitute this into (6) to obtain $2(1 + y) + 3y = 4$ or $5y = 2$, that is, $y = 2/5$. The rest is the same as before.

[1] $AD \neq BC$ is assumed.

Linear Algebra is the math branch studying such equations systematically. If you have not learned Linear Algebra (matrices and determinants, vector spaces, etc) yet, you should learn it as soon as possible. It is much more important than elementary physics.

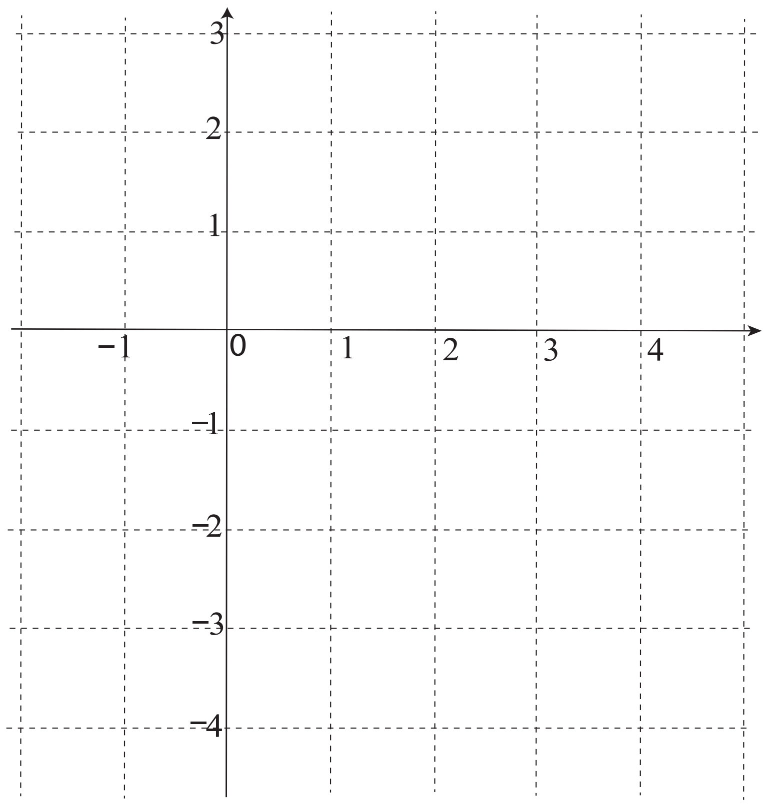
Let us solve some practice examples. Answers are in [ ].

1)

$$2x - y = 4, \tag{9}$$
$$3x + 2y = 2. \tag{10}$$

[$x = 10/7$, $y = -8/7$]

Draw the graphs of the above two linear equations below, and graphically locate the solution.



2)

$$3x - 4y = 5, \tag{11}$$
$$x + 4y = 3. \tag{12}$$

[$x = 2$, $y = 1/4$]

3)

$$2x - 2y = 5, \tag{13}$$
$$x + 3y = 3. \tag{14}$$

[$x = 21/8$, $y = 1/8$]

4)

$$
\begin{aligned}
3x - 2y &= 5, &(15)\\
-7x + y &= 8. &(16)
\end{aligned}
$$

$[x = -21/11, y = -59/11]$

5)

$$
\begin{aligned}
3x - 4y &= 5, &(17)\\
2x + 3y &= 7. &(18)
\end{aligned}
$$

$[x = 43/17, y = 11/17]$

In the following, set up the equations for unknown quantities and solve them.

6) There are some cranes (two-legged) and porpoises (without legs). The total number of legs is 100, and the total number of organisms is 60. How many porpoises are there?

7) An apple is 70 cents and a banana is 30 cents. You bought 30 fruits and paid 17 dollars. How many apples did you buy?

6) 50 cranes and 10 porpoises; $2x + 0 \times y = 100$, $x + y = 60$.
7) 20 apples; $70x + 30y = 1700$, $x + y = 30$.

Discussion 1-2 Quadratic Equations

In Physics 102 you may occasionally need the formula for the roots of a quadratic equation ($a \neq 0$):

$$ax^2 + bx + c = 0. \tag{1}$$

The formula for the roots of (1) is given by

$$x_\pm = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{2}$$

You should learn this by heart (you should be able to derive this as well; at least you should be able to demonstrate that $x_\pm$ indeed satisfies (1)).

Let us do some practice examples (the solutions are given below the problems). Simpler examples should be solved directly without the use of formula (2), i.e., with the use of factorization:

$$(x + \alpha)(x + \beta) = x^2 + (\alpha + \beta)x + \alpha\beta. \tag{3}$$

Solve the following examples:
1) $x^2 - 3 = 0$.

2) $x^2 - 3x + 2 = 0$.

3) $x^2 + x - 6 = 0$.

4) $2x^2 + 5x - 3 = 0$.

5) $6x^2 - x - 1 = 0$.

6) $x^2 + 2x - 3 = 0$.

7) $2x^2 + 3x - 5 = 0$.

8) $-3x^2 - 2x + 5 = 0$.

9) $4.9x^2 - 3.2x - 11 = 0$.

10) $-1.3x^2 + 5.1x - 2.2 = 0$.

1) $x = \pm\sqrt{3} = \pm 1.732\cdots$.
2) $= (x-1)(x-2)$, so $x = 1$ or 2.
3) $= (x-2)(x+3)$, so $x = 2$ or -3.
4) $= (2x-1)(x+3)$, so $x = 1/2$ or -3.
5) $= (2x-1)(3x+1)$, so $x = 1/2$ or $-1/3$.
6) $= (x-1)(x+3)$, so $x = 1$ or -3.
7) $= (2x+5)(x-1)$, so $x = -2.5$ or 1. In this case, using the formula for the root may be easier (but do not use the calculator).
8) $= -(x-1)(3x+5)$, so $x = 1$ or $-5/3$, In this case, using the formula for the root may be easier (but do not use the calculator).
9) $x = (3.2 \pm \sqrt{3.22 + 4 \times 11 \times 4.9})/9.8 = 1.857\cdots$ or $-1.20\cdots$.
10) $x = (-5.1 \pm \sqrt{5.12 - 4 \times 2.2 \times 1.3})/(-2.6) = 0.5\cdots$ or $-3.42\cdots$.

At this elementary level you must not use the formulas you cannot understand. Let us demonstrate the formula (2); this you must be able to do by yourself. Those who are not sure of their algebraic muscle, follow the calculation with a pencil and paper.

$$0 = ax^2 + bx + c = a\left[x^2 + \frac{b}{a}x\right] + c. \tag{4}$$

Now, we need a small trick called the *completion of square.* Note that (since $(x+B)^2 = x^2 + 2Bx + B^2$)

$$x^2 + Ax = x^2 + Ax + \frac{A^2}{4} - \frac{A^2}{4} = \left(x + \frac{A}{2}\right)^2 - \frac{A^2}{4}. \tag{5}$$

We utilize this relation:

$$x^2 + \frac{b}{a}x = \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}. \tag{6}$$

Therefore,

$$\begin{aligned} 0 &= a\left[x^2 + \frac{b}{a}x\right] + c = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] + c, & (7)\\ &= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}. & (8)\end{aligned}$$

This implies that

$$a\left[\left(x + \frac{b}{2a}\right)^2\right] - \frac{b^2 - 4ac}{4a} = 0 \tag{9}$$

or

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2} = 0. \tag{10}$$

Therefore,

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}. \tag{11}$$

Thus, we finally arrived at the desired formula:

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{12}$$

Discussion 1-3 Logarithms and Exponents

Logarithm of x in base B (> 0) is written as $\log_B x$. Notice that

$$B^{\log_B x} = x$$

or

$$\log_B B^x = x.$$

1) Assume $\log_{10} 2.1 = 0.3222$ is given. What is $\log_{10} 2100$? Answer *without* using a calculator.

2) We can calculate $1/2.1 = 0.7762$. What is $\log_{10} 0.7762$? Answer *without* using a calculator.

3) Define α and β as $\log_{10} 2 = \alpha$ and $\log_{10} 3 = \beta$. Express the following quantities in terms of α and β.

(i) $\log_{10} 96 =$

(ii) $\log_{10} 50 =$

(iii) $\log_{10} 120 =$

4) We will learn that the 'electric charge' Q stored in a 'capacitor' behaves (decays) as

$$Q(t) = Q_0 e^{-t/\tau},$$

where Q_0 is the charge at time $t = 0$, and τ (> 0) is called the 'time constant'.[2]
(i) Find t when $Q(t)/Q_0 = 1/2$ in terms of τ.

(ii) $Q(3)/Q_0 = 0.2$. Find τ.

[2]The terms with ' ' will be explained later in this course (Week 6).

Discussion 1-4 Vectors

1) **Scalar multiplication and addition**

∗ Draw $2.3\boldsymbol{v}$ and $-0.5\boldsymbol{v}$ in Fig. 1. Also draw $\boldsymbol{c}+\boldsymbol{d}$.

∗ Draw two arrows as you like them and construct their sum.

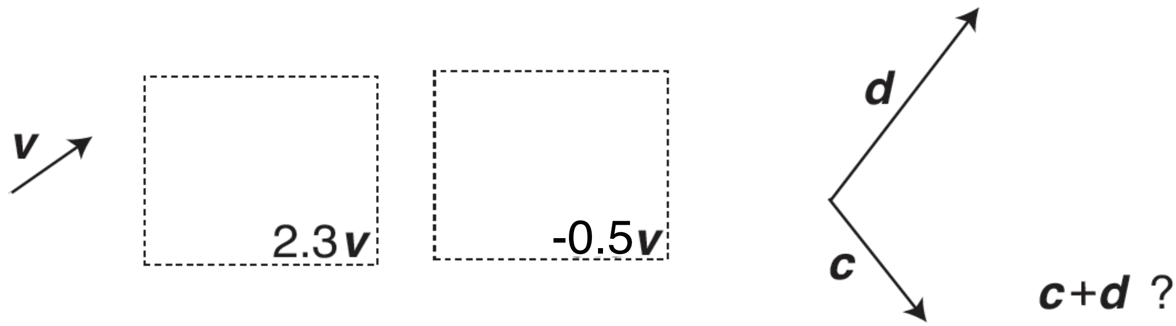


Figure 1: Scalar multiplication and addition

2) **Addition** does not care about the order: $\boldsymbol{a}+\boldsymbol{b}=\boldsymbol{b}+\boldsymbol{a}$ (commutative). You can add several vectors in any order. Construct the vectors asked in Fig. 2.

Figure 2: Construct the vectors asked in the figure.

3) Illustrate **subtractions** (Fig. 3):

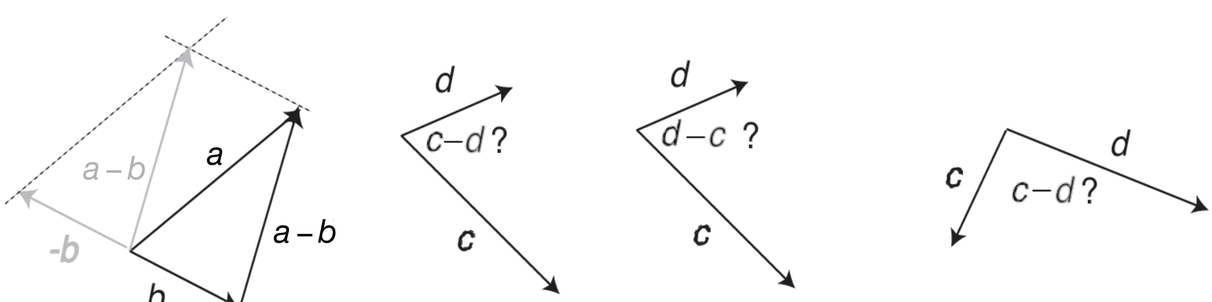


Figure 3: Draw vectors wanted in the figure.

4) If two distinct directions are given, we can **decompose** a given vector $\boldsymbol{v}$ as a sum of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ that are respectively pointing the given distinct directions (Fig. 4).

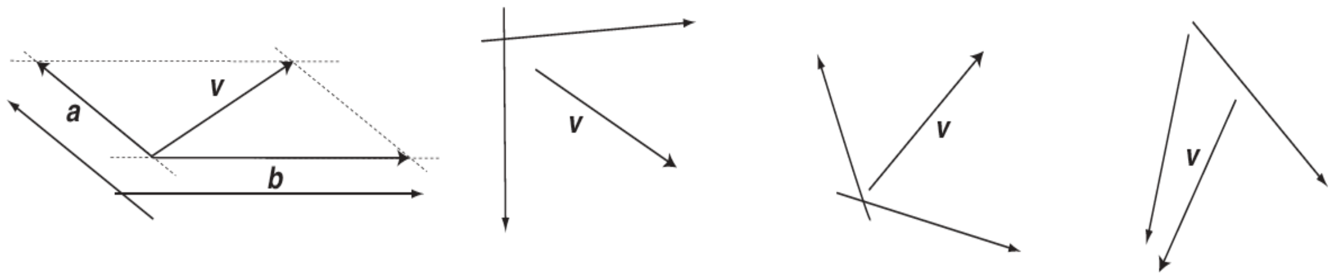


Figure 4: **Decomposition of a vector $\boldsymbol{v}$** into two prescribed directions as $\boldsymbol{v} = \boldsymbol{a} + \boldsymbol{b}$. Complete the right three figures, constructing '$\boldsymbol{a}$' and '$\boldsymbol{b}$'.

5) **Component-wise expression** of vectors (see Fig. 5).
(i) Find the x and y components in the figure.
(ii) For $\boldsymbol{a}$ and $\boldsymbol{b}$, find components. Draw a vector $\boldsymbol{c}$ given by $(-3, 1)$:

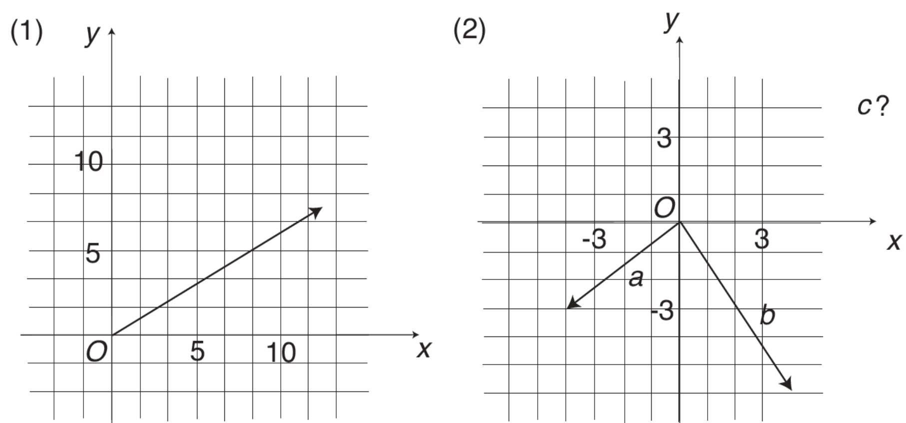


Figure 5: Coordinate expression of vectors

(iii) Visualize addition and subtraction of vectors.

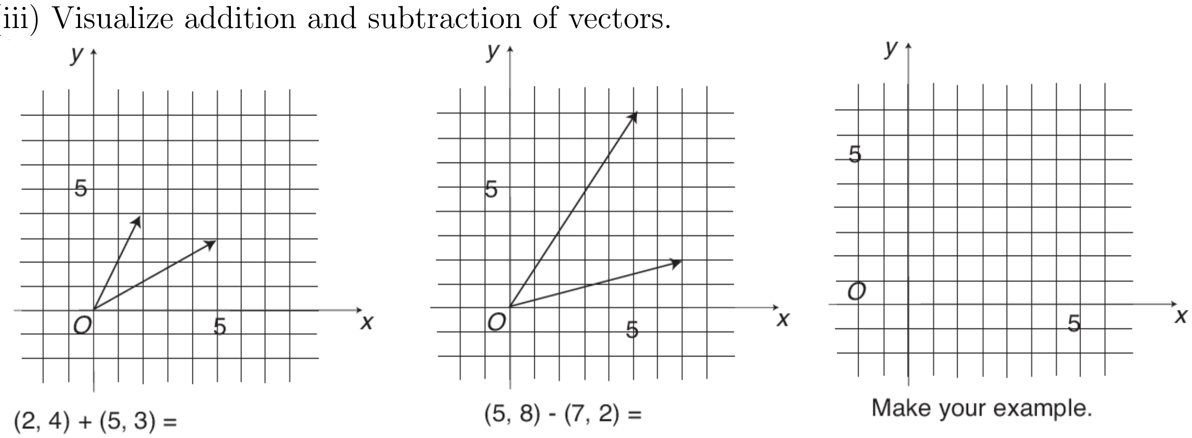


Figure 6: Draw the results of addition.

6) Find the magnitudes of the following vectors: $(2, 3)$, $(-1, 5)$, $(3, -4)$.

D1-4 Solutions

1)

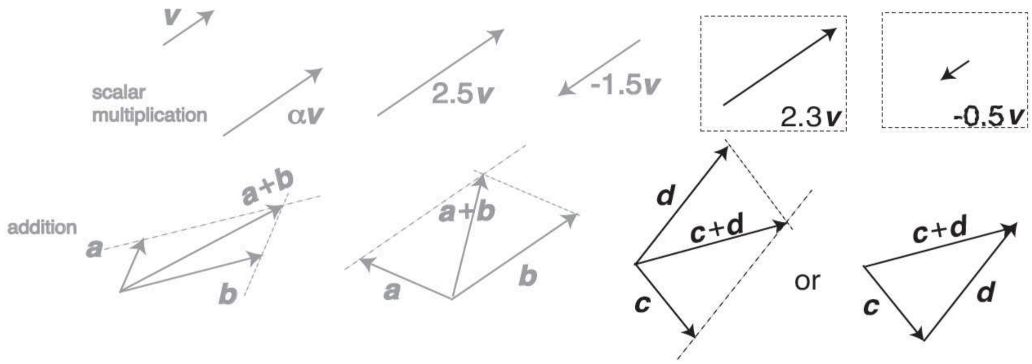


Fig. 1 Vector: scalar multiplication and addition. Here $\alpha > 0$ is assumed.

2)

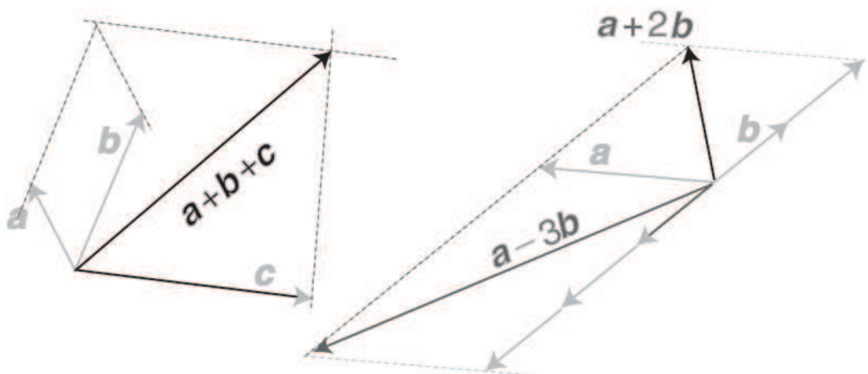


Fig. 2 Exercise. Construct the vectors asked in the figure.

3)

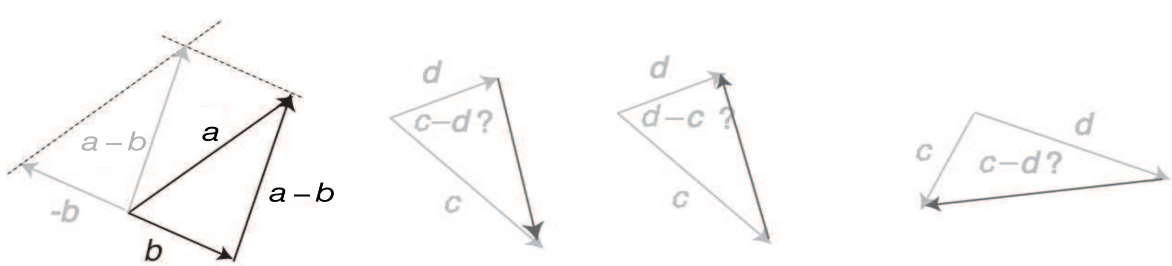


Fig. 3 Subtraction of vectors. Complete the figures.

$*$ $\boldsymbol{a}+\boldsymbol{b} = (1,9)$, $\boldsymbol{a}-\boldsymbol{b} = (3,-1)$, $2\boldsymbol{a}-\boldsymbol{b} = (5,3)$, $-2\boldsymbol{a}+5\boldsymbol{b} = (-9,17)$.

$*$ $|(2,5)| = 5.4 = |(-2,5)|$, $|(3,4)| = 5$.

4)

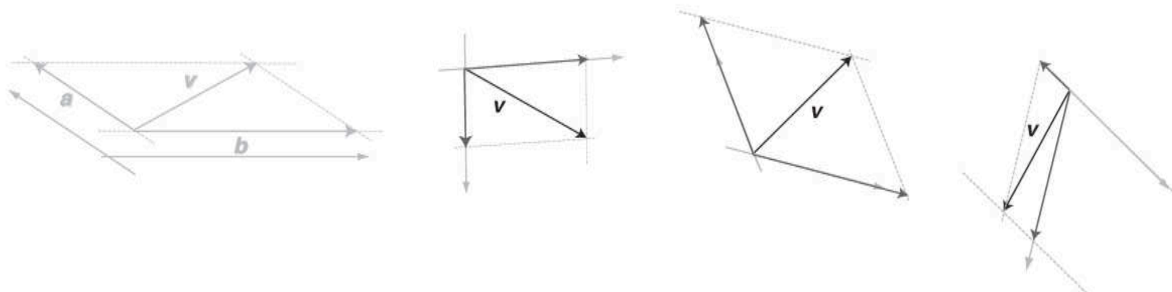


Fig. 4 Decomposition of a vector $\boldsymbol{v}$: Complete the right three figures, constructing '$\boldsymbol{a}$' and '$\boldsymbol{b}$'.

5) (i) (ii) omitted.

5) (iii)

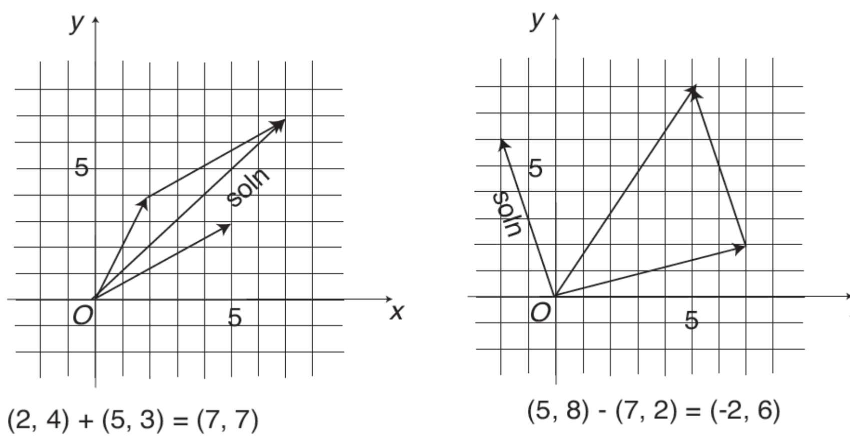


Fig. 5 Draw the results of addition examples.

6) $\sqrt{13}$, $\sqrt{26}$, 5.

Physics 101 Discussion Week 2 (Lectures 1 and 2)

Electric Charge, Coulomb's law, and Electric Dipole

Key Points

Electrostatic interactions among charges (Coulomb's law)

Empirically, we know that the following two basic rules **1** and **2** are enough to describe the electrostatic interactions among relatively stationary charges.

1. Coulomb force between two charges

Charge 1 is q_1 and charge 2 is q_2 (in **C** = **coulombs**, the unit of charge).[3] Let the distance between these two charges be r. Then, due to the **electrostatic interaction** between the charges (see Figure below)

Coulomb force $\boldsymbol{F}_{12}$ acts on charge 1 due to charge 2 (cf. Figure below):

its magnitude is give by

$$F_{12} = k\frac{|q_1||q_2|}{r^2} \quad \text{with } k = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 = \frac{1}{4\pi\epsilon_0},$$

if the force is measured in N (newtons), r in m (meters) and charges in C.

its direction is along the line connecting the two charges, and

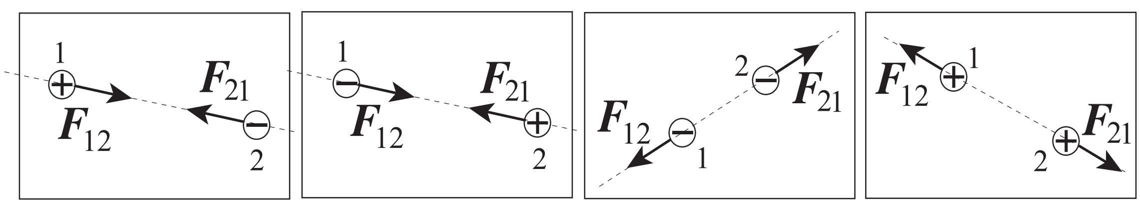
attractive, if the two charges have opposite signs (i.e., $q_1q_2 < 0$)

repulsive, if the two charges have identical signs (i.e., $q_1q_2 > 0$)

Coulomb force $\boldsymbol{F}_{21}$ acting on charge 2 due to charge 1 is $-\boldsymbol{F}_{12}$.

cf. the action-reaction principle: $\boldsymbol{F}_{12} + \boldsymbol{F}_{21} = 0$.

If not, what do you imagine to happen?



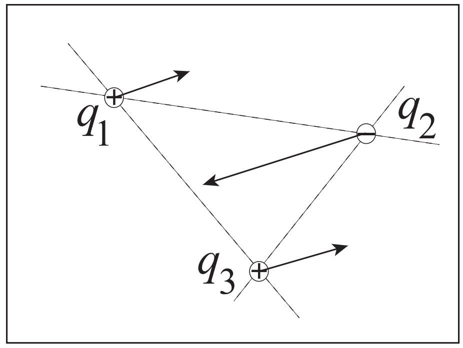
2. Superposition principle

If there are more than two charges,

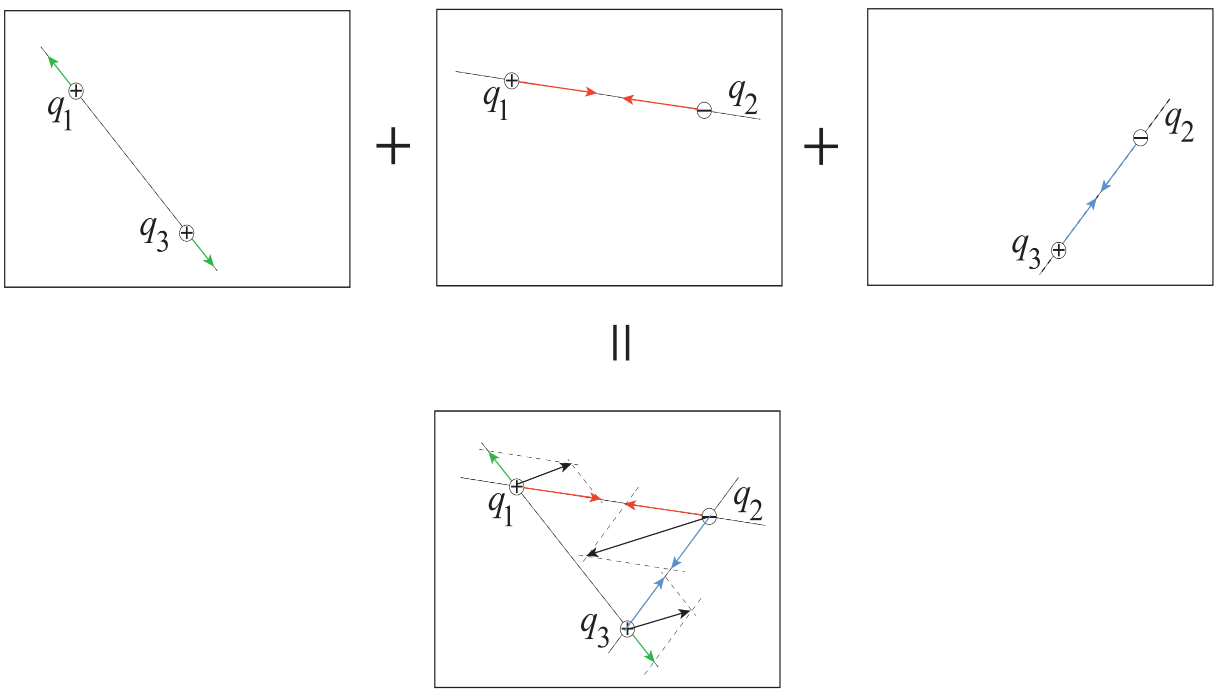
(i) decompose the whole system into pair Coulomb interactions,
(ii) compute the pairwise forces separately following **1** above, and then
(iii) sum all of them (as vectors) to get the total force on each charge.

The following example illustrates the principle:

[3] The units are always written in upright letters; the unit 'coulomb' commemorates Coulomb, and always starts with a lower case letter.



To study the system consisting of three charges in the above figure, decompose the system into pairs of charges as the top row of the following figure, and then add the resultant forces for individual pairs (green, red and blue) as vectors as illustrated in the bottom row:



Notice that the Coulomb forces do not change, even if all the charge signs are flipped simultaneously (charge conjugation symmetry).

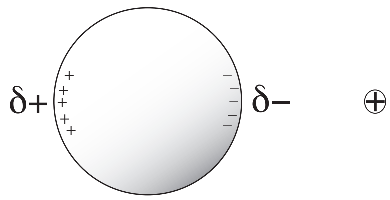
Conductor/Insulator

Conductor: a material in which charges can move freely.

Insulator: a material in which charges cannot move freely (however, 'polarization' is still possible due to small localized displacements of charges).

Induced charge and conservation of charge

If a charge (+ in the following figure) comes close to a conducting ball, since charges can move freely in the conducting ball, plus charges 'wish to go away from the plus charge,' and negative charges 'wish to come near the plus charge.' Thus, the **induced plus charge** $\delta+$ appears away from the external plus charge, and the **induced negative charge** $\delta-$ appears near it. Thus, an induced electric dipole (see below) is formed. (Even if the ball is an insulator, still charges can move slightly in it, and induced charges with the same signs as illustrated appear.)



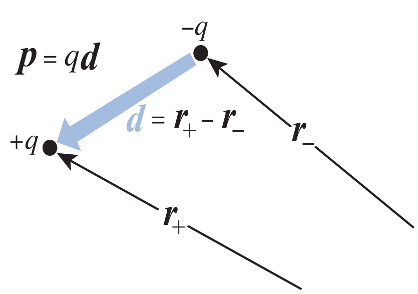
Initially, there is no net charge on the conducting sphere, so $(\delta+) + (\delta-) = 0$. This is one of the most fundamental laws of physics, the **conservation of electric charges**: you cannot create or annihilate electric charges.

Electric dipole

If charge $+q$ ($q > 0$) is at $\boldsymbol{r}_+$ and charge $-q$ is at $\boldsymbol{r}_-$, the system is called an **electric dipole**, and

$$\boldsymbol{p} = (+q)\boldsymbol{r}_+ + (-q)\boldsymbol{r}_- = q(\boldsymbol{r}_+ - \boldsymbol{r}_-) = q\boldsymbol{d}$$

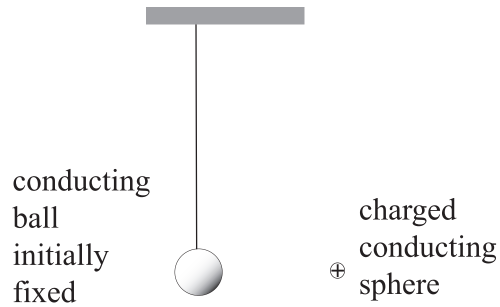
is called the **electric dipole moment**, where $\boldsymbol{d} = \boldsymbol{r}_+ - \boldsymbol{r}_-$. Notice that the vector $\boldsymbol{d}$ is the vector connecting the positive and the negative charges (from the negative to the positive charge).



If an electric dipole $\boldsymbol{p}$ is placed in an electric field $\boldsymbol{E}$, there is a torque $\boldsymbol{\tau}_{\rm dip} = \boldsymbol{p} \times \boldsymbol{E}$, whose magnitude is $\tau_{\rm dip} = pE\sin\theta$, where θ is the angle between $\boldsymbol{p}$ and $\boldsymbol{E}$.

Discussion 2-1 Induced charges

A conducting ball hangs from the ceiling with a flexible insulating string. Initially, the ball is fixed, and a positively charged small conducting sphere is placed near it as shown in the figure.



1) What happens if the ball is released (allowed to move) while still hanging from the ceiling?

2) What happens if the ball is released (allowed to move) while still hanging from the ceiling, but only after the positively charged small conducting sphere is allowed to make contact with the ball?

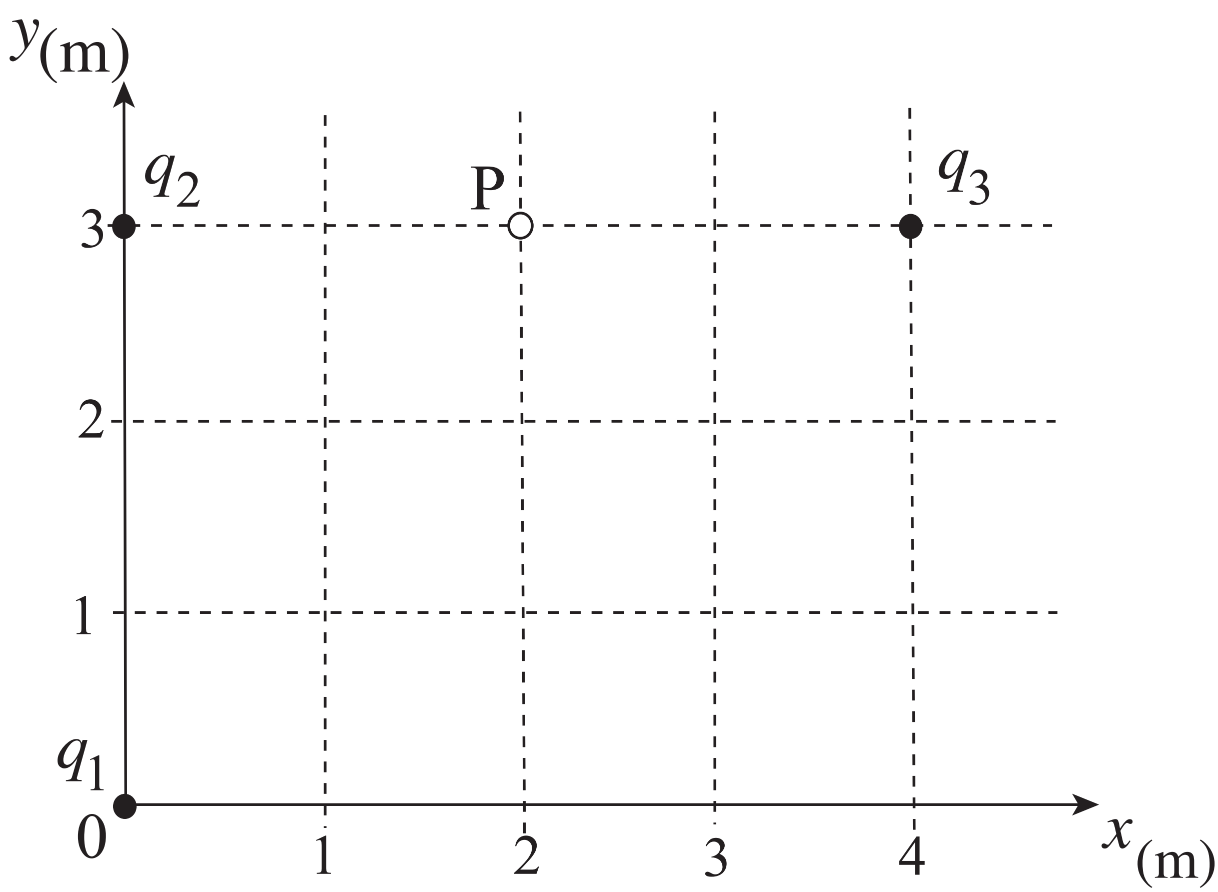
3) If the ball were made of an insulating material (say, a plastic ball), what would you expect to happen in the similar situation as 1)?

4) What do you expect to happen in 1)–3) above, if the positively charged small conducting sphere is replaced with a negatively charged small conducting sphere? Assume that the absolute values of the charge on the sphere do not change.

5) If the string hanging the ball is conducting and is (electrically) connected to the earth, then what difference do you expect from the situation 1) and 2)?

Discussion 2-2 Coulomb force

There are three charges q_1, q_2 and q_3 as shown in the figure. $q_1 = -0.1$ mC and $q_2 = q_3 = 0.3$ mC.



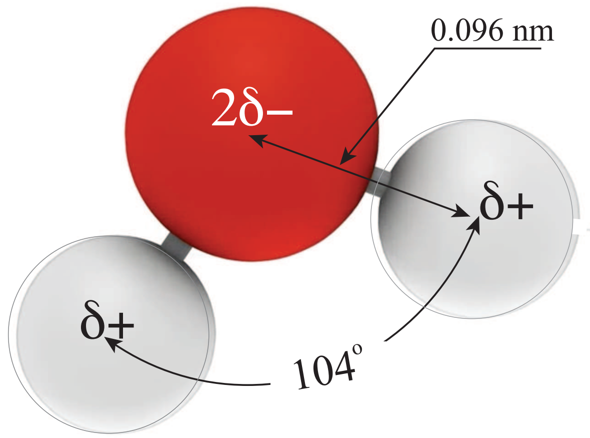
1) Find the force acting on charge q_2 (find its x and y components).

2) Find the force acting on charge q_1 (find its x and y components). Also determine its magnitude.

3) We place a new charge of -0.1 mC at point P. What is the magnitude of the total force acting on this new charge?

Discussion 2-3 Dipole moment of H_2O

The water molecule has the structure as illustrated below. The dipole charge is with $\delta+ = 0.35e$, where $e = 1.6 \times 10^{-19}$ C.



Obtain the magnitude and the direction (indicate it in the figure above) of the dipole moment of the water molecule.

Physics 102 (Quiz Sample) **Q1s**

Name: ____________ **Section:** ________ **Score:** ________/20

1. Very small and identical metal spheres A and B are with glass handles as in Fig. 1. Initially, A has no net charge and B has a net charge Q. After the metal spheres are connected, they are separated and placed as in the lower-right figure.

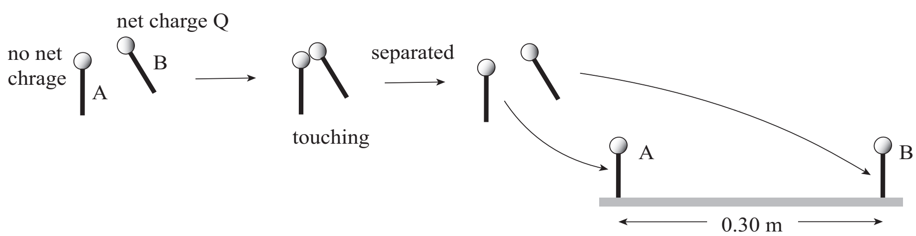


Figure 1:

(a) The magnitude of the force acting on charge at A is 10 N. What is the original net charge Q on B before touching with A? [5]

Notice that charge is conserved. After touching A and B must have equal charges due to symmetry. Both must have $Q/2$ charges due to conservation of charges.

In terms of Q, the force between A and B satisfies ($k = 9 \times 10^9$)

$$F = |\boldsymbol{F}| = k\frac{(Q/2)^2}{0.3^2} = \frac{Q^2}{4} \times 10^{11} = 10 \text{ N}.$$

Therefore, $Q^2 = 4 \times 10^{-10}$, or $|Q| = 2 \times 10^{-5}$. That is, $|Q|$ is 20 μC. We cannot determine its sign (recall the charge conjugation symmetry).

(b) If the above experiment is repeated with a doubled Q, what is the magnitude of the force acting on A? [5]

F is proportional to the product of charges, so F must be quadrupled, or 40 N.

3. Look at the configuration of three charges in the figure 2. A and C have $-2\ \mu$C and B $-3\ \mu$C.

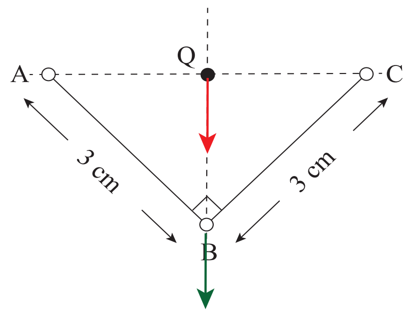


Figure 2:

(a) What is the total force acting on B from the other charges A and C? Compute its magnitude and indicate its direction in Figure 2. [5]

Recall :

Superposition principle,

Forces are vectors.

The x-components of the forces cancel each other, so we have only to consider the y-component. The forces are repulsive, so the resultant force must be in the $-y$-direction. The magnitude of the force between A and B is

$$|\boldsymbol{F}| = (9 \times 10^9)\frac{(2 \times 10^{-6})(3 \times 10^{-6})}{(0.03)^2} = 6 \times 10^{9-12+4} = 60\ \text{N}.$$

Therefore, $60 \times 2\cos 45° = 60 \times 1.414 = 84.9$ N (green arrow).

(b) Q is the point equidistant from all three points A-C and on the line connecting A and C. What is the total force acting on a 2 μC charge placed at Q? Compute its magnitude and indicate its direction in the figure 2. [5]

By symmetry forces due to A and C cancel each other, so we have only to consider the effect of B on Q. These charges have opposite signs, so the force is attractive and Q is pulled downward (red arrow). Its magnitude is

$$|\boldsymbol{F}| = (9 \times 10^9)\frac{(2 \times 10^{-6})(3 \times 10^{-6})}{(0.03\cos 45°)^2} = 12 \times 10^{9-12+4} = 120\ \text{N}.$$

Physics 102 Discussion Week 3 (Lecture 3)
Electric Field

Key Points

Electric field

Suppose charge q is located at the origin. If we place another charge away from the origin, then there is a Coulomb force acting on this charge.

We can interpret this informally as follows: the charge at the origin 'warps' the property of the space, and the new charge senses it and feels a force.

In more proper terms, we say the charge creates a **field** called the **electric field** $\boldsymbol{E}$ all over the space. If a charge q' is placed where there is an electric field $\boldsymbol{E}$, the following force acts on the charge q':

$$\boldsymbol{F} = q'\boldsymbol{E}.$$

Thus, the unit of electric field is N/C.

We know Coulomb's law, so we can find $\boldsymbol{E}$ created by a single charge.
The **electric field created by a system of charges** is determined by the following two basic rules, **1** and **2**.

1. Electric field due to a single charge

The electric field $\boldsymbol{E}$ created by a charge q (see Figure below):

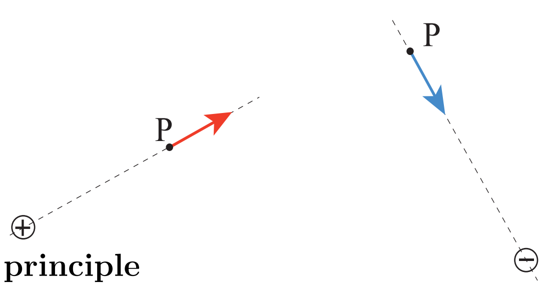
its magnitude E at a location P which is distance r away from the charge is given by ($k = 8.99 \times 10^9$ Nm^2/C^2)

$$E = k\frac{|q|}{r^2},$$

its direction is along the line connecting the charge and P

if $q > 0$, away from the charge to P,

if $q < 0$, toward the charge from P.

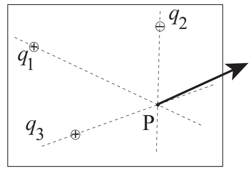


2. Superposition principle

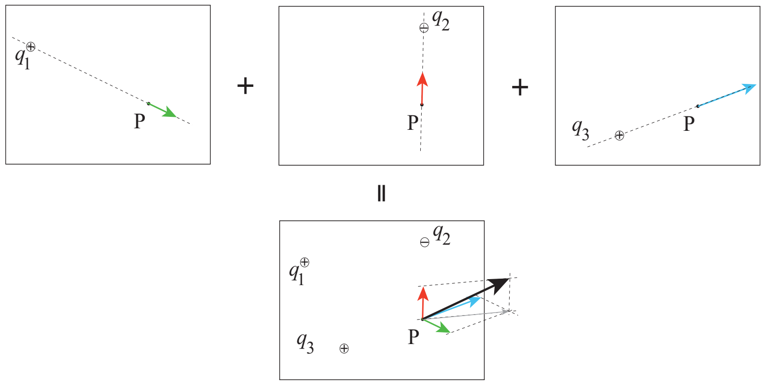
If there are more than one charges q_1, q_2, $\cdots$, the electric field at point P due to these charges is the vectorial sum (superposition) of all the electric fields $\boldsymbol{E}_i$ due to charge q_i determined separately according to **1**:

$$\boldsymbol{E} = \sum \boldsymbol{E}_i.$$

The following example illustrates the principle:



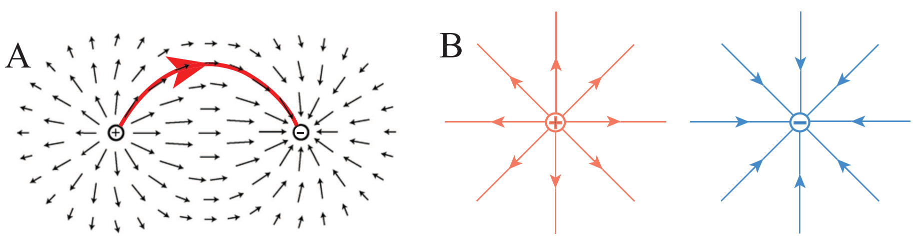
To study the system consisting of three charges illustrated above, decompose the system into individual charges as the top row of the following figure, and then the resultant electric fields due to individual charges are added as vectors (the bottom row):



Remark: No self interaction: There is no self interaction between the part of $\boldsymbol{E}$ created by a charge q and the charge itself.[4]

Electric field lines

If electric field $\boldsymbol{E}$ is given all over a region, we can imagine a curve whose tangent directions agree with $\boldsymbol{E}$ along it (see a red curve in A below). Such a curve is called an **electric field line**.



Sufficiently close to any charge, the field is almost determined by the charge alone, so field lines near a charge are solely determined by the charge and look as illustrated in B above.

You may imagine that (see B in Figure above; ϵ_0 below is defined by $\epsilon_0 = 1/4\pi k$)

q/ϵ_0 field lines emanate isotropically (i.e., evenly in all directions) from a positive charge q, and

$|q|/\epsilon_0$ field lines are sucked by a negative charge $-|q|$.

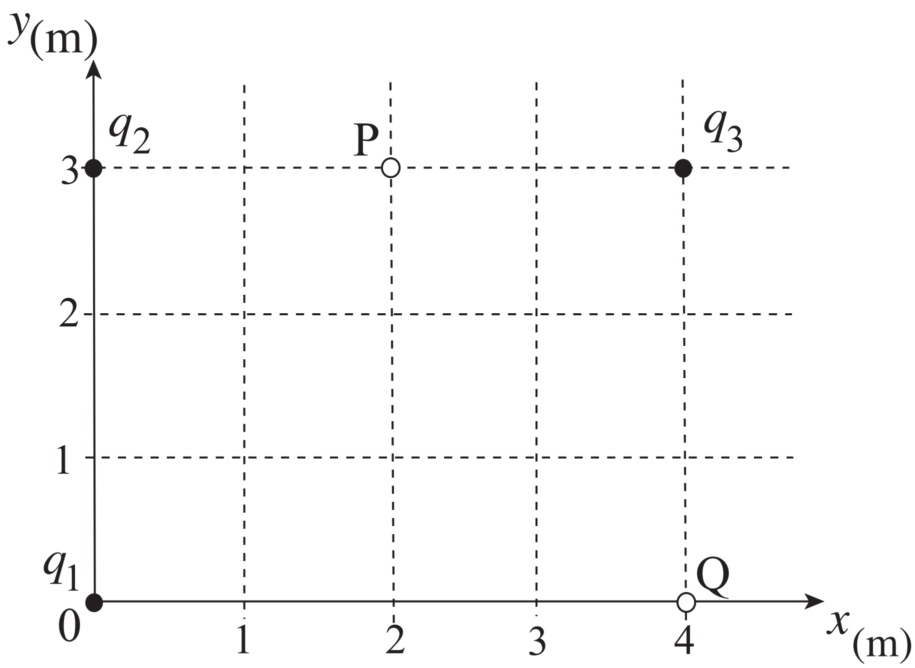
Thus, **the density of field lines can indicate the intensity of the electric field**. Needless to say, **the direction of the line (tangent direction) indicates the electric field direction** at the point. See Discussion 3-3 for an illustration.

Electric force lines never cross (except where $\boldsymbol{E} = 0$) and are never created nor annihilated except where charges are or at infinity.

[4]This 'infinite interaction' is already taken care of in the quantum field theoretical definition of the observed charge, so we need not worry about it.

Discussion 3-1 Electric field due to charges

We have three charges exactly as in **D2-2**: $q_1 = -0.1$ mC and $q_2 = q_3 = 0.3$ mC.



1) Find the electric field at P (i.e., E_x and E_y at P).

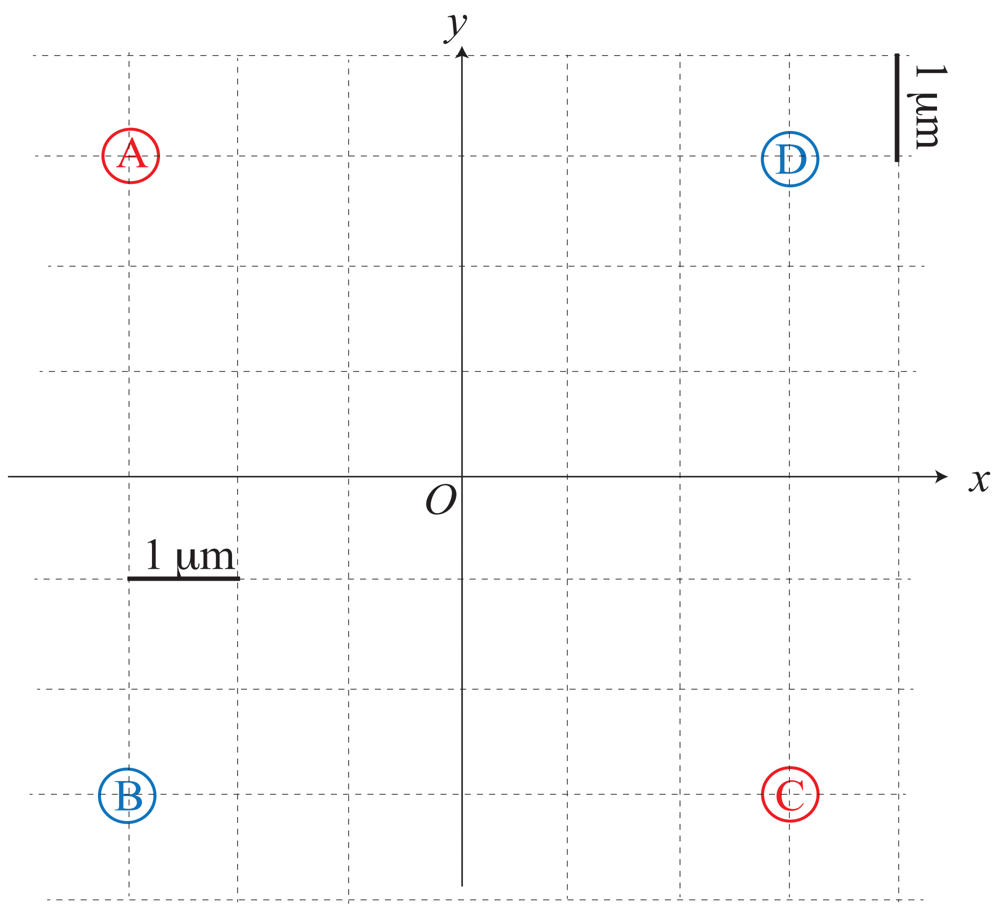
2) What is the force on the charge of $q' = -0.1$ mC placed at P?

3) Is there any place where $\boldsymbol{E} = 0$ on the rectangle $[0,4] \times [0,3]$?

4) If the sign of the charge q_1 is flipped to 0.1 mC, is there any place where $\boldsymbol{E} = 0$ on the rectangle $[0,4] \times [0,3]$?

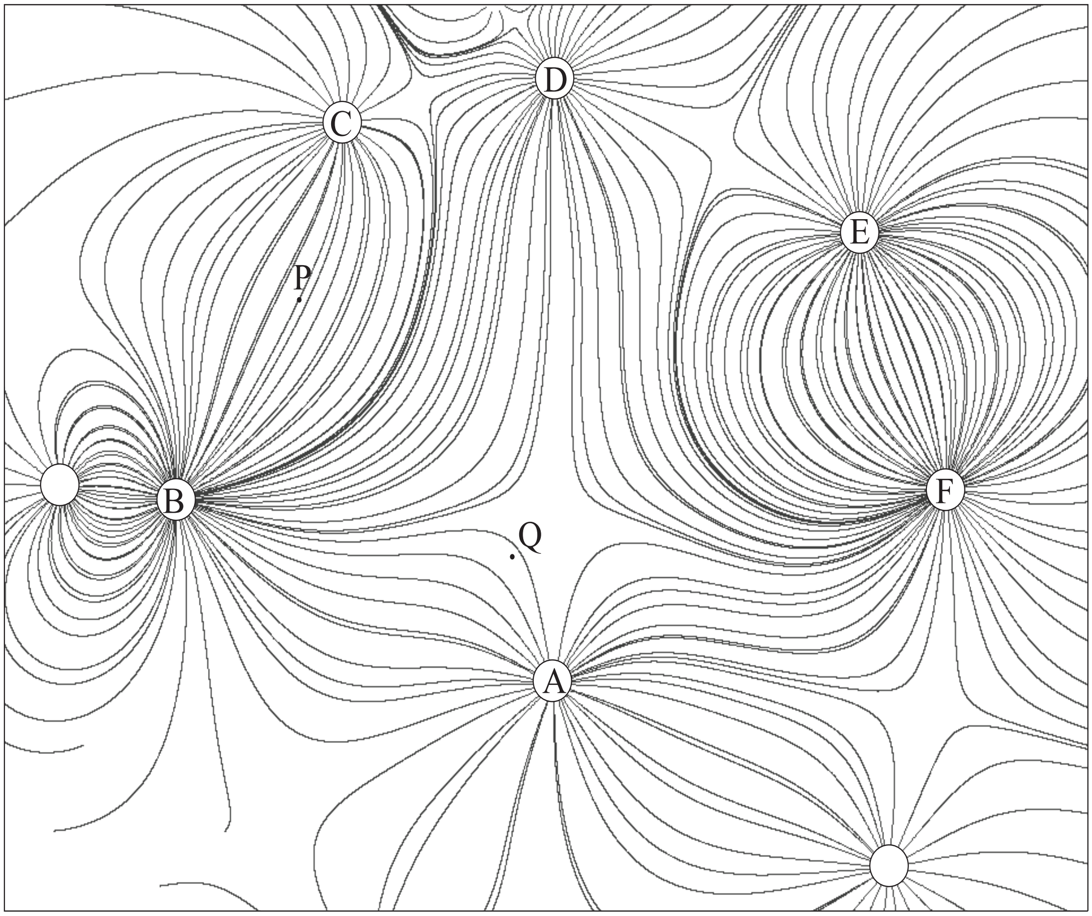
Discussion 3-2 Electric quadrupole

Sketch the electric field lines due to an electric quadrupole in the following figure. Assume charges A and C are positive, and B and D are negative.



Discussion 3-3 Field lines

There are several plus and minus charges on the plane defined by the sheet of this paper. You may assume that there are no other charges in the world.[5]



1) Suppose A is a positive charge. Choose positive charges among B - F.

2) Mark the locations with X where $\boldsymbol{E} = 0$.

3) Indicate the direction of the electric field at point P and Q.

4) Which point, P or Q, has a stronger electric field?

[5] The figure was created by using http://www.cco.caltech.edu/~phys1/java/phys1/EField/EField.html.

Physics 102 (Sample Quiz) Q2s

Name: ____________ **Section:** ________ **Score:** ________/20

1. As shown in Figure 1 several point charges are fixed on the sheet of this paper, making an electric field $\boldsymbol{E}$ in a plane. At the origin O the electric field is given by $\boldsymbol{E} = (-3.2, 1.2) \times 10^3$ N/C.

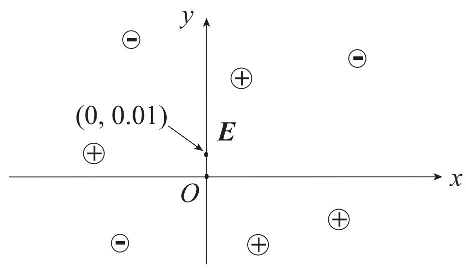


Figure 1:

(a) A charge $q = 1.5$ μC is placed at the origin. What is the magnitude of the force acting on the charge? [5]

Recall

$$\boldsymbol{F} = q\boldsymbol{E}.$$

$|\boldsymbol{E}| = 3.42 \times 10^3$ N/C.
$|\boldsymbol{F}| = q|\boldsymbol{E}| = (1.5 \times 10^{-6}) \times (3.42 \times 10^3) = 5.13 \times 10^{-3}$ N

(b) Another charge which is exactly $-q$ is placed at $(0, 0.01)$ m and connected to the $+q$ charge to make a dipole. What is the magnitude of the torque on this dipole? [5]

Recall

$$\text{torque } = |\boldsymbol{p}||\boldsymbol{E}|\sin(\text{angle from } \boldsymbol{p} \text{ to } \boldsymbol{E}).$$

Angle $= 180 - \tan^{-1}(3.2/1.2) = 180 - 69.4 = 110.6°$.
$|\boldsymbol{p}| = 1.5 \times 10^{-6} \times 0.01 = 1.5 \times 10^{-8}$ m·C.
Therefore, torque $= (1.5 \times 10^{-8}) \times (3.42 \times 10^3) \sin(110.6°) = 4.8 \times 10^{-5}$ m·N.

2. Electric field lines due to more than 10 charges on a plane are depicted in Fig. 2.

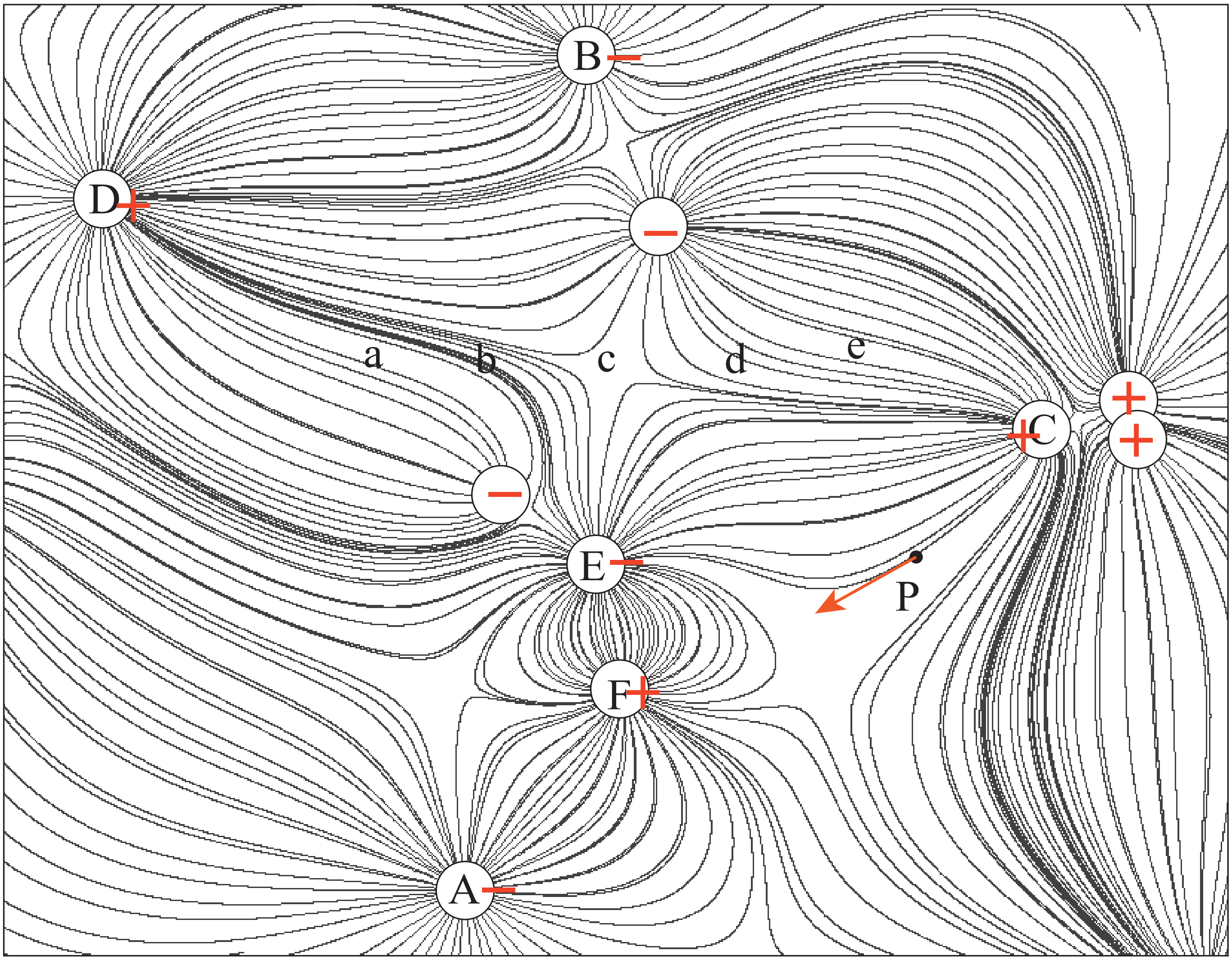


Figure 2:

(a) Suppose charge A is negative. Give all the negative charges among B-F. [4]

B and E are negative.

(b) Among the points a-e, where is the electric field zero? [2]

c.

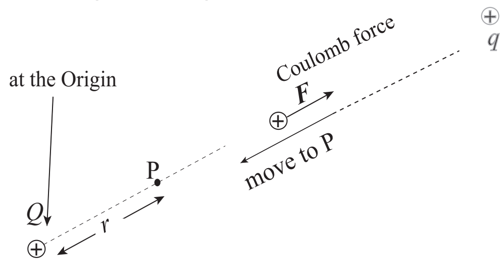
(c) Draw an arrow describing the direction of the electric field vector at P. [4]

Physics 102 Discussion Week 4 (Lectures 4 and 5)
Electric Potential and Work

Key Points

Electric potential

Background: Suppose charge Q is at the origin and charge q is initially very far away from the origin (at infinity). Now, let us drag the charge q from infinity to a point P which is distance r away from the origin (Figure below). If Q and q are with the same sign, we must do work against the repulsive Coulomb force $\boldsymbol{F}$ acting on charge q. It is known that the work you must perform is given by $W = kQq/r$.



This may be understood as the potential energy of charge q. If we exchange the charge q with another charge q', then this potential energy would become kQq'/r. Therefore, we interpret that the charge Q creates an **electric potential field** $V = kQ/r$, and a charge q placed at a point in this field has a potential energy $U = qV$, where V is the electric potential at the point.

The **electric potential field created by a system of charges** is determined by the following two basic rules, **1** and **2**.

1. Electric potential due to a single charge

The electric potential field V created by charge q at P which is distance r away from the charge is give by ($k = 8.99 \times 10^9$ Nm^2/C^2)

$$V = k\frac{q}{r},$$

If $q > 0$, then $V > 0$; if $q < 0$, then $V < 0$. Its unit is J/C (= V, volts).

2. Superposition principle

If there are more than one charges q_1, q_2, $\cdots$, the electric potential field V at a point P due to these charges is the sum of all the individual electric fields V_i due to charge q_i determined separately according to **1**:

$$V = \sum V_i.$$

Remark. No self interaction: Just as in the case of $\boldsymbol{E}$ there is no self interaction between the charge and the field created by it.

Electric potential energy

If charge q is placed at a position with electric potential V, its electric potential energy U reads

$$U = qV.$$

You may interpret U at a point as the needed work for you to bring q from infinity to the point. If U is due to charges q_1, q_2, $\cdots$, thanks to the superposition principle, you may interpret U as the sum ($U = \sum W_i$) of the needed work W_i to bring q to the point from infinity, assuming only q_i exists:

$$U = k\frac{qq_1}{r_1} + k\frac{qq_2}{r_2} + \cdots = q\left[k\frac{q_1}{r_1} + k\frac{q_2}{r_2} + \cdots\right],$$

where r_1 (resp., $r_2, \cdots$) is the distance between q and q_1 (resp., $q_2, \cdots$).

If you wish to move the charge q from a location with electric potential U_1 to another location with electric potential U_2, then you must supply work W given by

$$W = q(U_2 - U_1).$$

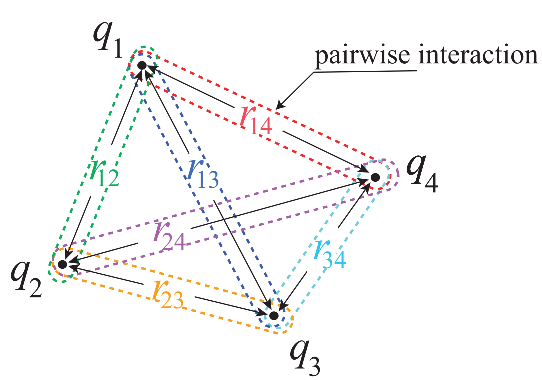
Electric energy stored in a charge system

To create a system with charges $q_1, q_2, \cdots$, located at $\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots$, respectively, starting with non-interacting charges, we can bring charges one by one from infinity to the specified locations. The total work you do to create the charge system may be interpreted as the total electric energy U stored in the charge system. The superposition principle and the formula for the pairwisely stored electric potential energy gives

$$U = \sum_{\text{all charge pairs}} k\frac{q_i q_j}{r_{ij}},$$

where r_{ij} is the distance between charge i and j. The following example illustrates the formula.

Illustration:
There are four charges in the space. What is the total electrostatic energy U stored in this system?



$$U = k\frac{q_1 q_2}{r_{12}} + k\frac{q_1 q_3}{r_{13}} + k\frac{q_1 q_4}{r_{14}} + k\frac{q_2 q_3}{r_{23}} + k\frac{q_2 q_4}{r_{24}} + k\frac{q_3 q_4}{r_{34}}.$$

We have only to add all the pair contributions.

Potential field as a landscape

If we place a positive charge q at a point with an electric potential V, its potential energy is $U = qV$. Thus, we can visualize the potential energy field as a potential energy landscape. Similarly, we can visualize the field V as a landscape: a plus point charge creates an infinitely tall peak, and a negative point charge an infinitely deep pit. Where there is no charge, there is no peak or pit.

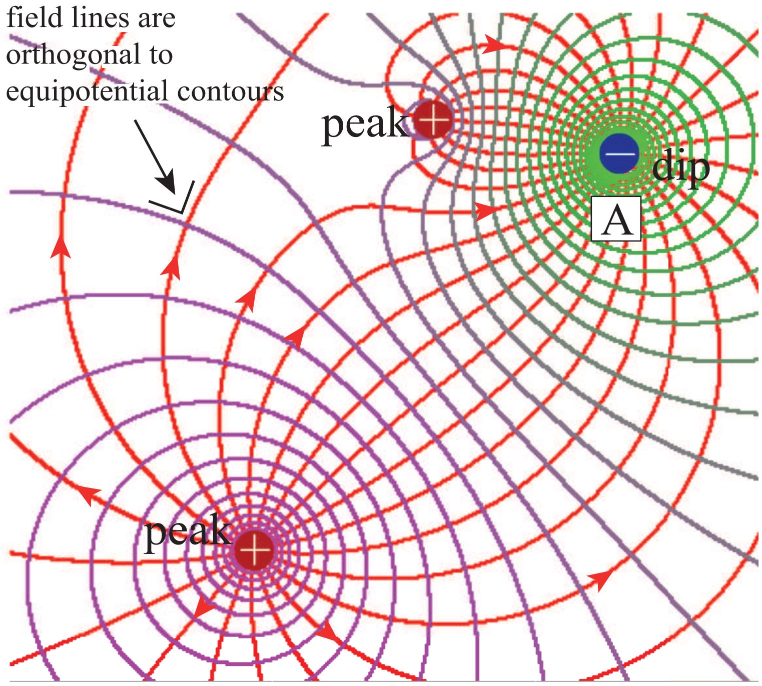
We can illustrate the V field in terms of equipotential curves (contours)[6] just as the usual geographic landscape (See Figure below).

Electric field denotes the steepest descending direction and the slope along it

At a point in an electric potential field V, the electric field $\boldsymbol{E}$ (vector) is given by

Magnitude: $E = |\boldsymbol{E}|$ is the slope of the V landscape,
Direction: The vector $\boldsymbol{E}$ points in the steepest descending direction.

Thus, equipotential contours and $\boldsymbol{E}$ vectors are orthogonal; the field lines are always orthogonal to the equipotential contours as illustrated below by an example.[7]



Notable qualitative features of equipotential contours and electric field lines

1) A **high contour density implies a strong electric field**; dense contours surrounding a charge indicate its magnitude. Larger charges are also connected with more numerous force lines. In the above the charge near **A** has the biggest absolute size.

2) **The electric field lines and contour curves must be orthogonal**; $\boldsymbol{E}$ points the steepest descending direction of the potential field V.

[6]Actually, they are cross sections of equipotential surfaces in 3D space.

[7]Field figures of this week are created with the aid of http://irobutsu.a.la9.jp/mybook/ykwkrEM/sim.html (Masahiro Maeno's *Comprehensible Electromagnetism* (in Japanese) support page).

Discussion 4-1 Electric potential energy due to two charges

You should try to illustrate the questions before answering them.

1) A positive charge **A** of 10 μC is fixed at a point O. Another charge **B** of -5 μC is found at point P which is 5 mm away from O. What is the electric potential energy stored in this charge configuration?

2) If initially the negative charge **B** in 1) is infinitely away from O, what work do you have to do to bring charge **B** to P?

3) Instead of the negative charge **B**, now you bring a positive charge **C** of 12 μC from infinity to P. What work do you have to do?

4) What is the electric potential energy stored in this two-positive-charge (**A** and **C**) configuration?

5) Next, you wish to bring the charge **C** currently at P to the point R that is 1 mm away from O. What work do you have to do?

6) The +12 μC charge **C** is allowed to move freely with the intial speed zero from R. When it is 18 mm away from O, what is its speed? Let us assume that the mass of this movable charge **C** is 1.0 g.

7) What happens if you switch the signs of all the charges (i.e., + charge to be $-$ charge of the identical magnitude and vice versa.) in the above questions?

Discussion 4-2 Electric potential energy of a quadrupole

In Fig. 1 we wish to bring two $+q = +100\ \mu$C charges to A and C, and two $-q = -100\ \mu$C charges to B and D from infinity one by one to make an electric quadrupole.

First, we fix one $+q$ at A $= (-3, 3)\ \mu$m.

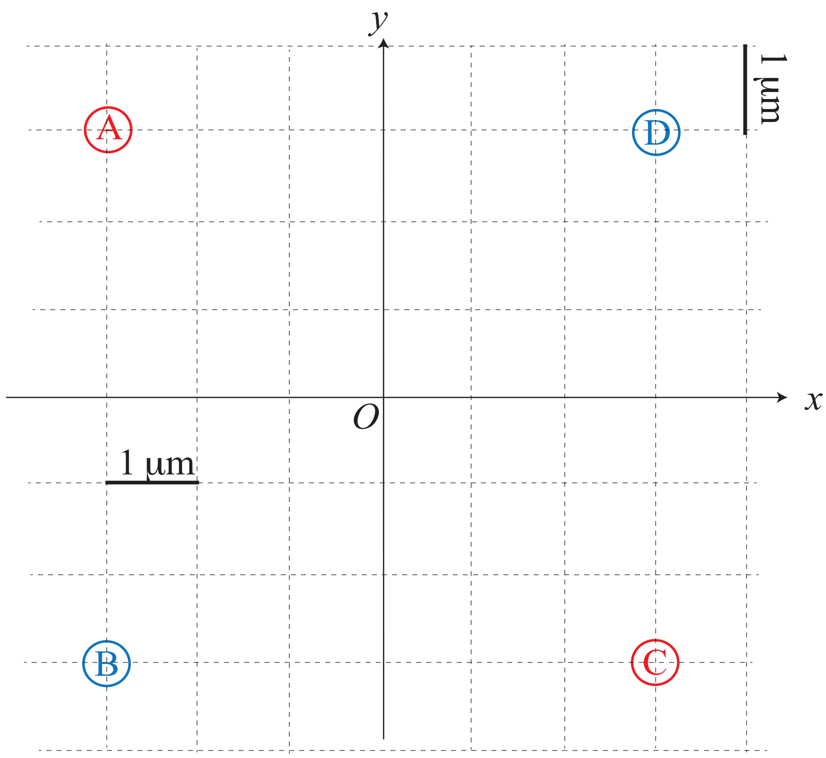


Figure 1:

1) $-q$ is brought to B from infinity and is fixed. What is the work you have to do? While performing this process what force do you feel?

2) Next, the other $-q$ is brought from infinity to D. What is the work you have to do?

3) Finally, the remaining $+q$ is brought to C from infinity. What is the work you have to do?

4) What is the electric potential energy stored in the completed electric quadrupole?

5) What is the electrical potential at O?

6) Is there any point within the square ABCD where the electric potential is more than 10 MV?

Discussion 4-3 Electric potential and work

Let us move a minus charge Q of -10 mC around a positive charge. The equipotential curves (surfaces) are depicted in the following figure (Fig. 2).

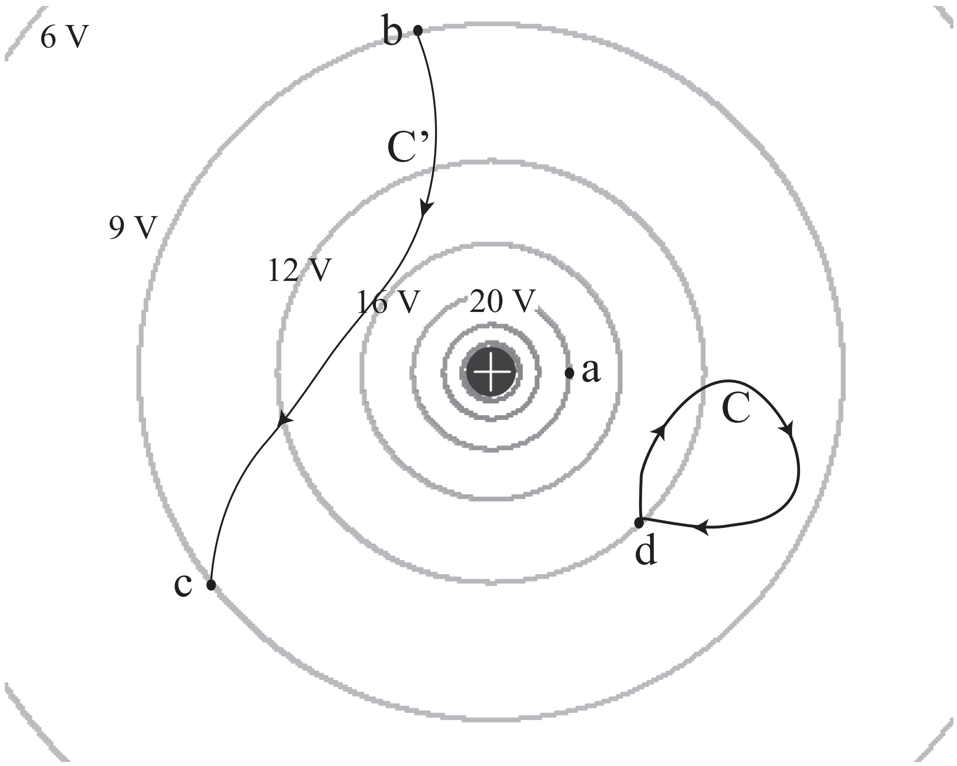


Figure 2:

1) What work do you have to do to move charge Q along the curve C' from b to c?

2) What is the work do you have to do to move charge Q from a to b?

3) What work do you have to do to move charge from d to the same point d along curve C?

Discussion 4-4 Electric potential 'landscapes'

1) What can you tell from the following four figures (Fig. 3; e.g., sign, magnitude, field lines, voltage, etc.)? Discuss with your friends.

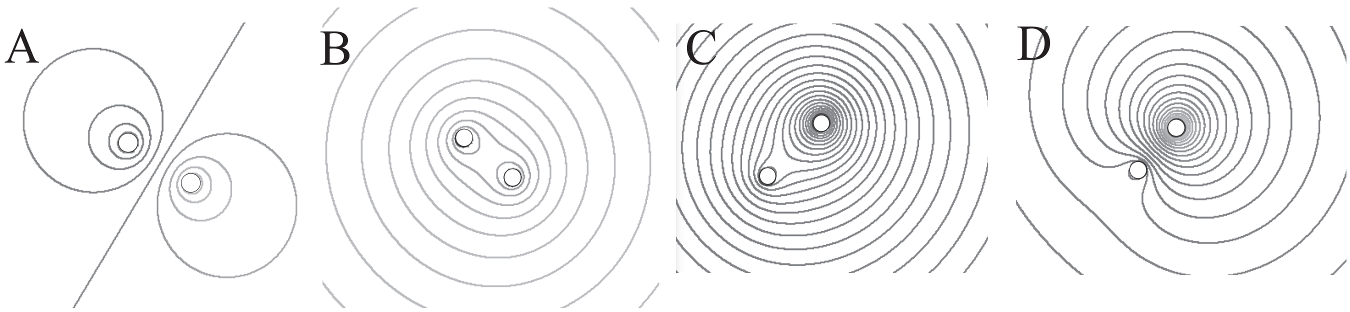


Figure 3:

2) Suppose A is positive in Fig. 4. Tell the signs of B-D and extend the field line starting from B.

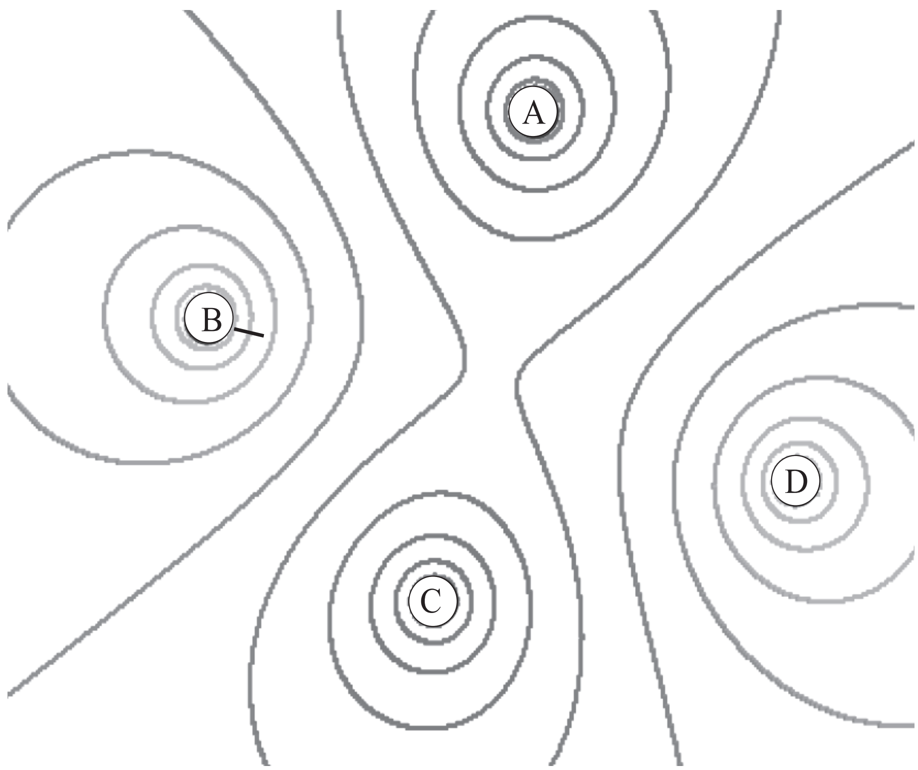


Figure 4:

3) Seven charges are on the plane and the equipotential surfaces (or contours) are described in Fig. 5. Let us assume charge A to be positive.

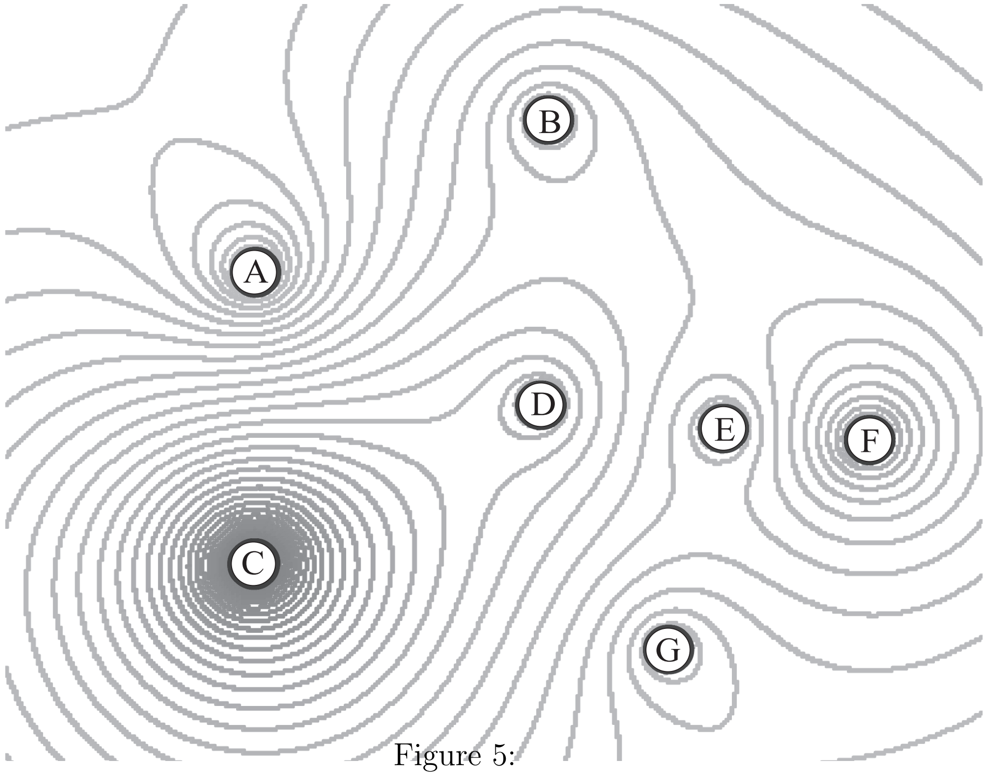


Figure 5:

(i) What charge is with the largest magnitude?

(ii) Write down all the negative charges.

(iii) Is there zero electric potential point in this figure?

Discussion 4-5 Electric potential and field lines with a conductor

A conducting circular ring is placed so that its center coincides with the center of the electric quadrupole constructed in **D4-2** as shown in Fig. 6.
1) Sketch the equipotential surfaces and the electric field lines in the figure.

Figure 6:

2) What is the electric potential of the conductor?

3) What is the electric potential at the center of the ring?

Physics 102 (Sample Quiz) **Q3s**

Name: ______________ **Section:** ________ **Score:** ________/**20**

1. A $-11\ \mu$C point charge **A** is fixed in the space.
(1) You bring a positive charge **B** of 8 μC from infinity to a point which is 7 cm away from charge **A**. What is the work you have to do? [5]

Recall $W = U_f - U_i$.

The initial potential energy is 0, because B is infinitely away from A.
The final potential energy is

$$k\frac{Q_A Q_B}{r} = (9\times 10^9)\frac{(-11\times 10^{-6})(8\times 10^{-6})}{0.07} = -11314\times 10^{9-6-6} = -11.3 \text{ J}.$$

Thus, you do -11.3 J of work (or you are dragged by the charge system and done $+11.3$ J of work by the system).

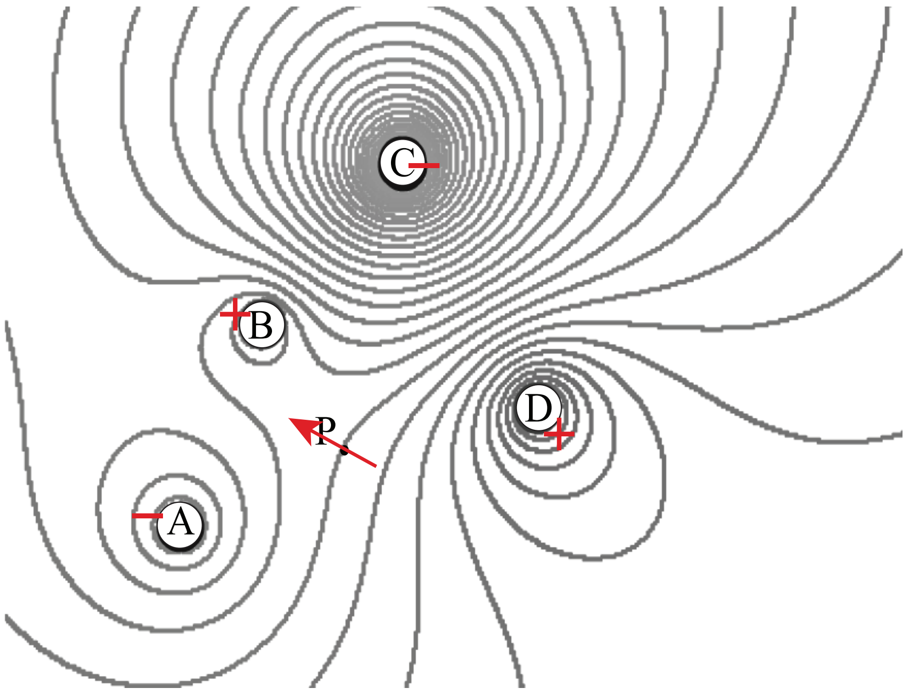
(2) Now, charge **B** is gently released and moves to a point 4 cm away from the fixed charge **A**. What is its speed, if its mass is 1.5 g? [5]

The initial potential energy is $U_i = -11.3$ J as computed in (1).
The final potential energy is $U_f = (9\times 10^9)(-11\times 10^{-6})(8\times 10^{-6})/0.04 = -19.8$ J (which is $= -11.3\times(.07/0.4)$).
Energy conservation:

$$-11.3 + 0 \text{ kinetic energy } = -19.8 + \frac{1}{2}mv^2,$$

so $v^2 = 2\times(19.8-11.3)/0.0015 = 11333.33$, or $v = 106.45$ m/s.

2. There are four charges on the plane. The equipotential curves are described in the following figure.



(1) Suppose A is negatively charged. State the signs of all the remaining charges B - D.

(2) Which charge has the largest magnitude?

C

(3) Indicate the direction of the electric field at P. You must briefly explain your choice.

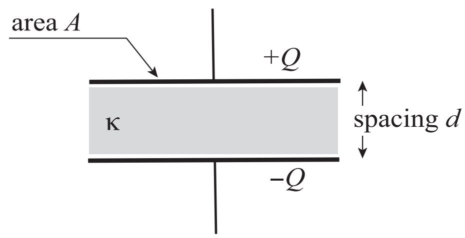
Since D is positive and since A is negative, the left side must be with lower potential. $\boldsymbol{E}$ must be perpendicular to the equipotential line.

Physics 102 Discussion Week 5 (Lectures 6 and 7)
Circuit elements and simple circuits

Key Points

Capacitors

A **capacitor** consists of two electrodes and the spacing d between them, which may or may not be filled with material (with a **dielectric constant** κ; Figure below). Positive and negative charges $\pm Q$ are separately stored on the electrodes with area A.



The stored charge Q is proportional to the voltage difference V across the spacing, and the proportionality constant C is called the **capacitance**:

$$Q = CV.$$

The unit of C is F = C/V and is called farad.

The electric energy U stored in the capacitor reads

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}. \tag{1}$$

The capacitance C may be computed as

$$C = \kappa\epsilon_0 \frac{A}{d},$$

where κ is a materials constant called the **dielectric constant** and $\epsilon_0 = 8.85 \times 10^{-12}$ $\mathrm{C^2/Nm^2}$.

Resistors

If charges move through a cross section at a constant rate such that charge Δq flows within a time span Δt, we say an **electric current** I given by

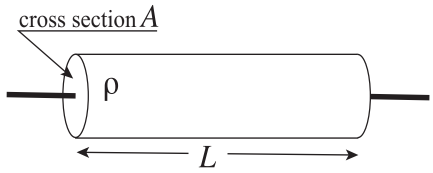
$$I = \frac{\Delta q}{\Delta t}$$

flows through the cross section. The unit of the current is A = C/s (ampere).

The current I through a material is proportional to the voltage V across it (**Ohm's law**);

$$V = IR.$$

The constant R is called the **resistance** and is measured in Ω = V/A (ohm).



The resistance R of a rod may be calculated as follows. Suppose the rod has a cross section A and length L (see Figure). The resistance is proportional to L/A:

$$R = \rho \frac{L}{A},$$

where the proportionality constant ρ is called the **resistivity**.

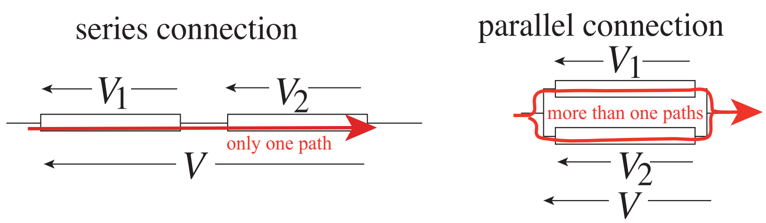
If a current I flows down the voltage difference V, electric energy is dissipated. The electric energy loss per unit time is the electric power consumed (= converted to heat) by the resistor and is given by

$$P = IV = I^2 R = \frac{V^2}{R}.$$

Parallel connection and series connection

See the figure below: Let V_1 and V_2 be the voltage drops across circuit element 1 and 2, respectively.

For a **series connection** $V = V_1 + V_2$,
For a **parallel connection** $V = V_1 = V_2$.



Parallel and series connections of resistors

(1) Series connection of resistors R_1 and R_2 (henceforth, the name of the resistor also denotes its resistance as well):

What quantity is shared by R_1 and R_2?
The current I is shared, so $V_1 = R_1 I$, $V_2 = R_2 I$.
The effective resistance R_S is defined by $V = R_S I$.
Since $V = V_1 + V_2$ (see Figure above),

$$R_S I = R_1 I + R_2 I \;\Rightarrow\; R_S = R_1 + R_2.$$

(2) Parallel connection of resistors R_1 and R_2.

What quantities are common for R_1 and R_2? The voltage drops V_1 and V_2 are, so $V = R_1 I_1 = R_2 I_2$.
Due to the charge conservation (or Kirchhoff's junction rule, see below) $I = I_1 + I_2$, so

the effective resistance R_P defined by $V = R_\mathrm{P} I$ implies

$$I = I_1 + I_2 \;\Rightarrow\; \frac{1}{R_\mathrm{P}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Although this is the standard formula, it is convenient to memorize

$$R_\mathrm{P} = \frac{R_1 R_2}{R_1 + R_2}.$$

Parallel and series connections of capacitors

(1) Parallel connection of capacitors C_1 and C_2 (henceforth, the name of the capacitor also denotes its capacitance as well).

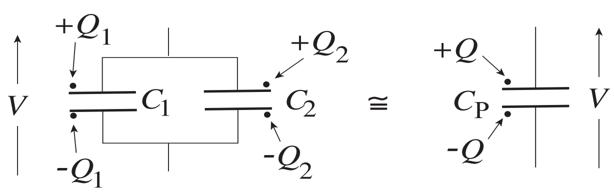
What are the common quantities for these capacitors?

The voltage drops V_1 and V_2 are, so $Q_1 = C_1 V$, $Q_2 = C_2 V$.

The effective capacitance C_P is defined by $Q = C_\mathrm{P} V$.

Since $Q = Q_1 + Q_2$ (see Figure below)

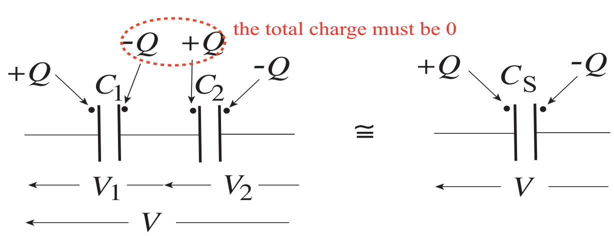
$$C_\mathrm{P} = C_1 + C_2.$$



(2) Series connection of capacitors C_1 and C_2.

What are common? The charges on the positive plates $+Q$ are due to the charge conservation.

Take C_1. If the positive plate has charge $+Q$, its other plate must have charge $-Q$. Before charging, the 'wire' between C_1 and C_2 has no net charge, so the charge conservation implies that charge $+Q$ must be on the plate of C_2 closer to C_1. That is, both the capacitors store the same charge Q. See Figure below.



$Q = C_1 V_1$, $Q = C_2 V_2$ and $Q = C_\mathrm{S} V$. Therefore,

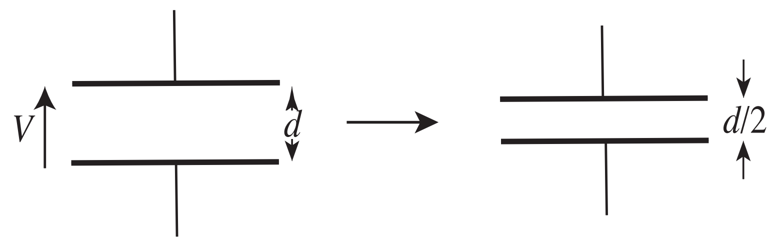
$$V = V_1 + V_2 \;\Rightarrow\; \frac{1}{C_\mathrm{S}} = \frac{1}{C_1} + \frac{1}{C_2}.$$

Although this is the standard formula, it is convenient to memorize

$$C_\mathrm{S} = \frac{C_1 C_2}{C_1 + C_2}.$$

Discussion 5-1 Capacitors and work

(A) We wish to halve the spacing of the parallel plate capacitor as illustrated below.

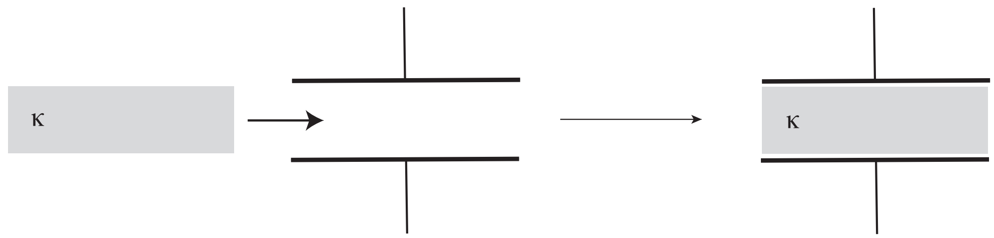


1) Initially, the voltage across the capacitor is V. What is the voltage across the spacing after halving it? Assume that the terminals of the capacitor are kept open.

2) By this halving process how does the stored energy change?

3) The energy is not conserved. Explain why the energy is not conserved. Who did the required work?

(B) We wish to insert a dielectric material with dielectric constant κ into the spacing as illustrated below.



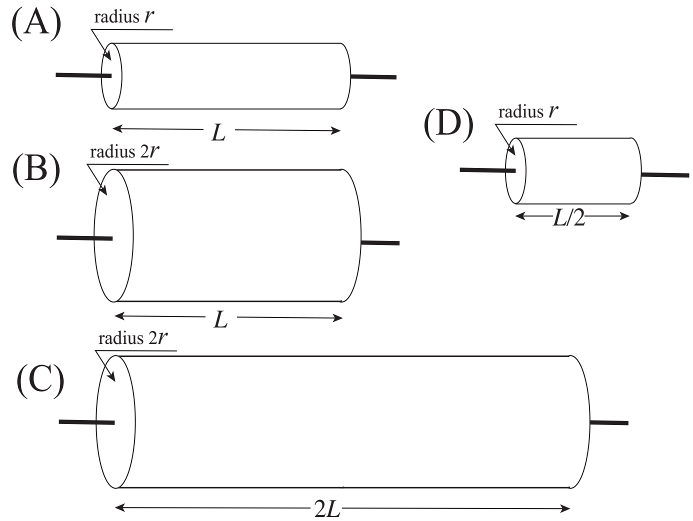
4) Initially, the voltage across the capacitor is V. What is the voltage across the spacing after inserting the dielectric material? Assume that the terminals of the capacitor are kept open.

5) By this insertion of the dielectric material how does the stored energy change?

6) The energy is not conserved. Explain why the energy is not conserved. Who did the required work?

Discussion 5-2 Power dissipation due to resistors

There are four resistors made of the same material as illustrated in the figure below. The resistance of (A) is R.

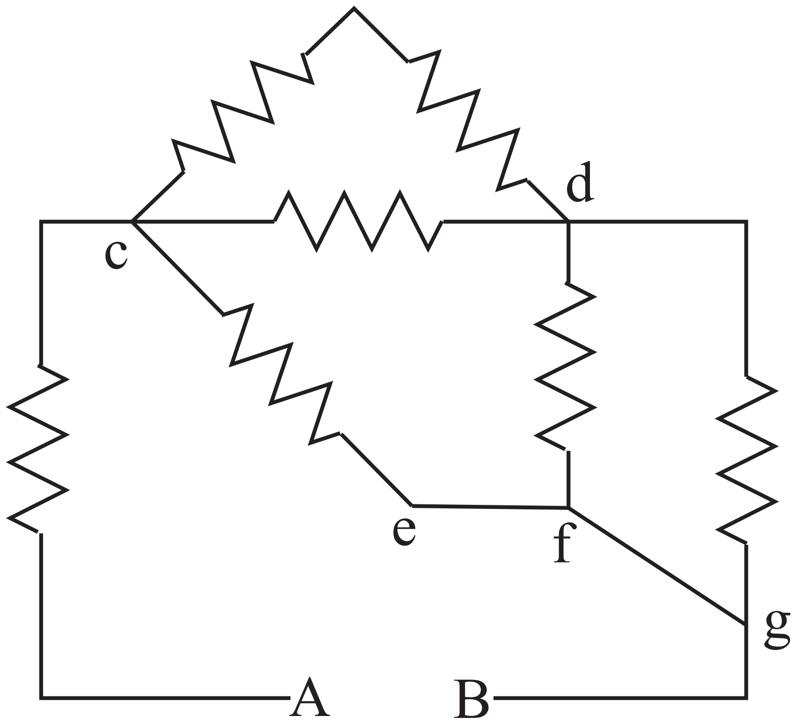


1) Find the resistances of (B), (C) and (D) in terms of R.

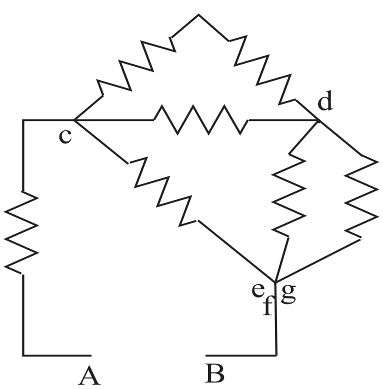
2) If the voltage across the resistors are identical, which resistor dissipates the electric energy most?

Discussion 5.3 Finding effective resistance

Consider the following resistor network. All the resistors are with $R = 3\ \Omega$.



1) Let us identify junctions that need not be distinguished before starting the reduction. Check that the above circuit is equivalent to the following circuit.



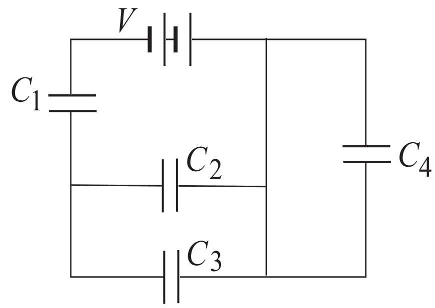
2) Find the effective resistance between A and B.

3) Suppose a 15 V power supply was connected between A and B. What is the potential difference between c and g ?

4) If these resistors were all identical light bulbs, which one would be the brightest? Which one would be the least bright?

Discussion 5-4 Capacitor circuit

Consider the capacitor network connected to a battery as illustrated below. You have already measured the quantities numerically given in the table in 1):



1) Fill the empty boxes in the following table and determine V of the battery.

Capacitor	Capacitance	Stored Charge	Voltage	Stored Energy
C_1	6.0 μF	6.0 μC		
C_2				
C_3	0.5 μF			25 μJ
C_4				

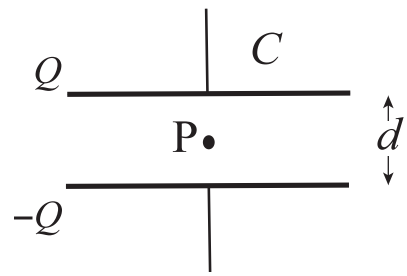
2) What is the total electrostatic energy stored in this capacitor system?

Physics 102 (Sample Quiz) Q4s

Name: ________________ **Section:** ________ **Score:** ________/**20**

1. Initially, between the plates of area A of the capacitor is vacuum and the magnitude of the electric field at P is E.

(1) What is the amount of charge Q stored in this capacitor? [4]

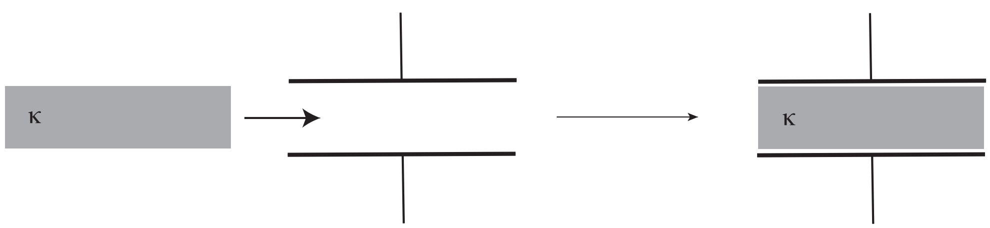


$E = V/d$

$Q = CV = \epsilon_0 AV/d$

$Q = \epsilon_0 AE.$

(2) Into the spacing of the capacitor considered in (1) is inserted a dielectric material with dielectric constant $\kappa = 3/2$ that exactly fills the space (see the figure). The terminals are kept open throughout the process. What is the magnitude of the work W you do in terms of U, which is the stored energy in the case of (1) (i.e., without the dielectric material)? [6]



Notice that the charge is conserved.

$U = Q^2/2C$. Also note that $C \propto \kappa$.

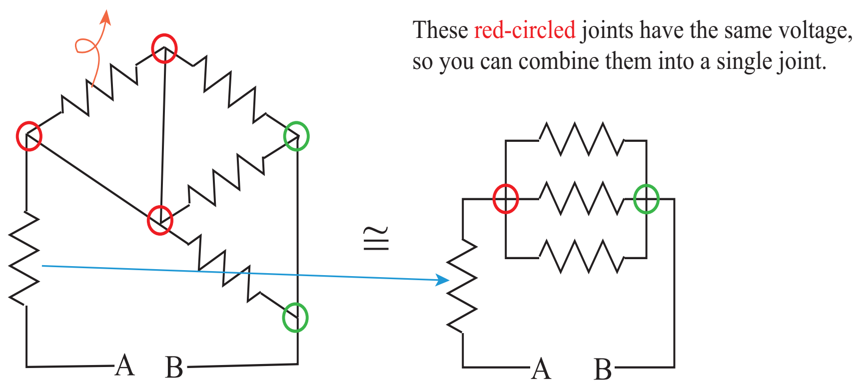
Therefore, $U \propto 1/\kappa$.

The initial energy is $U_i = U$. The final energy is $U_f = U/\kappa = 2U/3$.

$W = U_f - U_i = U/3$. Thus, the system has lost $U/3$ of energy. That is, you have been done a work of $U/3$ by the capacitor system.

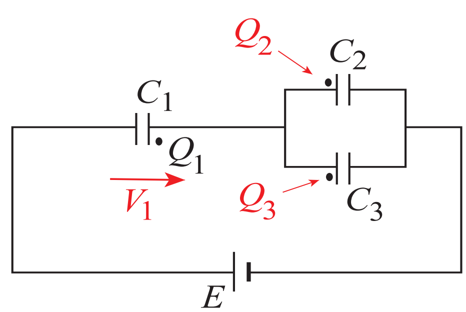
This must be intuitively understandable, because the dielectric material is actually sucked into the gap.

2. Find the resistance between A and B in the following resistor system. All the resistors are identical with resistance R. [5]



Thus, R and $R/3$ are in series $\Rightarrow$ the total resistance is $4R/3$.

3. In the following capacitor circuit $E = 12$ V. The capacitors are $C_1 = 1.5$ μF and $C_2 = 0.5$ μF with $Q_1 = -9$ μC. What is the capacitance C_3? [5]



$Q = CV$.

The voltage across C_1 is $V_1 = 9/1.5 = 6$ V.
Therefore, the voltage across $C_2 = 12 - 6 = 6$ V $\rightarrow Q_2 = 0.5 \times 6 = 3$ μC.
$Q_1 + Q_2 + Q_3 = 0$ due to charge conservation. This means $Q_3 = 6$ μC.

We know the voltage across $C_3 = 6$ V. Therefore, $C_3 = Q_3/6 = 1$ μF.

OR

$C_2 + C_3$ must be equal to C_1, because the voltage across $C_2 + C_3$ is also 6 V. This should imply $C_3 = 1$ μF.

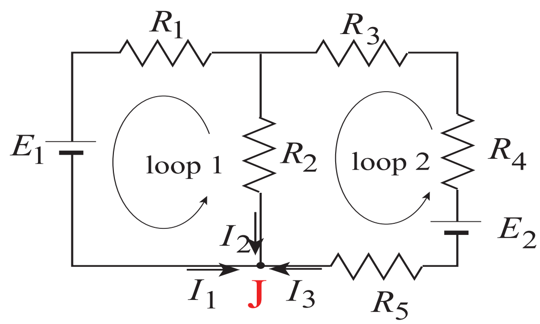
Physics 102 Discussion Week 6 (Lectures 8-9)

Kirchhoff's rules and RC Circuits

Key Points

Kirchhoff's rules

Kirchhoff's rules are reviewed here with the following example:



Algebraic expression of the currents

To determine the current along a branch, in order to express the current (say, I_1 in the above), we must assign the current direction *before* knowing the actual current. If the value of the current so assigned turns out to be positive, it means indeed the actual current flows in the direction you assigned (with an arrow). If negative, the actual current flow is in the opposite direction. *Algebraically, you need not worry about whether the true flow direction is as assigned or not. Simply, calculate algebraically, assuming everything is as assigned.* At the end, your wrong assumptions (if any) would *automatically corrected* with negative signs that are algebraically obtained.

Kirchhoff's rules

Kirchhoff's rules consist of (1) the junction rule and (2) the loop rule:

(1) **Junction rule**: The algebraic sum of all the in-coming currents and that of the out-going currents are identical (due to the conservation of charge).

For the junction J in the above figure, $I_1 + I_2 + I_3 = 0$.[8]

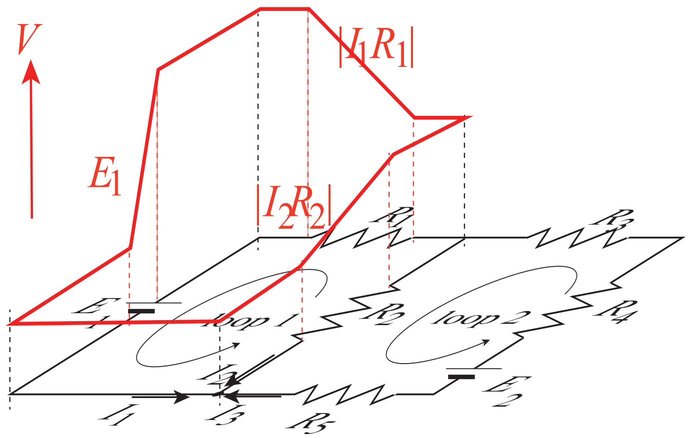
(2) **Loop rule**: Around any loop the voltage returns to the original value (see Figure below). Thus, around a loop algebraic sum of all the voltage drops and electromotive forces add up to zero.

Illustration using the above circuit.

Junction rule:

If applied to J, we have $I_1 + I_2 + I_3 = 0$ as already noted above.

[8] Clearly, at least one of them must be negative according to our choice of the arrows.



Loop rule:

Notice that I_1 flows from right to left through R_1 (due to the conservation of charge) *irrespective of the true direction of the current.* Assume everything is positive and set up the equations. Solve them. If some values are negative, simply flip your assigned directions. That is all. **Trust algebra**.

Loop 1: $-E_1 + R_2I_2 - R_1I_1 = 0$,

The loop direction (which you may choose as you wish, but here we adopt the one in the figure) goes from the positive to the negative terminals of the battery (downhill); the (algebraic) voltage change through the battery: $-E_1$.

The loop direction is against I_2 through R_2, so it is (algebraically) uphill; the (algebraic) voltage change through R_2: $+I_2R_2$.

The loop direction is in the same direction as I_1 flowing through R_1, so the (algebraic) voltage change through R_1: $-I_1R_1$.

Loop 2: $-I_2R_2 + R_3I_3 + R_4I_3 + R_5I_3 + E_2 = 0$.

The loop direction goes from the negative to the positive terminals of the battery (uphill); the (algebraic) voltage change through the battery: $+E_2$.

The loop direction is against I_3 through R_3, R_4 and R_5, so these contributions are all (algebraically) uphill, so

the (algebraic) voltage changes through R_3, R_4, and R_5: $+I_3R_3$, $+I_3R_4$, $+I_3R_5$, respectively.

The loop direction is in the same direction as I_2 flowing through R_2, so the (algebraic) voltage change through R_2: $-I_2R_2$.

RC circuits

The most important fact about RC circuits is:

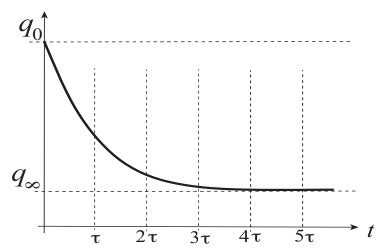
the amount of charge in any capacitor cannot change instantaneously at all. This of course implies that the voltage across any capacitor cannot change at all instantaneously. In contradistinction, currents can change instantaneously.

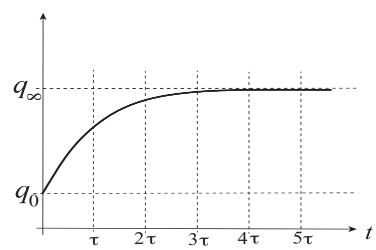
Time dependence: In a RC circuit the time dependence of the charge $q(t)$ of a capacitor is always the following form:

Let $q(0) = q_0$ (the initial charge) and q_∞ be the final charge after a sufficiently long time. Then,

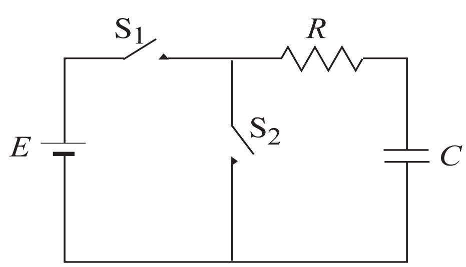
$$q(t) = q_\infty + (q_0 - q_\infty)e^{-t/\tau},$$

where τ is the time constant obtained from the effective circuit as $\tau = R_{eff}C_{eff}$. Two examples (case $q_0 > q_\infty$ and case $q_\infty > q_0$) are illustrated





Example:
In the following RC circuit with a battery E with two switches let us assume that the capacitor is uncharged initially.



1) At $t = 0$, switch S_1 is closed. What is the current immediately after $t = 0$?

Since $q_0 = 0$, so must be the charge stored in the capacitor immediately after $t = 0$. Therefore, the voltage across R is E, so $I = E/R$.

2) What is $q(t)$, the charge stored in the capacitor at t?

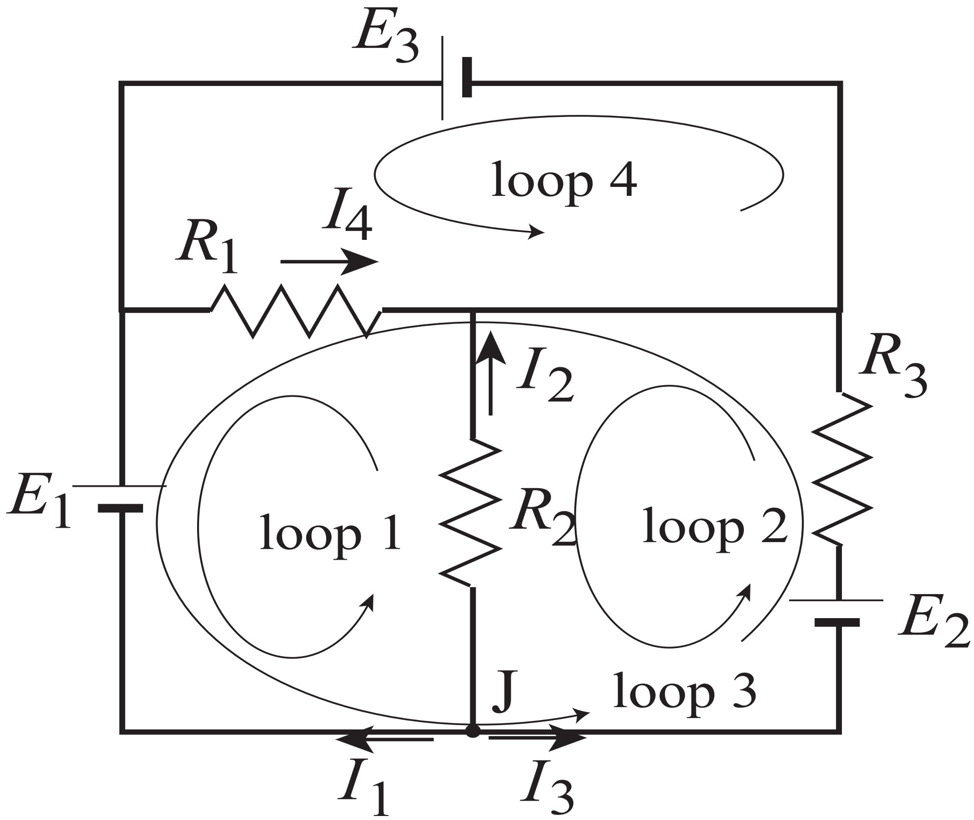
$q_0 = 0$ and $q_\infty = CE$, so the above formula gives $q(t) = (CE)\left(1 - e^{-t/RC}\right)$.

3) Now, after a long time we open S_1. Then, resetting the clock, at $t = 0$ switch S_2 is closed. Find $q(t)$.

$q_0 = CE$. Obviously, $q_\infty = 0$. Therefore, $q(t) = (CE)e^{-t/RC}$.

Discussion 6-1 Kirchhoff's rules

Consider the following circuit.



1) Apply the junction rule at J.

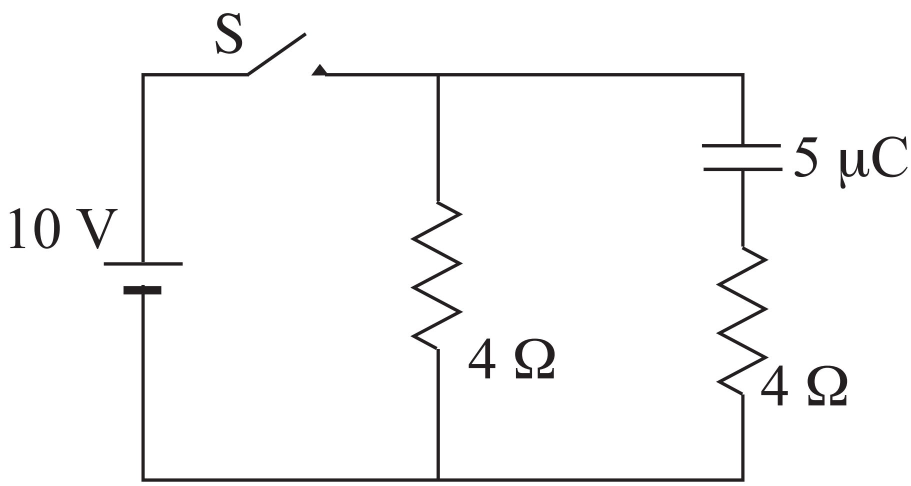
2) Apply the loop rule to loop 1, loop 2 and loop 3.

3) What is the relation among these three loop equations?

4) What is I_4?

Discussion 6-2 RC circuit, time dependence

Initially, there is no charge in the capacitor and the switch is open.



1) At time $t = 0$, switch S is closed. What is the current through switch S immediately after $t = 0$?

2) What is the current through switch S a long time later? What is the charge stored in the capacitor?

3) Now, switch S is opened. What is the current in the circuit immediately after S is opened?

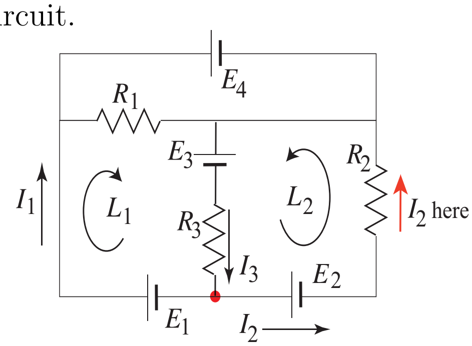
4) How long does it take for the current to fall to $1/e$ of the initial value after switch S is opened?

Physics 102 (Sample Quiz)

Q5s

Name: ______________ **Section:** __________ **Score:** __________/20

1. Consider the following circuit.



(1) What is the relation among I_1, I_2 and I_3? [3]

Apply the junction rule at the red spot:
in: I_3,
out: I_1 nd I_2.
$\Rightarrow I_1 + I_2 = I_3$.

(2) Write down the loop equation for loop L_2 [3].

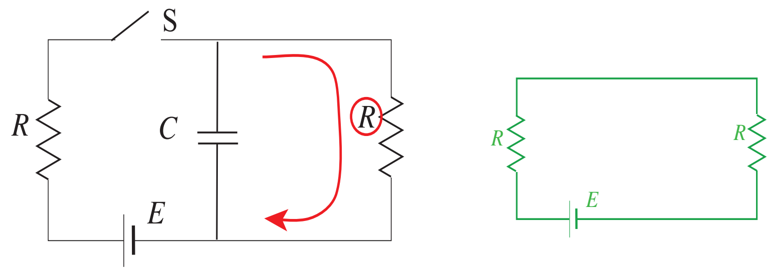
Apply the loop rule along L_2:
down: E_2, I_2R_2, E_3, I_3R_3.
Therefore,

$$-E_2 - I_2R_2 - E_3 - I_3R_3 = 0.$$

(3) Find the current through R_1. Assume all resistors are 2 Ω and all the batteries supply 5 V. [4].

The voltage across R_1 is solely determined by E_4, so the current is $E_4/R_1 = 5/2 = 2.5$ A.

2. The voltage $E = 20$ V, $R = 1$ kΩ and the capacitance $C = 3$ μF. [Hint: Recall the formula in the Key Points: $q(t) = q_\infty + (q_0 - q_\infty)e^{-t/\tau}$).



(1) Switch S has been closed for a long time. What is the charge in the capacitor? [5]

'For a long time' $\Rightarrow$ C is full $\Rightarrow$ no current through C.

All the current is along the red arrow, whose magnitude is $E/2R$. (as illustrated by the equivalent circuit in green).
Therefore, the voltage across marked R is $r \times (E/2r) = E/2$. This must be the same voltage across C.

Therefore, $Q = (E/2)C = 30$ μC.

(2) Now, at $t = 0$ switch S is opened. After 2 ms what is the charge in the capacitor? [5]

Now, at $t = 0$ $q(0) = 30$ μC from the answer to (1).
We can know $q(\infty) = 0$, because all the charge in C would drain through 'marked' R.
$\tau = CR = 3$ ms. (note that k $\times$ μ = milli)

$$q(t) = q(0)e^{-t/\tau} = 30\, e^{-2/3} = 30 \times 0.5134 = 15.4\ \mu\text{C}.$$

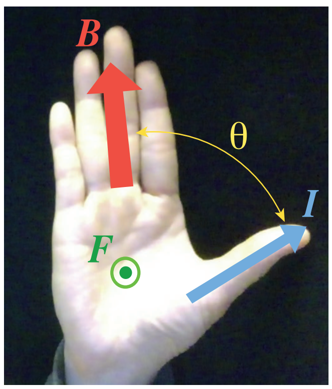
Physics 102 Discussion Week 7 (Lectures 10 and 11)
Magnetic force

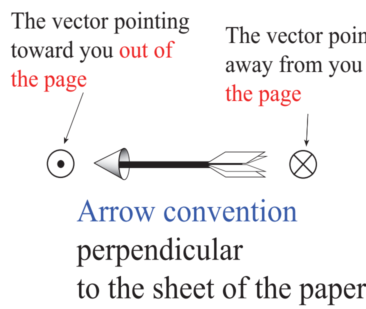
Key Points

Magnetic force

If a charge q moves at a velocity $\boldsymbol{v}$ in a uniform magnetic field $\boldsymbol{B}$, there is a **magnetic force** $\boldsymbol{F}$ on the charge:

Its magnitude is: $F = |q|vB\sin\theta$, where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{B}$.
Its direction is determined by the **right-hand rule** (Figure just below):
The fingers point in the direction of $\boldsymbol{B}$, the magnetic field.
The thumb points in the direction of the current due to the motion of the charge.[9]
The 'pushing direction' by the palm denotes the direction of $\boldsymbol{F}$.

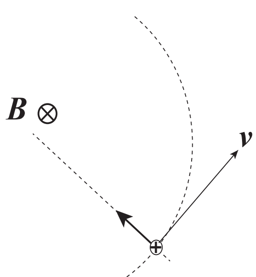




Cyclotron motion

You must remember how to write down the radial equation of motion:
$\boldsymbol{m}$ × **centripetal acceleration = centripetal force**

$$m\frac{v^2}{r} = F \quad\Rightarrow\quad m\frac{v^2}{r} = qvB.$$



Since the magnetic force is always perpendicular to the velocity, magnetic field does not do any work to the particle.
Thus, the kinetic energy of the particle is conserved.

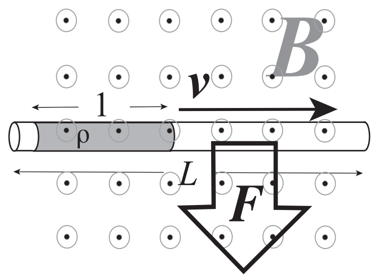
This implies the radius of the circular motion (called cyclotron motion) is given by

$$r = \frac{mv}{qB}.$$

[9]If $q > 0$, it is the direction of $\boldsymbol{v}$, but if $q < 0$, it is the direction of $-\boldsymbol{v}$. HOWEVER, if you use the right-hand rule in terms of the current, you can *always use the same rule*.

Magnetic force on current

Suppose there are movable charges ρ per unit length of a wire, and the movable charges move with the mean velocity v. Then, the current is $I = \rho v$. Therefore (see the figure below), if the magnitude of the magnetic field is B, the magnitude of the magnetic force per unit length of the wire is IB ($\times \sin\theta$ if not perpendicular, where θ is the angle between I and $\boldsymbol{B}$).



Thus, the **magnetic force** acting on the portion of length L of a wire carrying current I is a vector with:

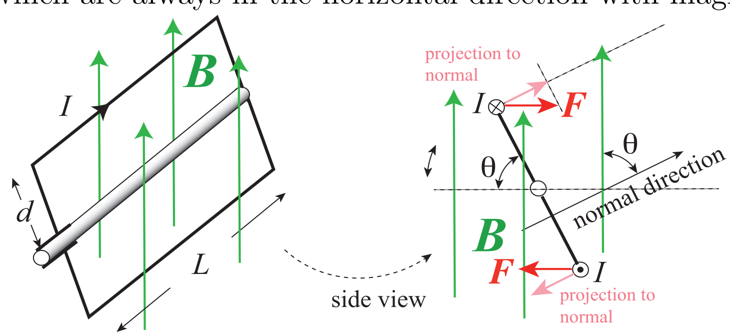
the magnitude given by

$$F = ILB\sin\theta.$$

the direction just specified by the right-hand rule we have discussed.

Torque on a loop current

There is a rectangular loop of wire of size $2d \times L$ that can rotate around the horizontal axle as illustrated below. A uniform magnetic field $\boldsymbol{B}$ is vertical. The magnetic forces $\boldsymbol{F}$ act on the longer edges, which are always in the horizontal direction with magnitude ILB.



The magnitude of the torque is the arm length $\times$ the force component (pink arrows) orthogonally projected onto the normal direction to the plane determined by the arm and the rotation axis, so there is a torque $Fd\sin\theta$ due to the force on one long edge. There are two edges, so the magnitude of the total torque is given by

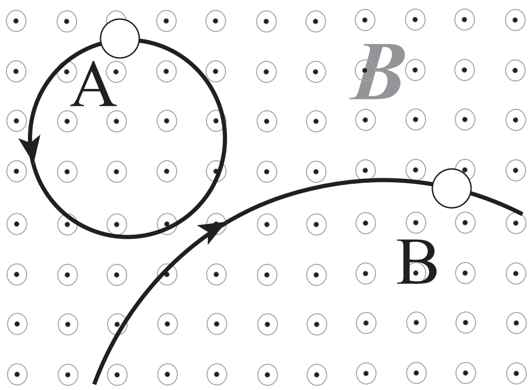
$$\tau = 2Fd\sin\theta = 2dLIB\sin\theta = IAB\sin\theta,$$

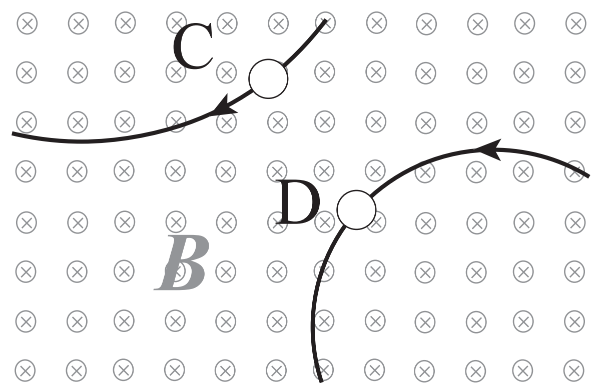
where A is the area of the loop. Next week, we will discuss the magnetic moment due to the current loop: $\mu = IA$ and $\tau = \mu B\sin\theta$. **If the loop consists of N turns** of the wire instead of one, N is multiplied to I as $\tau = NIAB\sin\theta$ (and $\mu = NIA$, etc).

Discussion 7-1 Cyclotron motion

[Qualitative]

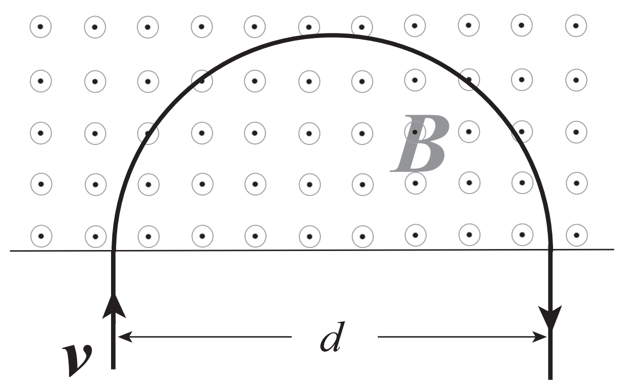
1) Find the signs of charges A - D in the following situations consistent with the cyclotron motion. The arrows denote the moving directions of the charges. The background magnetic fields $\boldsymbol{B}$ are indicated by gray arrow heads/tails.





[Quantitative]

A proton, traveling with speed $v = 3.0 \times 10^6$ m/s, enters a region with a uniform magnetic field of intensity $B = 1.5$ T described below. After completing a semicircular orbit, the proton exits the field.



2) Calculate the distance d.

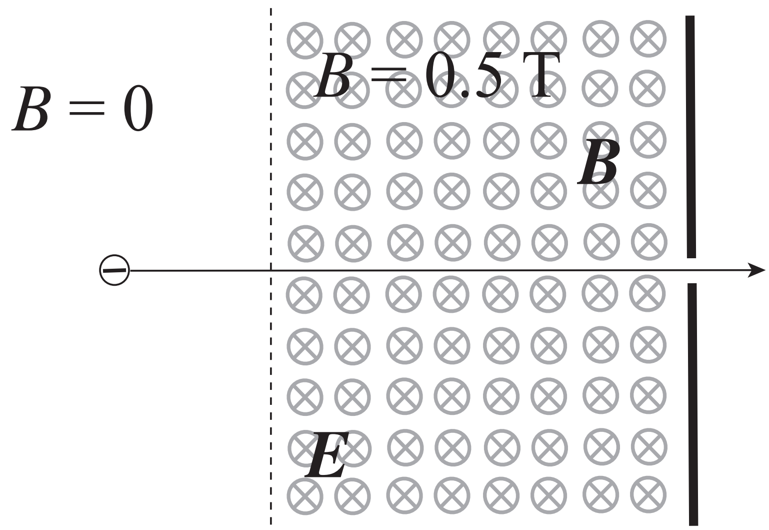
3) How long does it take for the proton to complete this semicircular orbit?

4) Let the answer to 2) be d_0 and that to 3) t_0. Find the following distances d and times t in terms of d_0 and t_0.
(i) For the proton, if the initial speed is doubled, but if the magnetic field is the same, what is the distance d, and the needed time t to complete the semicircular orbit ?

(ii) What is the distance d if the proton is replaced with a deuterium ion (its mass is twice as large as that of a proton)? What is the needed time t to complete the semicircle? Assume that the initial velocity and the magnetic field are the same as the proton case.

Discussion 7-2 The velocity selector

A uniform magnetic field of 0.5 T into the page is applied in the region shown below. A beam of electrons is directed through the region. Without an electric field, the path of the electrons will be a circle.

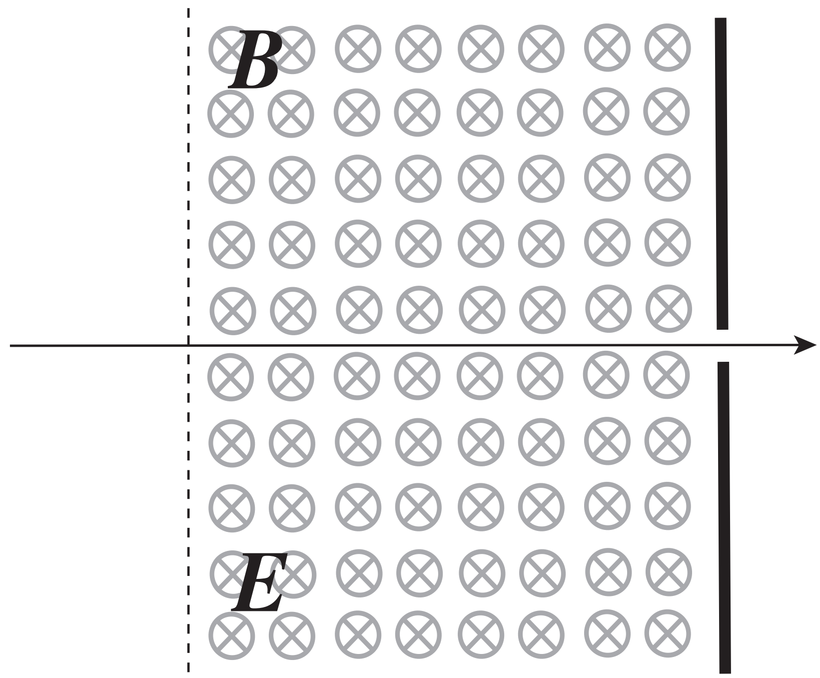


1) What direction will the electron beam bend when only the magnetic field is present? Indicate it in the figure above.

2) In what direction does the electric field need to point for the electron beam to travel along a straight line, as shown? Indicate in the figure above with an arrow marked with E.

3) If we want a beam of electrons with speed $v = 600$ m/s to exit through the slit on the right, how strong should the electric field be?

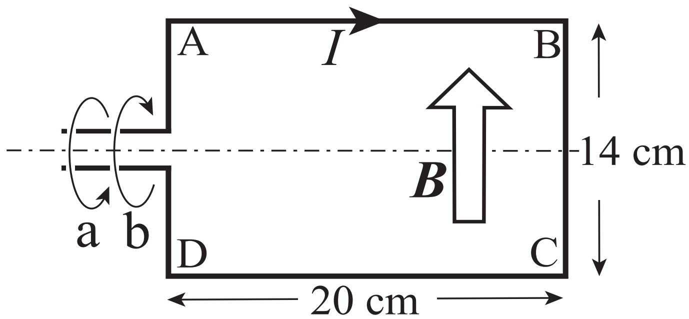
4) What happens if an electron in the beam has a different speed? Sketch the trajectories for the slower and the faster cases in the following figure with $\boldsymbol{E}$ and $\boldsymbol{B}$ being fixed as in 3).



5) How would the answers change if the charge is positive with all other quantities unchanged?

Discussion 7-3 Current loop and torque

A square wire can rotate freely around the center line in the figure. A current $I = 8$ A flows in the direction indicated by the arrow along the wirework. A uniform magnetic field $\boldsymbol{B}$ of magnitude 4 mT is applied in the plane of the sheet of this paper as shown in the figure. Initially, the orientation of the loop is flat on the page.



1) What is the magnitude and direction of the force on wire segment CD?

2) Which way does the loop rotate? Choose a or b.

3) What is the magnitude of the torque on the loop in this position?

4) If the magnetic field were, instead, oriented into the page, would the loop feel a torque?

Physics 102 (Sample Quiz)

Q6s

Name: ____________________ **Section:** __________ **Score:** __________/**20**

1. In the figure below the gray zone has a uniform magnetic field $\boldsymbol{B}$ perpendicular out of the page whose magnitude is 0.12 T. A negatively charged particle is already rotating in the plane of this page as illustrated below.

(a) What is the rotational direction of the charge of this particle, a or b? Justify your choice briefly. [2]

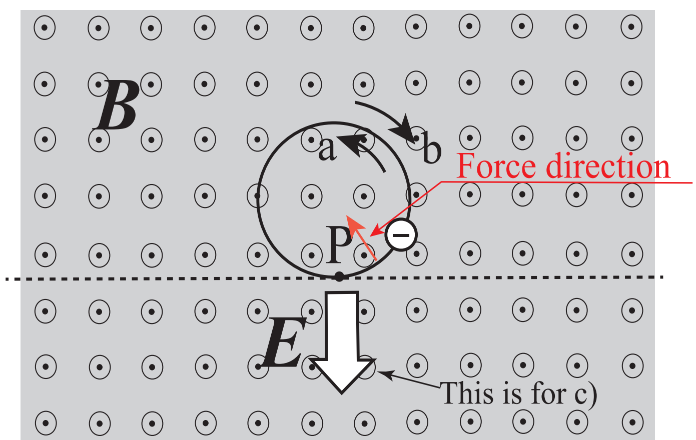
Keys: Rotational motion is due to centripetal force; Right-hand rule

Thus the current direction is b. However, the charge is negative, so it must run in the opposite direction: a.

(b) The radius of the circular motion is 1.3 mm. The particle mass $m = 15 \times 10^{-27}$ kg, and the magnitude of its charge is $e = 1.6 \times 10^{-19}$ C. What is the speed of the particle? [4]

Key: $mv^2/r = qvB \Rightarrow v = rqB/m$

$v = 1.3 \times 10^{-3} \times 1.6 \times 10^{-19}$
$\times 0.12/15 \times 10^{-27}$
$= 0.0166 \times 10^{-3-19+27}$
$= 1.66 \times 10^3 = 1.66$ km/s



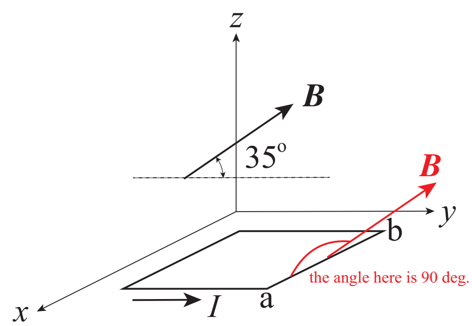
(c) When the particle comes to point P, the electric field $\boldsymbol{E}$ is turned on, which is perpendicular to the tangent line at P (dotted line) and in the sheet of the paper. The magnitude of the electric field is $|\boldsymbol{E}| = 150$ V/m. Can the particle stay above (in the paper) the dotted line? [Yes or no, but you must quantitatively justify your choice.] [4]

Magnetic force = qvB, Electric force = qE, so we have only to compare vB and E

$vB = 1660 \times 0.12 = 199.2$ V/m,
$E = 150$ V/m.
Therefore, the magnetic force wins! $\Rightarrow$ Yes, the particle stays above the dotted line. The particle cannot bend into the domain opposite to the dotted line (at least for a short time according to the above logic. To show this is true for all $t > 0$ is beyond P102).

2. A metal square frame with edge 0.5 m lies in the xy-plane. A uniform magnetic field $\boldsymbol{B}$ of magnitude 2.5 T is parallel to the yz-plane, making an angle 35° with the xy-plane as illustrated below.

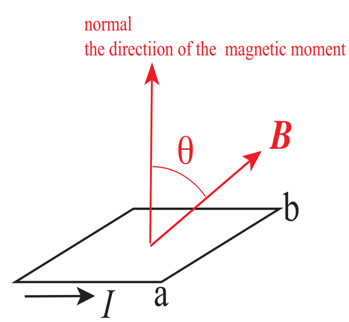
(a) The metal frame carries a permanent current of $I = 2$ A in the direction of the arrow. What is the magnitude of the total force acting on the edge ab, which is parallel to the x-axis? [5]



Right-hand rule:
Magnitude $= ILB\sin\theta$

Notice that in this case $\theta = 90°$.
$F = ILB = 2 \times 0.5 \times 2.5 = 2.5$ N.

(b) What is the magnitude of the torque on the square? [5]



$\tau = IAB\sin\theta$, where θ is the angle between B and the normal.

The angle $\theta = 90° - 35° = 55°$, so
$\tau = IAB\sin\theta = 2 \times 0.5^2 \times 2.5 \times \sin 55 = 1.0239$ N·m.

Physics 102 Discussion Week 8 (Lectures 12 and 13)
Magnetic Field due to Currents, Magnetic Dipoles

Key Points

Magnetic field due to a single straight wire

Empirically, we know the following two basic rules **1** and **2** to understand the magnetic field due to currents through several straight wires.

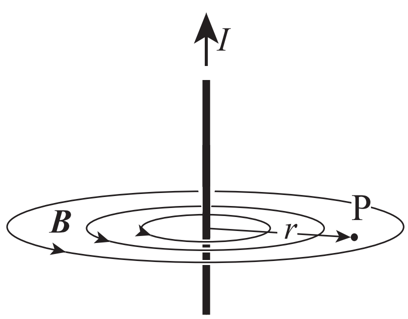
1. Magnetic field due to single wire carrying current

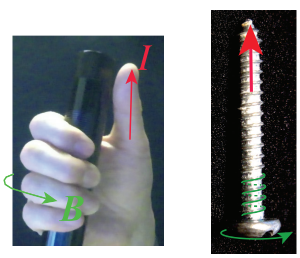
The magnetic field $\boldsymbol{B}$ created by a wire with current I (see Figure below):

its magnitude B at a location P which is distance r away from the wire is give by ($\mu_0 = 4\pi \times 10^{-7}$ T·m/A)

$$B = \frac{\mu_0 I}{2\pi r}.$$

its direction is given by the right-hand(ed) screw rule





right-handed screw rule

2. Superposition principle

If there are more than one wires with current, each wire contributes a magnetic field $\boldsymbol{B}_i$ according to **1**, and the total magnetic field is simply the (vector) sum of all the individual magnetic fields $\boldsymbol{B}_i$:

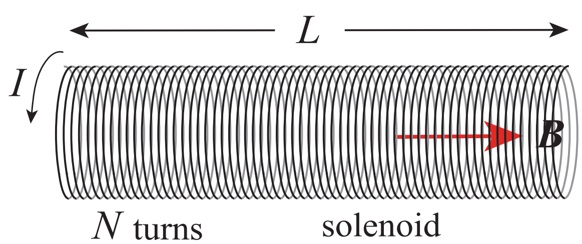
$$\boldsymbol{B} = \sum \boldsymbol{B}_i.$$

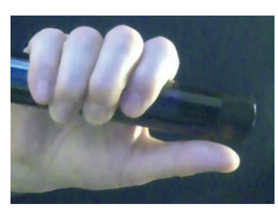
Magnetic field $\boldsymbol{B}$ inside a long solenoid with $\boldsymbol{n}$ turns per meter

its magnitude B inside is uniform and is given by $B = \mu_0 n I$ ($B = 0$ outside).

Here, $n = N/L$.

its direction is given by the right-hand(ed) screw rule.



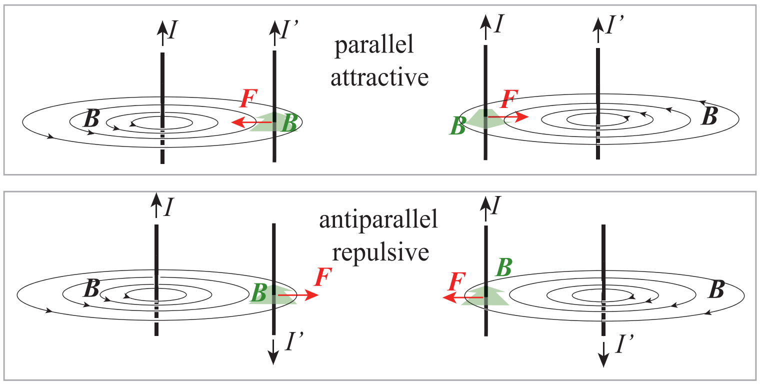


right-handed screw rule

Forces between currents

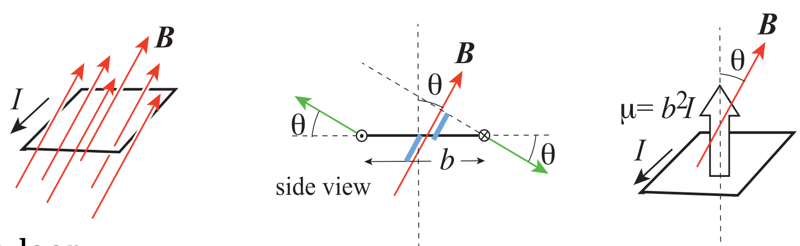
Parallel currents attract each other; antiparallel currents repel each other.

This can be understood by combining the magnetic field due to the current summarized above and the magnetic force on the current discussed in Week 7. (No self-interaction is considered.)



Current loop and magnetic dipole

If we place a small square loop with edge length b carrying a current I in a magnetic field $\boldsymbol{B}$ (see Figure below), it feels a torque as illustrated in Figure below (center).



Torque on the loop

The magnitude of the forces (green arrows) are bIB. The (magnitude of the) torque on the loop due to each force is $bIB \times (b/2)\sin\theta$, where $(b/2)\sin\theta$ is the 'arm length' projected to the plane perpendicular to the force (blue bars). Therefore, the total torque on the loop has a magnitude $b^2IB\sin\theta$. This is equal to $AIB\sin\theta$, where A is the area of the square.

Definition of magnetic dipole moment

We know the magnitude of the torque τ felt by an electric dipole $\boldsymbol{p}$ placed in an electric field $\boldsymbol{E}$ is given by $\tau = pE\sin\theta$.[10] Therefore, we can define a magnetic dipole $\boldsymbol{\mu}$ analogously such that the torque is given by

$$\tau = \mu B \sin\theta.$$

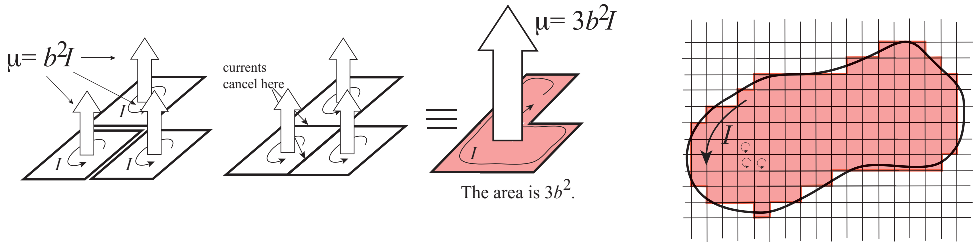
Comparing this with $b^2IB\sin\theta$, we say that a small square in the figure has a magnetic dipole moment with a magnitude given by b^2I with the direction as illustrated in the right figure above. The right-hand(ed) screw rule applies.

General formula of magnetic moment

The following illustration tells us that the magnetic moment of a current loop can be written in terms of the area of the loop *irrespective of its shape* as

$$\mu = AI.$$

[10] If you know the vector product, $\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{E}$.



The above figure illustrates that joining small square loops can approximate any shape if the squares are small enough. When, e.g., three squares carrying the same current are joined, the currents along the joined edges cancel. Only the outermost current survives and the moment vectors are added as illustrated. Thus, the shape of the loop does not matter; only the area matters as can be guessed from the rightmost figure.

N-turn coil

If the loop is a coil with N turns, we may regard it as a stack of N loops, so the general formula for the (magnitude of the) magnetic moment of a coil reads

$$\mu = NAI.$$

Potential energy stored

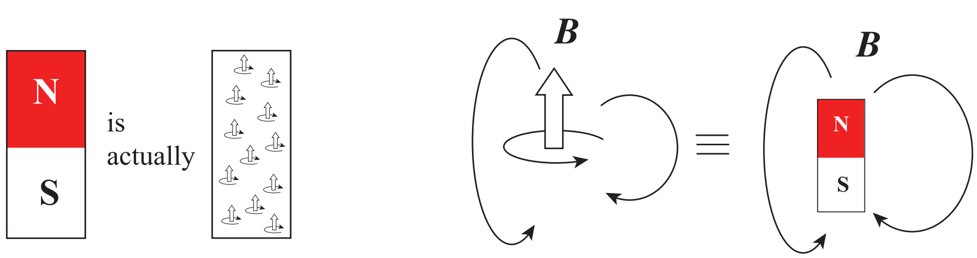
The work we must do to change θ to $\theta + \Delta\theta$ is $bIB \sin\theta \times b\Delta\theta$,[11] so the work you have to do to rotate the dipole from $\theta = 0$ (parallel to the magnetic field) to θ is $-b^2 IB \cos\theta$. That is, the energy of a magnetic dipole in a magnetic field can be written as

$$U = -\mu B \cos\theta,$$

if the energy origin is chosen to be at $\theta = 90°$ (perpendicular to the magnetic field).

Bar magnet is a collection of loop currents

A bar magnet is actually a collection of microscopic loop currents:

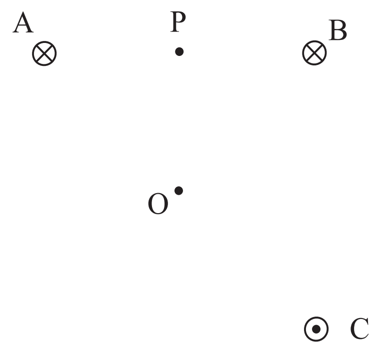


Therefore, we have a macroscopic correspondence between a loop current and bar magnet as illustrated above right.

[11]Notice that the displacement of the point where the force acts upon is always perpendicular to the loop, so the work done by the force to the system is $\Delta W = -bIB \times (b\Delta\theta/2)\cos(90° + \theta) = (b^2/2)IB\Delta\theta \sin\theta$. We have a pair of forces, so the work done to the loop is doubled: $\Delta W = b^2 IB\Delta\theta \sin\theta$. That is, to increase θ by $\Delta\theta$, we must do a work $\Delta W = b^2 IB\Delta\theta \sin\theta$. Summing all these pieces from $\theta = 0$ (the lowest energy state) to θ, we can calculate the energy U stored in the magnetic dipole-magnetic field system as $U = -b^2 IB \cos\theta$, because $\Delta \cos\theta/\Delta\theta = -\sin\theta$.

Discussion 8-1 Magnetic field due to straight wires

Three parallel long wires are perpendicular to the page. They carry currents whose magnitudes are identical and $I = 3$ A, but in various directions as noted in the figure. **ABC** makes an isosceles right triangle and the length of the edge **AB** $= 2$ m.



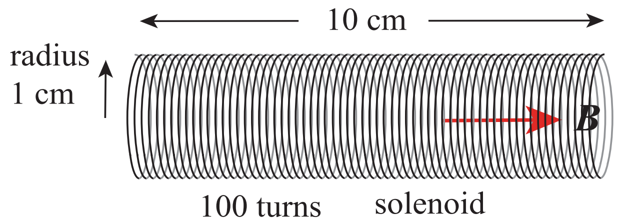
1) What is the magnetic field at the midpoint P of edge **AB**? Draw the directional arrow in the figure and compute its magnitude.

2) Suppose the magnitude of the magnetic field due to **A** at O (the midpoint of edge **AC**) is b. What is the magnitude of the total magnetic field at O?

3) What is the magnetic force acting on wire **B** per 1 m due to the other two? Draw an arrow indicating the force direction in the figure.

Discussion 8-2 Solenoid

A 100 turn solenoid has a radius 1 cm and length 10 cm.



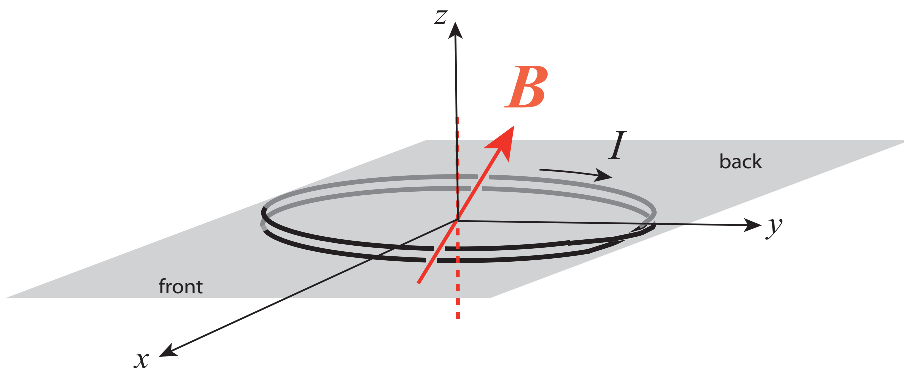
1) What is the magnetic field strength inside the solenoid if a current of 1 A is flowing? First, write a general equation, and then calculate the value.

2) When a magnetic field of intensity B is perpendicular to a cross section of area A, we can imagine something is flowing through the cross section. Recall electric field lines were imagined as flow lines of some fluid whose velocity field is the electric field; this time $\boldsymbol{B}$ is imagined as a velocity field of some fluid. The amount of this fluid is called the *magnetic flux*. If the magnetic field is uniform across the cross section, we say there is a magnetic flux of $\Phi = BA$ through the cross section. Its unit is Wb = T·m^2 (= weber). As we will learn next week Φ or its changing rate is critical to the generation of electricity by using mechanical work.

What is the magnetic flux Φ through the cross section of the solenoid?

Discussion 8-3

A two turn circular coil of radius 0.5 m is on the xy-plane. The wire carries a permanent current of $I = 2.5$ A as shown in the figure below. The magnetic field $\boldsymbol{B}$ whose magnitude is 0.3 T is in the yz-plane and makes an angle of 30° with the z-axis.



1) What is the magnetic moment of this two-turn coil? You must calculate its magnitude and indicate its direction in the figure.

2) What is the magnitude of the torque on the coil? In the figure you must specify the axis of the rotation of the coil due to the torque and indicate which direction the coil turns around the axis of your choice.

3) If you wish to turn the coil 20° from y to z around the x axis, how much work do you have to do? Before any computation you should tell whether you have to do a positive work or not.

Physics 102 (Sample Quiz)

Q7s

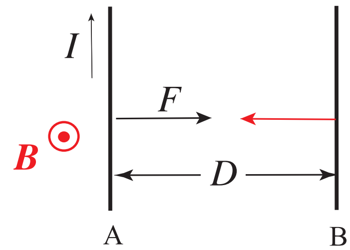
Name: ______________________ **Section:** __________ **Score:** __________/20

1. Two parallel wires are $D = 0.2$ cm apart as illustrated below. The current through wire A is $I = 20$ A, and the force F acting on wire A is 0.2 N/m. There is a constant current through wire B as well. You may assume that there are only these two wires. [Note that **the current in B is not given**.]

(a) What is the force acting on wire B per unit length? Also **show its direction** in the figure with an arrow. [3]

Action-Reaction!

If $-\boldsymbol{F}$ (the red arrow in the figure) were not the answer, you could have a free lunch.



The force must be 0.2 N/m in the opposite direction thanks to the action-reaction principle.

(b) What is the magnetic field at the position of wire A due to the current through wire B? Also **indicate its direction** in the figure with an arrow (or its head/tail symbol $\otimes$ or $\odot$). [3]

Magnetic force: RH rule and $F = ILB\sin\theta$.

$B = F/IL$ (because B must be perp to I in A), so $B = 0.2/20 = 0.01$ T, because $F/L = 0.2$ N/m.

(c) What is the current in wire B? [4]

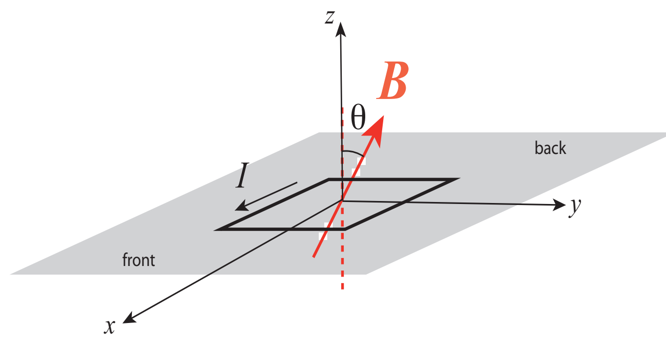
Ampere's law: RH screw rule and $B = \mu_0 I/2\pi r$.

$$I = 2\pi DB/\mu_0$$
$$= 2\pi \times 0.2 \times 10^{-2} \times 0.01/4\pi \times 10^{-7}$$
$$= 2 \times 0.2 \times 10^{-2} \times 0.01/4 \times 10^{-7} = 0.1 \times 10^3 = 100 \text{ A}.$$

2. On the xy-plane is a closed square loop of wire enclosing an area $A = 2.3 \times 10^{-4}$ m^2. The wire carries a permanent current of $I = 12$ A as shown in the figure.

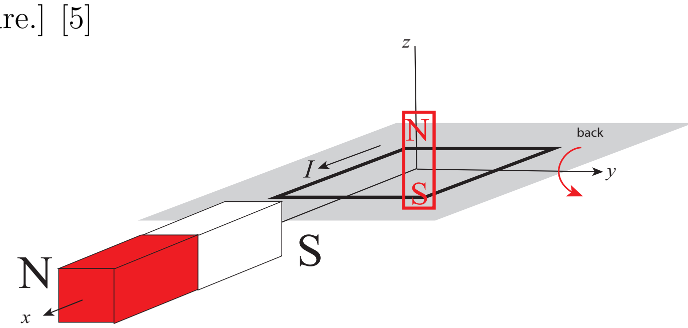
(a) A uniform magnetic field $\boldsymbol{B}$ makes an angle $\theta = 30°$ with the positive z-axis as illustrated twists the loop with a torque of 4×10^{-3} N·m. What is the magnitude of $|\boldsymbol{B}|$ of the magnetic field? [5]

$\tau = NIAB \sin\theta$.



$$
\begin{aligned}
B &= \tau / IA \sin\theta \\
&= 4 \times 10^{-3}/12 \times 2.3 \times 10^{-4} \times 0.5 \\
&= 0.2898 \times 10^{-3+4} = 3.0 \text{ T.}
\end{aligned}
$$

(b) After turning off the magnetic field, a bar magnet parallel to the x-axis is placed on the x-axis as illustrated below while the loop with the same current as in (a) is fixed. Then, the loop is gently released so that it can freely rotate around the origin. What happens to the loop shortly after it is released? [e.g., it starts to rotate around $\cdots$ as shown by the arrow (that you draw) in the figure.] [5]



The easiest way is to replace the loop with a bar magnet and
use your common experience:
The same poles repel; opposite poles attract.

N is attracted to S, so the loop turns around the y-axis as shown in the figure (or positively/counterclockwisely).

Physics 102 Discussion Week 9 (Lecture 13 and 14)

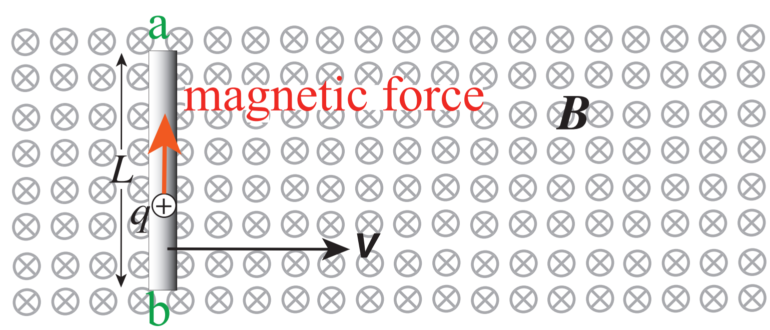
Motional emf, Lenz's law and Faraday's law

Key Points

Motional emf

Motional emf can be understood easily, if you know the magnetic force (p72, Week 7) on a moving charge.

Let us hold a metal bar perpendicularly to a uniform magnetic field as illustrated below, and then move the metal bar in the direction of the arrow with velocity $\boldsymbol{v}$.



(i) Imagine a positive charge q in the bar. The charge would feel a magnetic force of magnitude qvB in the direction indicated by the red arrow. Analogously, a negative charge in the bar feel a force of magnitude $|q|vB$ in the opposite direction.
(ii) Thus, due to the presence of the magnetic field $\boldsymbol{B}$, moving charges with velocity $\boldsymbol{v}$ in the metal bar feel an electric field $\boldsymbol{E}$ whose magnitude is $E = vB$ and whose direction is determined by the right-hand rule p71). Plus charges accumulate at a, so, seen from the outside, the bar end a has a higher voltage compared to end b.

(iii) If the ends of the moving bar are connected to, e.g., a resistor, the charges can move without building up at the ends, and the metal bar acts as an electric battery. Its electromotive force (emf) must be equal to LE, or .

$$\textbf{motional emf} = \boldsymbol{vBL},$$

because the electric field $\boldsymbol{E}$ is present uniformly along the moving metal bar in the magnetic field.

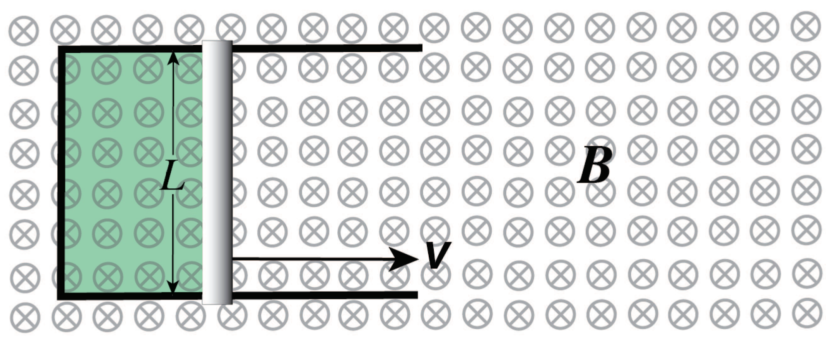
If the sign of the emf were opposite (i.e., if 'b' had a higher voltage than 'a'), what could happen?
(i) Immediately after you start to move the metal bar, there must be an electric current (until charges build up at the ends and polarize the metal bar).
(ii) What force do you expect to act on this current?
(iii) What is the direction to the force the bar feels relative to its velocity? You should realize that eventually horrible things happen. Such things never happen in the benevolent world in which we have evolved. This topic is related to the next topic: Lenz's law.

Lenz's law tells us that our world is stable

We need a new concept (there was a preview in **D8-2**): magnetic flux. Imagine the magnetic vector field $\boldsymbol{B}$ as a flow field (just we did for $\boldsymbol{E}$ when we considered electric force lines in Week 3). Then, the total amount of flow of this fluid through a cross section $A = A \times$ normal component of $\boldsymbol{B}$ is called **magnetic flux** (unit = T·m^2 $\equiv$ Wb (weber)).

Lenz's law tells us that the direction of the current due to motional emf is to reduce the effect of changes (perturbations) applied to the system. Imagine the opposite. What would happen? If you apply a small perturbation to the system, its effect on the system would be amplified indefinitely, implying the system is unstable. Since (intelligent) organisms can evolve only in the stable world, the world we live satisfies Lenz's law.

Imagine a closed circuit made by connecting both ends of the conducting bar with a conducting wire (see Figure below). As the metal bar slides to the right, the green-shaded area increases. Lenz says that the current flows to oppose the increase of the magnetic flux through this shaded area.



Since the magnetic field is constant and uniform, and since the area increases, the current should flow to oppose the increase of the magnetic flux into the page. Then the right-handed screw rule tells us that the current must flow in the counterclockwise direction through the closed circuit. Thus, the current flows upward through the metal bar (as we know).

If you understand the spirit of Lenz's law just explained, even without imagining the magnetic flux through the loop, you can tell which way the current should flow as follows (this is a bit of the repetition of the discussion at the bottom of the preceding page):
(i) If there is a current through the metal bar, we know there is a magnetic force on the bar.
(ii) This force direction must be in the direction to decelerate the metal bar (otherwise, you can make a gun; even if the metal bar does not move initially, it starts to move and then would be accelerated indefinitely, but our world is stable, so such things cannot happen.)
(iii) Thus, if a conducting bar is moving to the right (in the B field into the page), the magnetic force must be to the left. The right-hand rule for the magnetic force tells us that the current must be upward through the metal bar.

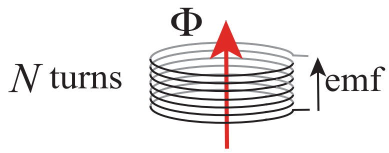
Faraday's law

If a magnetic flux Φ through an N-turn coil (see Figure on the next page) is time-dependent, then there is an electromotive force $\mathcal{E}$ in the coil (Faraday's law):

Its magnitude is given by

$$\mathcal{E} = N\left|\frac{\Delta\Phi}{\Delta t}\right|.$$

Its sense is specified by Lenz's law.



Φ may be changed by changing the magnetic field and/or the loop shapes or directions. The motional electromotive force discussed above may be understood in terms of Faraday's law (as we have already noted).

Transformer

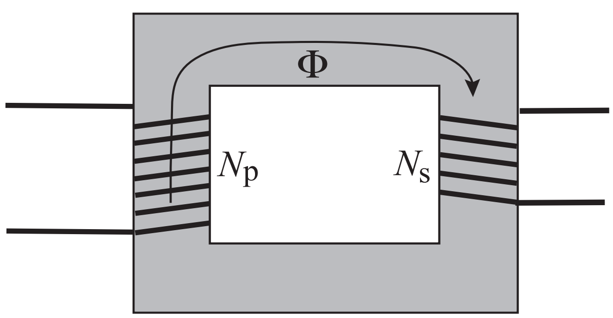
Two key points of (ideal) transformers:
(1) No flux leak (the magnetic flux is totally confined to transformer material) $\Rightarrow$ Φ per single turn is the same for any single turn, irrespective of the
primary side or the secondary side.
(2) Conservation of energy (or power).

(1) and Faraday's law implies

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

(2) implies that $P = IV$ is the same on the primary and the secondary sides. Thus,

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

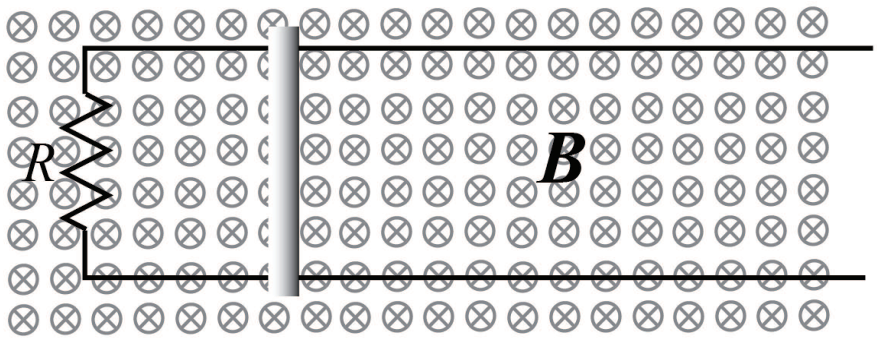


Remark. Root-mean-square values and maximum values:
For a quantity changing sinusoidally in time (as $A\sin\omega t$), its maximum (or peak) value is A, the amplitude itself: $A_{\rm max} = A$. If $|A\sin\omega t|^2$ is averaged over time, the average is equal to $A_{\rm rms}^2$, and $A_{\rm rms} = A/\sqrt{2}$. Therefore,

$$A_{\rm rms} = \frac{A_{\rm max}}{\sqrt{2}}.$$

Discussion 9-1 Bar sliding on parallel wires

There are two parallel wires as shown in the figure, whose left ends are connected to a resistor R. There is a magnetic field $\boldsymbol{B}$ into the page. Along the wires we can slide a conducting rod. You may assume the wires and the rod have only negligible resistance. Thus,



1) If the rod is pulled to the right on the wires, what kind of current I can we expect along the rod, upward or downward in the figure above?

2) Suppose the current is I. What force F (magnitude) do you need to maintain the speed of the rod to be v?

3) The distance between the wires is now known to be L. What is the magnitude of the magnetic field B in terms of I, L, R and v?

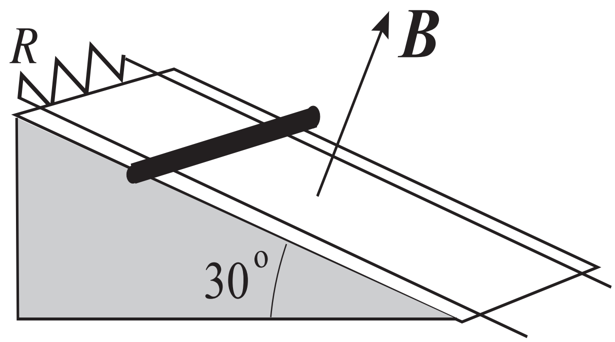
4) Suppose we remove the resistor, but the rod continues to slide on the wires at speed v to the right. What is the voltage difference between the two wires? Which wire is at a higher voltage, the upper or the lower wire?

5) In the original configuration $R = 20\ \Omega$, $L = 0.3$ m and the force $F = 15$ N is required to move the rod at speed 2 m/s. What is the magnitude of the magnetic field B?

6) What is the rate of change of the magnetic flux through the closed loop made by the rod, wires and the resistor? Does it agree with the answer to 4)?

Discussion 9-2 Conducting rod is sliding down a slope

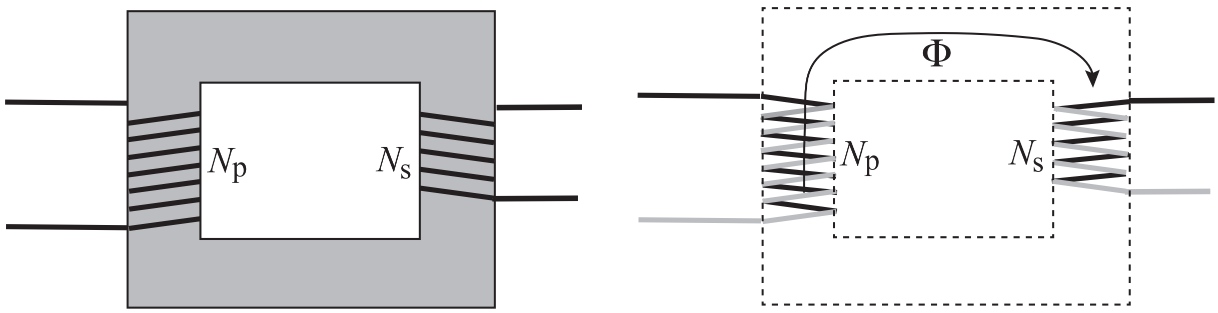
[This is not a very easy question, so you may treat this as an extra question.]
A rod of mass $m = 3$ g is sliding down frictionlessly along a pair of parallel wires on the slope making angle 30° from the horizontal ground. The wires are 0.5 m apart. There is a 1 T magnetic field perpendicular to the slope surface. The wires are connected with a resistor of resistance $R = 2\ \Omega$. Assuming that the slope is sufficiently long, determine the terminal speed. [Hint. Assuming the current I through the rod, write down the force balance and power balance equations.]



Discussion 9-3 Faraday's Law and Transformers

A simple transformer is made of two solenoids sharing the magnetic flux through them.

In the figure Left the primary side of the transformer is a solenoid of $N_p = 100$ turns around an iron core (the gray portion) of cross section 1 cm^2, covering 5 cm length of the core. Since the space inside the solenoid is not a vacuum for the actual transformer, the actual magnetic flux (and magnetic field) in the transformer through the solenoids is different from the case without the core (Right in the figure), but the flux is just proportional to the amount of magnetic flux (and magnetic field) we get when there is no iron core. Therefore, let us consider the solenoid without the core (Right) for simplicity (but assuming there is no flux leak).



1) What is the magnetic flux Φ through the solenoid, if the primary current is $I_p = 15$ A?

2) Suppose the primary current I_p changes as described in the following figure.

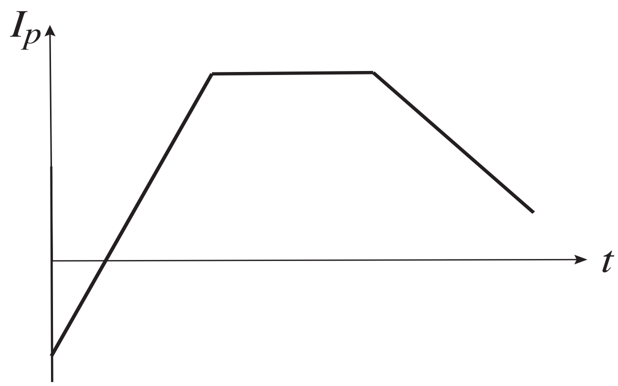


Figure 1:

Qualitatively sketch the magnetic flux Φ through the solenoid on the secondary side, and also the emf that appears in the secondary coil. You may choose your own sign convention.

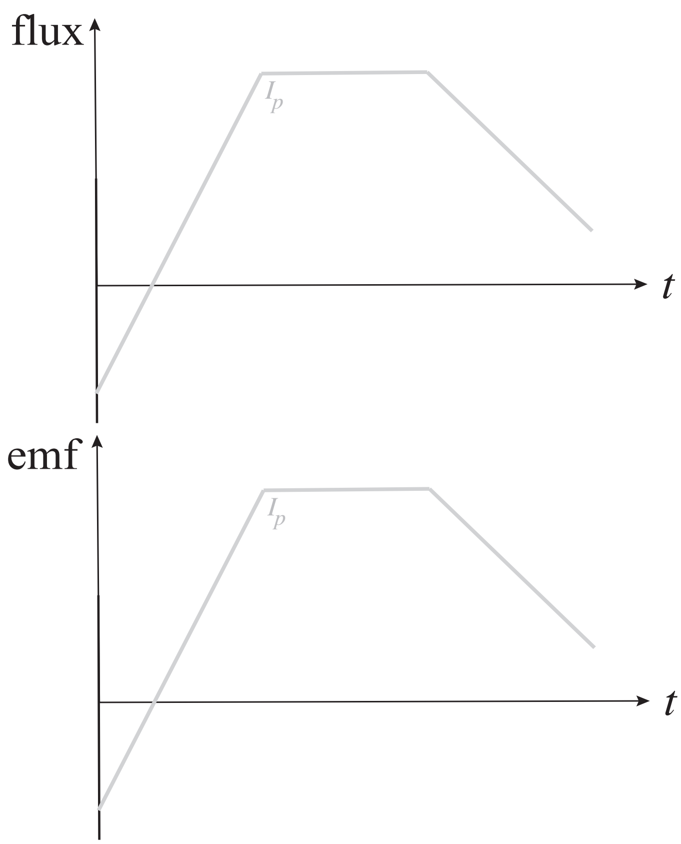


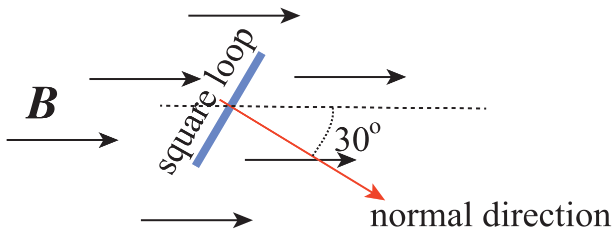
Figure 2:

3) $N_s = 15$ and the current through the primary coil is given by $I_p = 12(1 - 0.5t)$ A, where t is time in seconds. What is the electromotive force at time t in the secondary coil?

4) Now, 60 Hz AC current of power 120 W is supplied to the primary side. The root mean square current on the secondary side is 1.1 A. What is the peak voltage supplied to the primary side?

Discussion 9-4 Field change through a coil

A square loop with side $L = 0.2$ m has its normal direction making 30° with the magnetic field $\boldsymbol{B}$ as illustrated below:



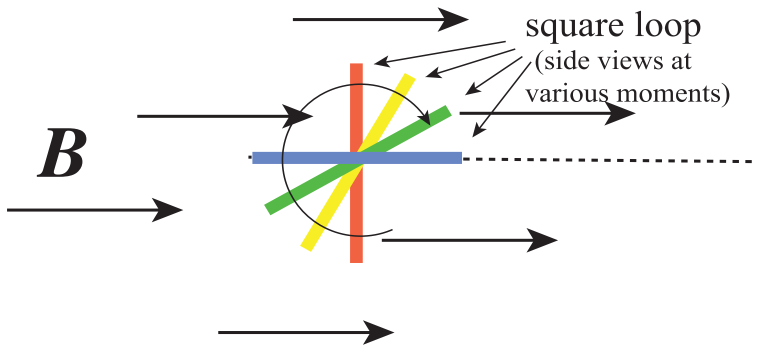
1) The formula for the magnitude of the time-dependent magnetic field is $B = 8 - 0.3t$ T, where t is measured in seconds. What is the magnetic flux through the square loop at $t = 1$ s?

2) Suppose the resistance of the square loop is 12 Ω. Find the magnitude of the current I through the loop as a function of time.

Discussion 9-5 Rotating coil in a magnetic field

Now, the $\boldsymbol{B}$ field is constant ($|\boldsymbol{B}| = 1.5$ T) but the square loop with side $L = 0.2$ m (the same as in **D9-4** above) is rotating at an angular velocity $\omega = 1500$ rad/s. The electromotive force in the loop is sinusoidal.

1) In which angular position of the loop does the magnitude of the electromotive force maximum?



2) The angle between the magnetic field and the normal direction of the loop is given by $\theta = \omega t$. Find the magnetic flux $\Phi(t)$ through the loop as a function of time. Then, using Faraday's law, find the maximum electromotive force along the square loop. Notice that

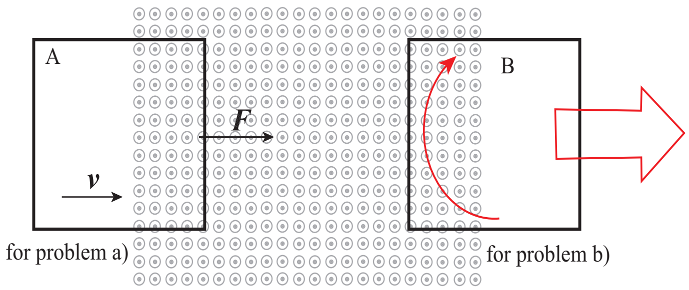
$$\frac{\Delta \cos \omega t}{\Delta t} \simeq -\omega \sin \omega t.$$

Physics 102 (Sample Quiz)

Q8s

Name: ________________ **Section:** ________ **Score:** ________/20

1. In a rectangular domain is a uniform magnetic field $\boldsymbol{B}$ perpendicularly out of the page as illustrated below. A rectangular loop in the figure is a square with edge length $L = 0.3$ m. It is a conductor with the total resistance $R = 12\ \Omega$ and can move frictionlessly in the plane of the sheet of this paper.



a) The square is pulled into the magnetic field with a constant force $\boldsymbol{F}$. It has a constant speed in the situation depicted by A in the above figure. $\boldsymbol{B}$ is constant and $|\boldsymbol{B}| = 5$ T. Find the ratio $|\boldsymbol{F}|/|\boldsymbol{v}|$. [5]

power balance $Fv = V^2/R$
emf $V = LvB$

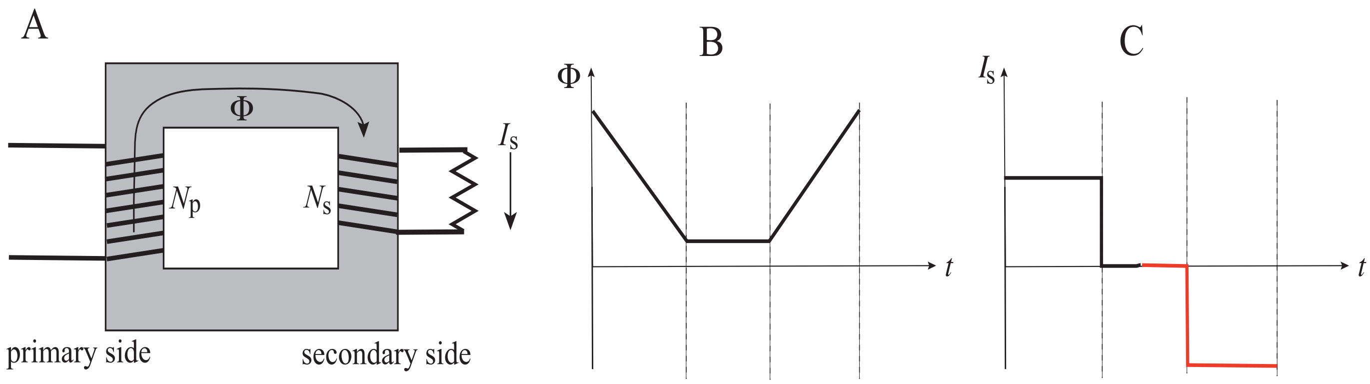
Therefore, $Fv = (LvB)^2/R \Rightarrow F/v = (LB)^2/R$.
That is $F/v = (0.3 \times 5)^2/12 = 0.1875$ N·s/m.

b) After the pulling was stopped, the square moved to the position described in situation B and stopped. Now, you increase the intensity of the magnetic field at a constant rate, maintaining its direction. Then, the square started to move. In which direction does it start to move? Indicate your answer with an arrow in the figure, and justify your arrow briefly. [5]

Lenz tells us that we would have a clockwise current. Upon this current works a magnetic force to the right. Thus, the loop is pushed out.

A more direct argument is: to keep the magnetic flux through the loop, it should get out of the magnetic field.

2. In the figure below **A** describes a(n ideal) transformer.



a) Suppose the magnetic flux Φ in the secondary coil changes as illustrated in **B**. Sketch the secondary current I_s in **C** (i.e., continue the current already drawn in the graph). Briefly justify your choice. [5]

Because $\Delta\Phi/\Delta t$ changes its sign, the current also flips its sign.

b) The primary solenoid of the transformer has $N_p = 300$ turns, and the secondary side $N_s = 120$ turns. A sinusoidal voltage of 240 V (rms voltage) is applied to the primary side. What is the peak value of the voltage on the secondary side? [5]

V/N is constant.
(peak) $= \sqrt{2} \times$ rms.

$240/300 = x/120 \Rightarrow x = 24 \times 4 = 96$ V.
This is rms, so to get the peak value we must multiply $\sqrt{2}$: 136 V.

Physics 102 Discussion Week 10 (Lectures 15, 16 and a part of 17)
Electromagnetic Wave, Polarizer and Plane Mirror.

Key Points

Electromagnetic wave

If a charge undergoes an accelerated motion, **electromagnetic waves** (EM waves) are generated. For example, if a dipole moment oscillates, an EM wave is produced; this is the working principle of many antennas. If a fast moving charged particle is stopped or changes its direction suddenly, very short-wave length EM waves (e.g., γ-rays, X-rays) may be produced.

As shown in the following table, γ-rays, X-rays, (visible) light, microwaves, etc., are examples of electromagnetic waves.

CLASS: γ, HX, SX, EUV, NUV, NIR, MIR, FIR, EHF, SHF, UHF, VHF, HF, MF, LF, VLF, VF/ULF, SLF, ELF

FREQUENCY	WAVELENGTH	ENERGY
300 EHz	1 pm	1.24 MeV
30 EHz	10 pm	124 keV
3 EHz	100 pm	12.4 keV
300 PHz	1 nm	1.24 keV
30 PHz	10 nm	124 eV
3 PHz	100 nm	12.4 eV
300 THz	1 μm	1.24 eV
30 THz	10 μm	124 meV
3 THz	100 μm	12.4 meV
300 GHz	1 mm	1.24 meV
30 GHz	1 cm	124 μeV
3 GHz	1 dm	12.4 μeV
300 MHz	1 m	1.24 μeV
30 MHz	10 m	124 neV
3 MHz	100 m	12.4 neV
300 kHz	1 km	1.24 neV
30 kHz	10 km	124 peV
3 kHz	100 km	12.4 peV
300 Hz	1 Mm	1.24 peV
30 Hz	10 Mm	124 feV
3 Hz	100 Mm	12.4 feV

Legend:
γ = Gamma rays

HX = Hard X-rays
SX = Soft X-Rays

EUV = Extreme-ultraviolet
NUV = Near-ultraviolet

Visible light (colored bands)

NIR = Near-infrared
MIR = Moderate-infrared
FIR = Far-infrared

EHF = Extremely high frequency (microwaves)
SHF = Super-high frequency (microwaves)

UHF = Ultrahigh frequency (radio waves)
VHF = Very high frequency (radio)
HF = High frequency (radio)
MF = Medium frequency (radio)
LF = Low frequency (radio)
VLF = Very low frequency (radio)
VF = Voice frequency
ULF = Ultra-low frequency (radio)
SLF = Super-low frequency (radio)
ELF = Extremely low frequency(radio)

PHz (petahertz) = 10^{15} Hz; EHz (exahertz) = 10^{18} Hz (source: Wikipedia)

Speed of light

All the electromagnetic waves travel at the same speed c in vacuum; c is the speed of light = 3×10^8 m/s. The wave length λ and the frequency f of an electromagnetic wave in vacuum are related as

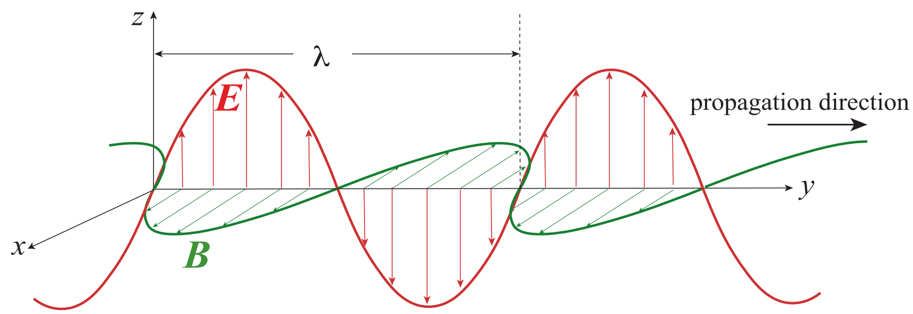
$$c = f\lambda.$$

Speed of light v may be different in different materials: $v = c/n$, where n is called the **index of refraction** ($n \geq 1$). Notice that the **frequency of light is invariant**.

Plane electromagnetic wave

Take a plane electromagnetic wave of wavelength λ propagating in the $+y$ direction (see Figure below). 'Plane' means that at any moment in any plane perpendicular to the propagation direction (y-axis in our case) the electric field $\boldsymbol{E}$ is identical everywhere. The same is true for the magnetic field $\boldsymbol{B}$ as well. That is, for a plane electromagnetic wave, the fields change in unison in any plane perpendicular to its propagation direction.

Therefore, to visualize a plane electromagnetic wave we have only to know the fields along one line pointing in the propagation direction (along the y-axis in our case as illustrated below):



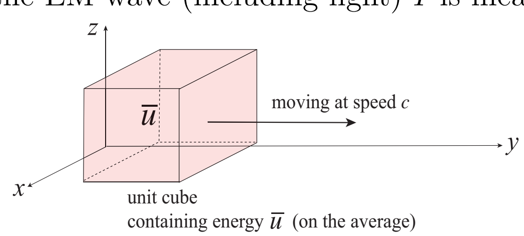
Notice the following points:
(0) EM waves are transverse waves.
(1) $\boldsymbol{E}$ and $\boldsymbol{B}$ fields are in phase.
(2) The propagation direction is determined by the right-handed screw rule: rotating from $\boldsymbol{E}$ to $\boldsymbol{B}$ pushes the "screw" in the propagation direction as in the figure.
(3) $E = cB$, where c is the speed of light (if the wave is propagating through vacuum).
(4) Electromagnetic waves carry energy at the speed of light.

The (time-averaged) energy density of an electromagnetic wave reads

$$\overline{u} = \frac{1}{2}\epsilon_0 E_{\rm rms}^2 + \frac{1}{2\mu_0}B_{\rm rms}^2 = \epsilon_0 E_{\rm rms}^2 = \frac{1}{\mu_0}B_{\rm rms}^2.$$

Note that the amplitudes (max or peak values) and rms values are related as $E_{\rm rms} = E_{\rm max}/\sqrt{2}$ and $B_{\rm rms} = B_{\rm max}/\sqrt{2}$.

Thus (see Figure below) the energy flux S (energy through a unit area per unit time) is $S = c\overline{u}$. The intensity of the EM wave (including light) I is measured by S: $I = S$.



EM radiation from a point source

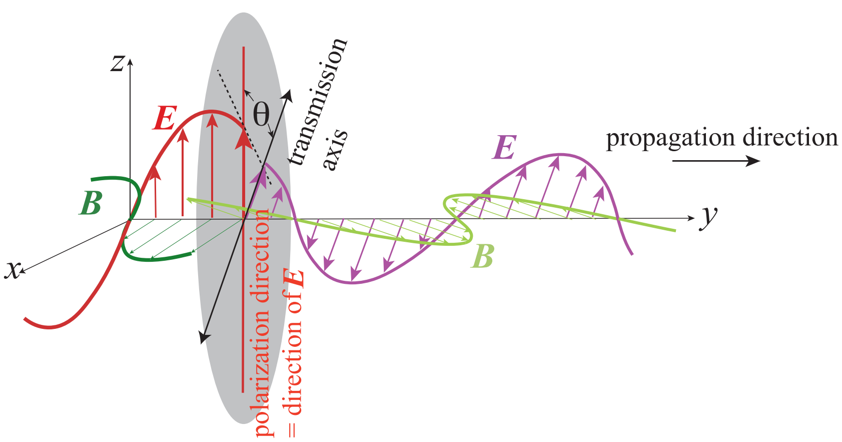
If an EM wave is produced from a point source (say, light bulb, star, etc.), then its flux $S(r)$ (also called intensity I) decays as $1/r^2$, where r is the distance from the source. This is due to energy conservation: $4\pi r^2 S(r)$ must be constant (total power supplied by the source).

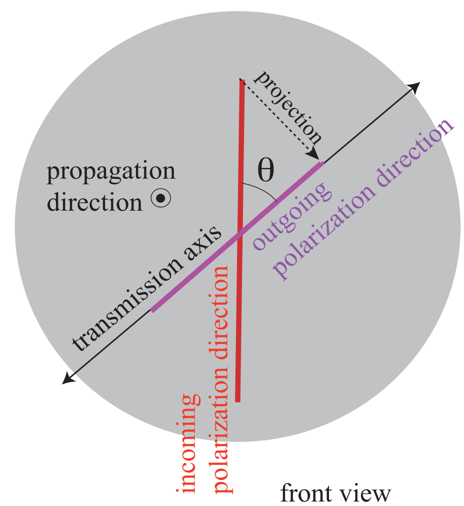
Polarization

An EM wave is a transverse wave of $\boldsymbol{E}$ and $\boldsymbol{B}$, and sometimes its $\boldsymbol{E}$ field may be spatially regular. In such cases we say the EM wave is **polarized**, and its $\boldsymbol{E}$-vector direction (which need not be constant in time) is called the **polarization direction**. If the polarization direction is fixed in one direction, we call the light is **linearly polarized**.

Although lasers usually produce polarized lights, ordinary light bulbs produce unpolarized light. We can produce linearly polarized light from such light by selective absorption. A **linear polarizer** is a device that absorbs the component of the electric field vector perpendicular to its **transmission axis**.

The following figure illustrates what happens when a linearly polarized light ($\boldsymbol{E}$ and $\boldsymbol{B}$) is incident upon a linear polarizer whose transmission axis makes an angle θ with the polarization direction of the incident light. Only the $\boldsymbol{E}$ component parallel to the transmission direction can go through the polarizer. Therefore, an EM wave with $\boldsymbol{E}$ and $\boldsymbol{B}$ leaves the polarizer.





Therefore, the amplitude of the electric field changes from the incoming E_0 to the outgoing $E_0 \cos\theta$ through a linear polarizer. This derives the **law of Malus**, because the light intensity $S = I$ is proportional to the amplitude of the electric field squared:

$$I = I_0 \cos^2\theta.$$

If the incident light is unpolarized, θ in the above formula is random and evenly distributed, so the intensity after passing a linear polarizer would become $I = I_0 \times$average of $\cos^2\theta = I_0/2$.

Geometrical optics

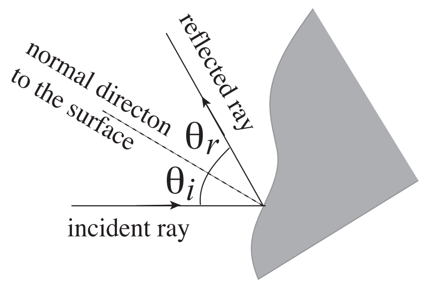
As we will learn more clearly later (p141), if the wavelength of the electromagnetic wave is small enough compared to the systems we are interested in, we may assume the waves propagate along lines (rays). Thus, we can analyze the paths of the wave geometrically. This is geometrical optics.

Law of reflection

The angle θ_i between the incident ray and the normal direction of the reflecting surface (called the **incident angle** or the angle of incidence) and the angle θ_r between the outgoing (reflected) ray and the normal direction of the reflecting surface (called the **reflection angle** or the angle of reflection) are identical:

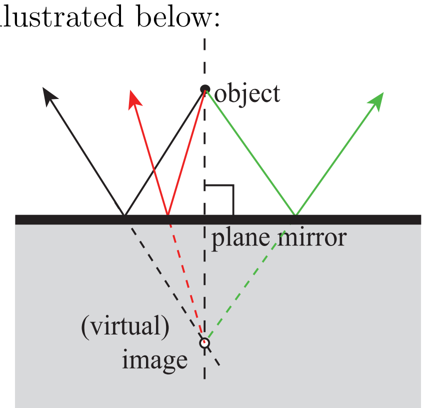
$$\theta_i = \theta_r.$$

Remember that whenever a surface is involved, angles are measured from its normal direction.



Plane mirror

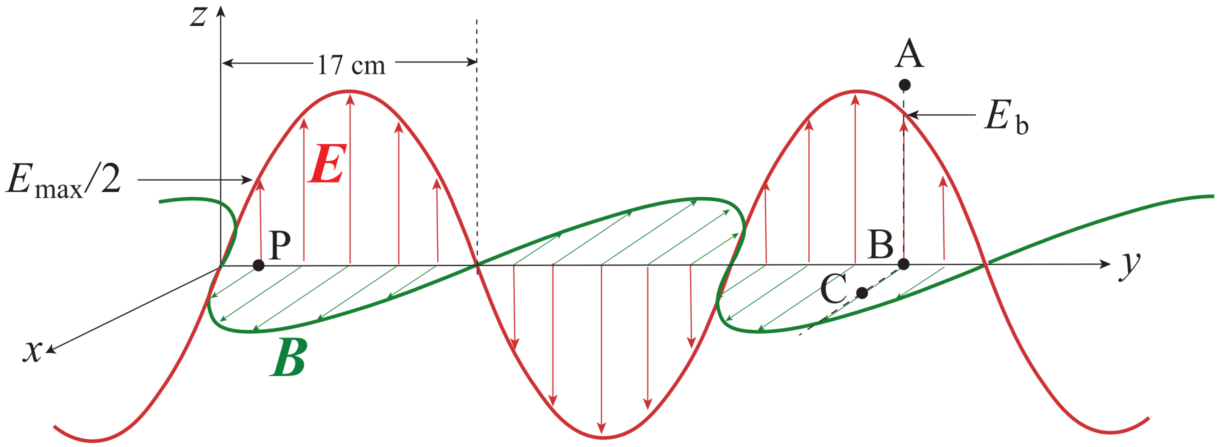
The image of an object in front of a plane mirror (a real object) is located “behind” the mirror as illustrated below:



All the reflected rays appear to originate from a single point behind the mirror: the location of the (virtual) image.

Discussion 10-1 The Plane Electromagnetic Wave

Consider the electromagnetic wave shown in the diagram below. The wave is propagating through a vacuum. Along the y-axis the electric field of the wave is parallel to the z-axis and is a sine wave of amplitude 3.3 V/m. The accompanying magnetic field is parallel to the x-axis. See Figure (which is a snapshot at a particular instant).



1) In which direction is this wave propagating?

2) Is this wave polarized? If so, in what direction is it polarized?

3) What is the frequency of this electromagnetic wave?

4) What are the intensities of the electric field at point A, B and C? (note: this is only a question about the convention to illustrate plane waves.)

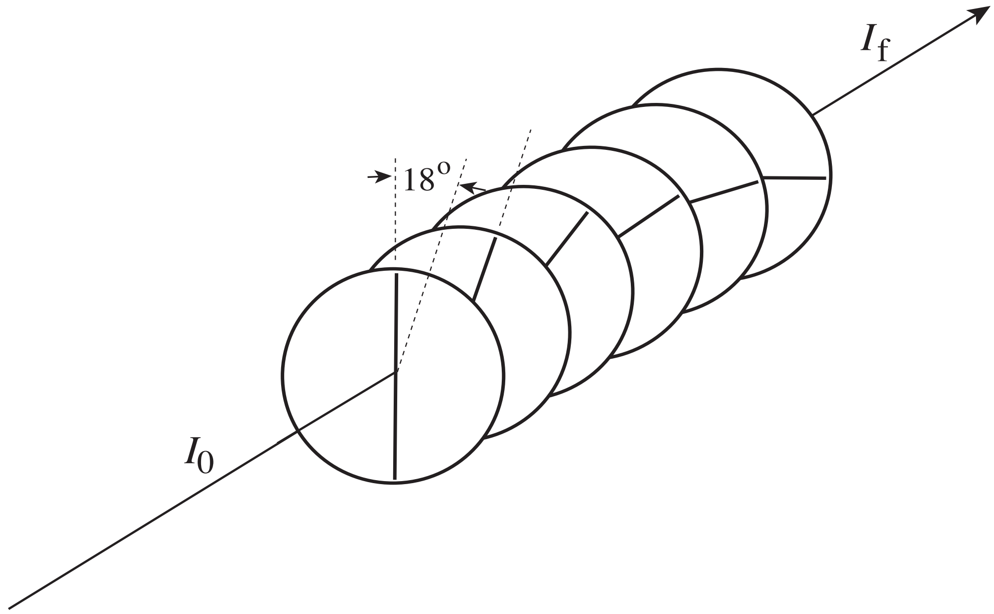
5) What is the intensity of the magnetic field at P, where the magnitude of the electric field is $E_{max}/2$?

6) What is the amount of energy going through a square with edge length 2 m perpendicular to the y-axis in 1 hr?

7) This wave goes through a slab made of a material with the index of refraction $n = 2.1$. What are the wavelength, the propagation speed and the frequency of this wave in the slab?

Discussion 10-2 Polarizer

Unpolarized light, of intensity I_0, is incident on a series of 6 linear polarizers, each with its transmission axis tilted by the same amount relative to the previous polarizer, $\theta = 18°$, as illustrated below.



1) What is the intensity of the light after the first polarizer in terms of I_0?

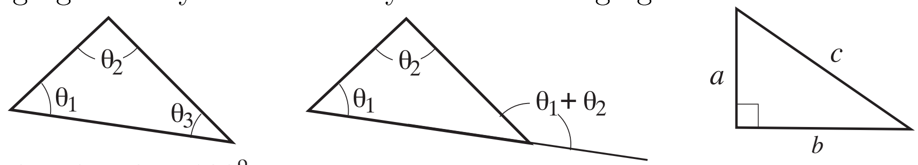
2) What is the intensity of the light after the 2nd polarizer in terms of I_0?

3) What is the final intensity I_f in terms of I_0?

4) If the angle between the transmission axes of adjacent polarizers is halved ($\theta = 9°$) and if the number of the polarizers is doubled (or, more precisely, is increased from 6 to 11), what happens to the final intensity? No quantitative calculation is needed. Guess your answer and produce some justifying argument for your guess.

Elementary geometry of triangles:

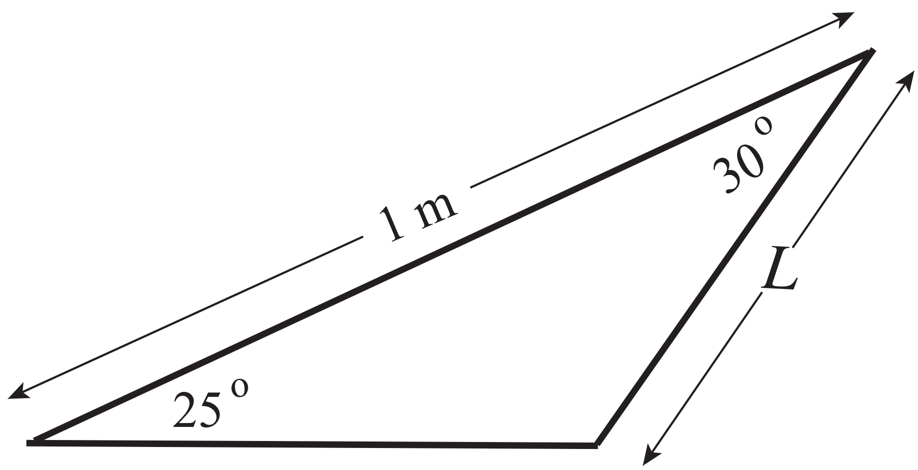
The following figures may refresh what you learned long ago:



$\theta_1 + \theta_2 + \theta_3 = 180^\circ$ $\qquad$ $a^2 + b^2 = c^2$

Discussion 10-3 Elementary geometry

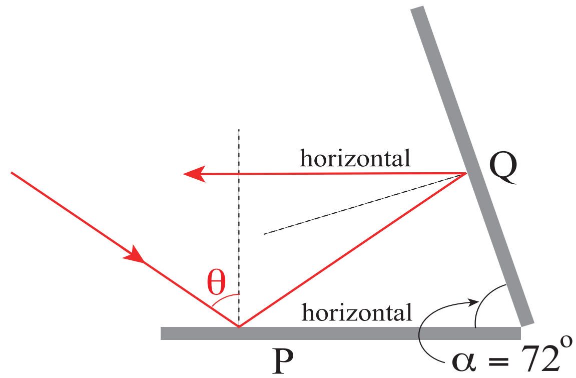
1) Find the length L in the following figure.



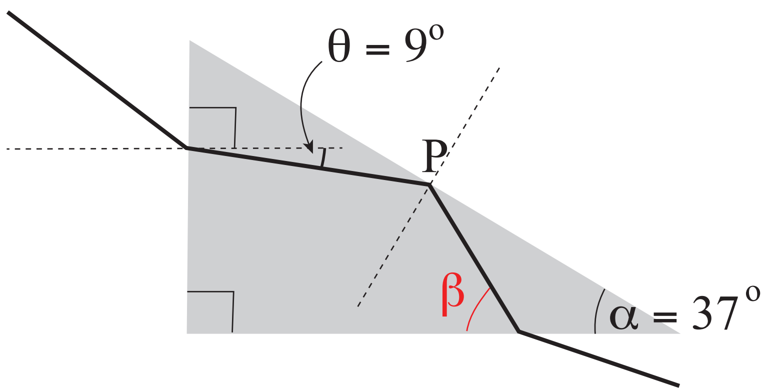
2) What is the area of this triangle?

Discussion 10-4 Reflections

(1) Two flat mirrors are joined at one end at an angle $\alpha = 72°$ as shown in the figure. The bottom mirror is horizontal. A beam of light hits the horizontal mirror at an incidence angle θ and reflects off of both mirror surfaces at P and Q, emerging horizontally from the two mirrors. What is the angle θ?

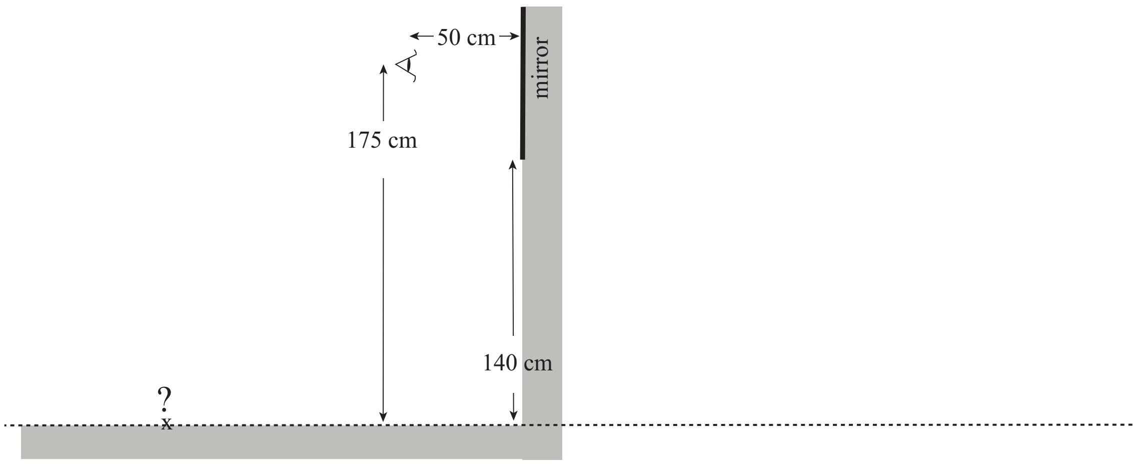


(2) Inside an orthogonal triangular prism shown in the figure (internal) reflection occurs at point P. The angle $\theta = 9°$ and $\alpha = 37°$. What is the angle β?



Discussion 10-5 Plane mirror

You (represented by your right eye in the figure) stand on a horizontal floor and the height of your eyes is 175 cm. 50 cm away from you is a vertical wall on which is a large vertical plane mirror whose lower edge is 140 cm from the floor. What is the closest point ('×') on the floor to the wall you can see without moving your head?



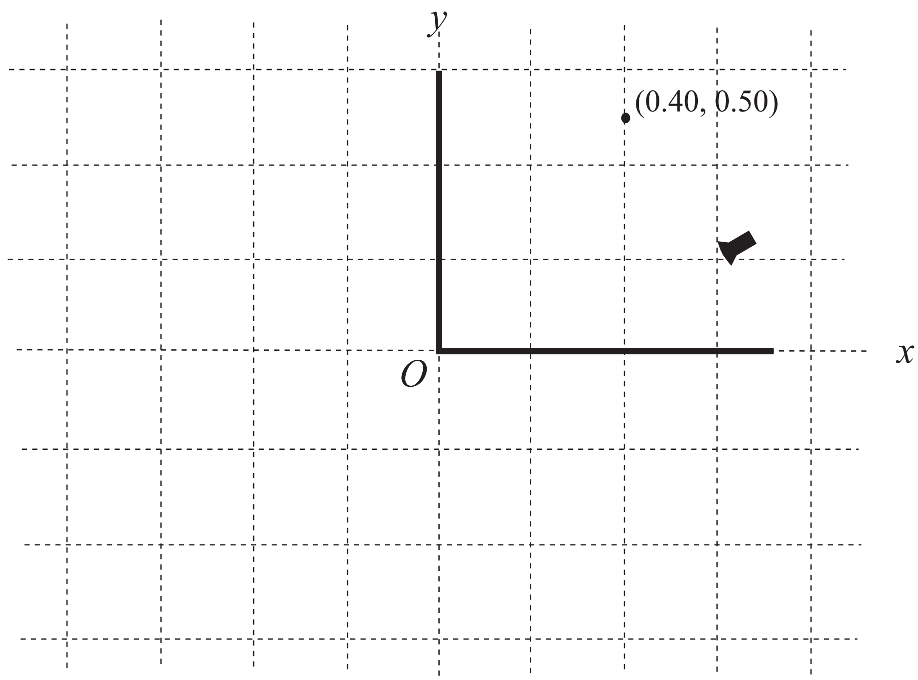
1) There may be several ways to solve this problem. One way is to locate the image of your right eye by the mirror. Locate it in the figure above, and also give its height and the distance from the wall. Is it a real or a virtual image?

2) Then, imagine that you are at the image and from there peek into the room through the mirror. Where is the closest point on the floor you can see?

3) Or, you can look into the lowest edge of the mirror. If the light ray comes from that edge into your eyes, where must the ray before reflection have started? This also answers the question.

Discussion 10-6 Two orthogonal mirrors

Two plane mirrors are connected vertically as in the figure. One mirror is in the xz-plane and the other in the yz-plane (the z-axis is out of the page toward you). A small object is at $(0.4, 0.5)$ (m).



1) How many images are made by these mirrors? Mark the image positions in the above figure and then write down the xy-coordinates of these images.

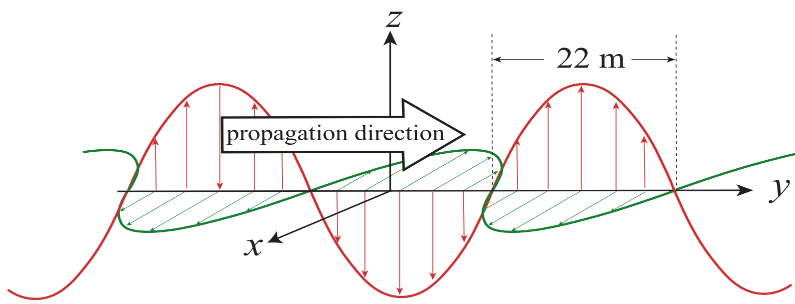
2) Now, you wish to use a flashlight at $(0.6, 0.2)$ (m) to shine the object (indirectly), pointing the flash light downward as in the figure. What is the x-coordinate of the location on the bottom mirror you must shine? [There are two possibilities, but answer the one with two reflections or the point closer to the vertical mirror.]

Physics 102 (Sample Quiz)

Q9s

Name: ____________________ **Section:** __________ **Score:** __________/20

1. A plane electromagnetic wave in a vacuum propagating in the $+y$-direction is illustrated below [5].



(1) Is the propagating field parallel to the ***z*-axis** the magnetic field of this electromagnetic wave, Yes or No? [2]

$\boldsymbol{E}$ to $\boldsymbol{B}$ RH screw!

Therefore, $\boldsymbol{E}$ is parallel to z, NO.

(2) The peak value of the electric field of this wave is 1.2 V/m. How much energy can this wave on the average send in 12 s through an area of 5 m^2 perpendicular to the y-axis? [5]

$I = c\overline{u}$,
$\overline{u} = (1/2)\epsilon_0 E_{\rm max}^2$,
total energy $= IAt$

$$\text{total energy} = c \times (1/2)\epsilon_0 E^2 At = (1/2)3 \times 10^8 \times 8.85 \times 10^{-12} \times 1.2^2 \times 5 \times 12$$
$$= (1/2) \times 3 \times 8.85 \times 1.2^2 \times 5 \times 12 \times 10^{8-12}$$
$$= 1147 \times 10^{-4} = 0.1147 \text{ J}.$$

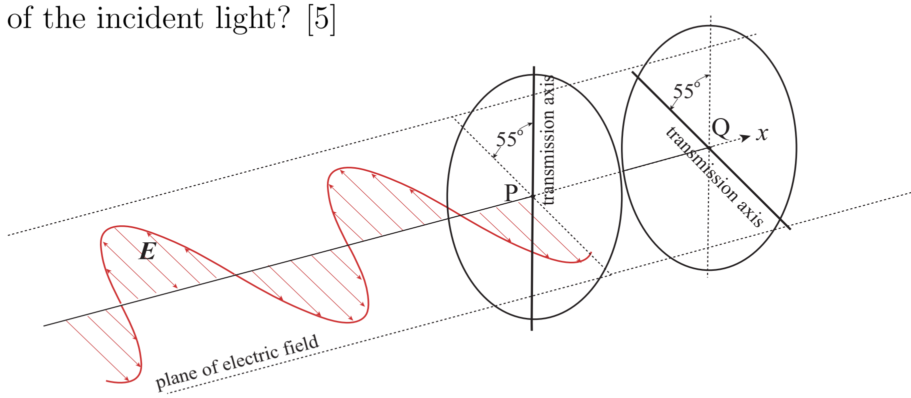
(3) This wave goes into a medium with the index of refraction $n = 1.4$. What is its wavelength and frequency in the medium? [3]

$c = \lambda \times f$
$c \to c/n$, but f is invariant.

$\lambda = 2 \times 22 = 44$ m in vacuum.
$f = c/\lambda = 3 \times 10^8/44 = 300 \times 10^5/44 = 6.82 \times 10^5$
$= 682$ kHz. This is the same in the medium. λ in the medium $= 44/1.4 = 31.4$ m.

2. A plane electromagnetic wave is incident on a polarizer as depicted in the figure. The 'plane of electric field' indicates the plane in which the electric field of the wave along the x-axis lies. It makes an angle of 55° with the transmission axis of the polarizer at P. The whole system is in a vacuum.

What is the intensity I of the light beyond the second polarizer at Q in terms of the intensity I_0 of the incident light? [5]



The plane of $\boldsymbol{E}$ is the plane of polarization.

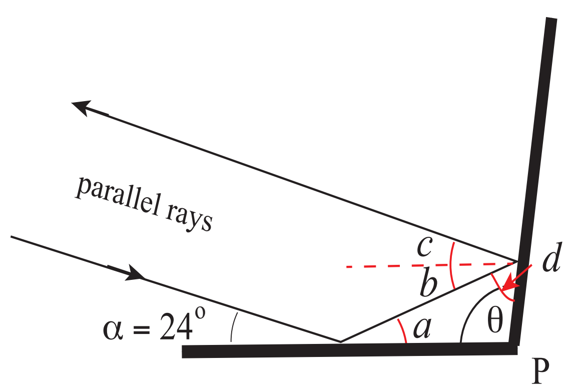
The law of Malus: $I = I_0 \cos^2\theta$.

Beyond the first polarizer, the plane of polarization makes an angle 55° with the polarizer at Q.

$$I = I_0(\cos 55°)^4 = 0.108 I_0.$$

3. Two plane mirrors (indicated by the two thick line segments in the figure) are joined at P. A ray coming from the left is reflected twice by the mirrors in this sheet of paper and leaves in the same direction as it came. What is the angle θ between the two plane mirrors? [5]

Let the red broken line be parallel to the bottom mirror.
Then, $a = b$. $\alpha = a$ due to the law of reflection.
Also $\alpha = c$. Thus, $b = c$.



Therefore,
the broken red line must b perpendicular to the upright mirror due to the law of reflection.
Thus $b + d = a + d = 90° = \theta$.

Physics 102 Discussion Week 11 (Lectures 17-19)
Refraction, Mirrors and Lenses

Key Points

Ray optics

Short wavelength electromagnetic waves propagate as if they consist of particles moving straight, so tracing (piecewise) straight rays, we can analyze many optical devices. This is geometrical optics (also called **ray optics**). Rays travel in straight lines in the uniform space or materials. At the boundary of different optical media **reflection** and/or **refraction** occurs (see below).

Optical reciprocity

"If I can see you, you can see me."

If a ray can go from A to B (with any reflections, refractions, etc.) in space, then a ray can reach from B to A retracing the ray from A to B.

Refraction

Speed of light v may be different in different materials (as noted last week): $v = c/n$, where n is called the **index of refraction** ($n \geq 1$).

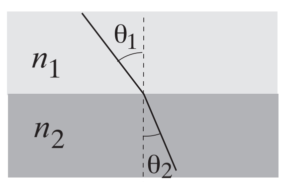
Notice that the **frequency of light is invariant**, so $\lambda_n = \lambda/n$, where λ_n is the wavelength in a material with the index of refraction n.

Snell's law

The angle of incidence θ_1 and the angle of refraction θ_2 are related as (see Figure below)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \ .$$

The angles are measured from the normal direction of the interface.



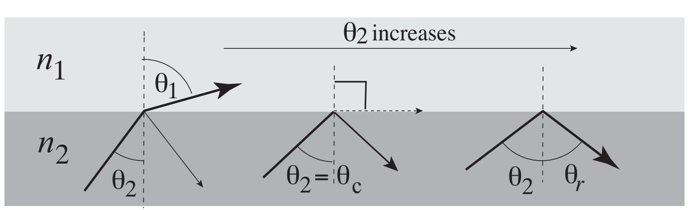
Thanks to optical reciprocity, incident and reflective rays may be switched with the same Snell's law. Also, do not forget that at any interface there is reflection, albeit faint.

Total internal reflection

Suppose $n_1 < n_2$ and a ray is incident from the '2' side. Since $\theta_1 > \theta_2$, $\theta_1 = 90°$ occurs for $\theta_2 = \theta_c < 90°$. This angle θ_c is called the **critical angle** (for this interface). Clearly,

$$\sin\theta_c = n_1/n_2.$$

If θ_2 is further increased, no light comes out of medium '2' (see Figure below). This phenomenon is called **total internal reflection**.



critical angle total internal reflection

Ray optics of thin lenses and mirrors

Some terminologies of ray optics may be summarized as follows:

Object: the source from which light rays appear to originate.

real object: an object from which actual rays (light particles) are coming.

virtual object: an imaginary object to which coming rays extrapolated forward converge, but from which no actual rays are coming.

Image: the point where rays apparently converge.

real image: the point where actually light rays converge.

virtual image: an imaginary point at which coming rays extrapolated backward converge, but at which no actual rays arrive.

Principal axis (or optical axis): line through the center of the optical system (usually the line around which the system is symmetric).

Tracing rays (see **D11-3∼11-8**)

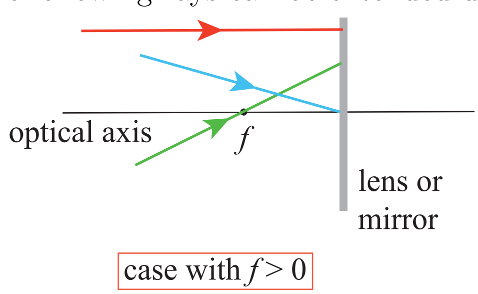
You can unambiguously draw the following rays only (see the following figures; colors are corresponding):

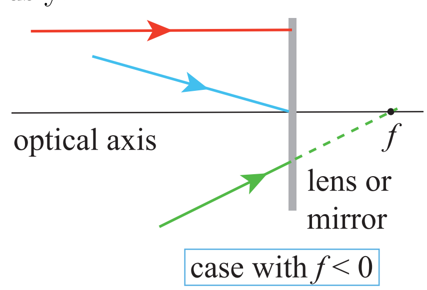
(1) Rays incident parallel to the optical axis

(2) Rays going through focal points

(3) Rays going through the center of the lenses or mirrors.

Only the following rays can be extended unambiguously:[12]





Lens formula

$$\frac{1}{f} = \frac{1}{d_\mathrm{o}} + \frac{1}{d_\mathrm{i}}.$$

The (signed) **magnification** is give by

$$m = -\frac{d_\mathrm{i}}{d_\mathrm{o}}.$$

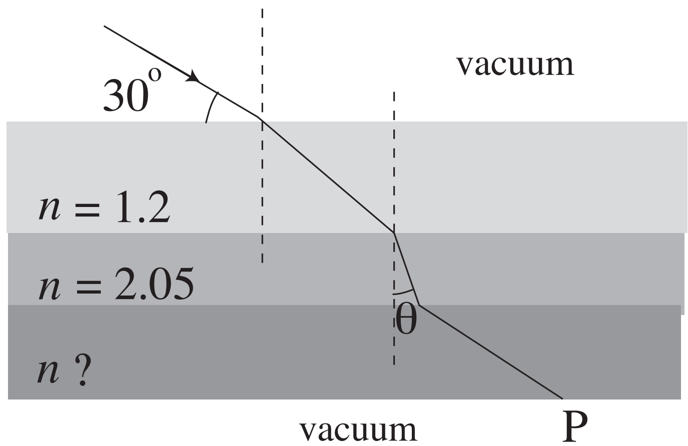
Here, the symbols with their sign conventions and interpretations are as follows:

		+	−
f	focal length	can actually gather light energy	cannot gather light energy
d_o	object distance	real object (actually rays come from it)	imaginary object (rays extrapolated forward converge)
d_i	image distance	real image (actually light energy gathers)	imaginary image (rays extrapolated backward converges)
m	magnification	upright	inverted

[12]**For mirrors, the rays going through their centers of curvature retrace themselves.**

Discussion 11-1 Parallel layers with distinct refractive indices

There are three horizontal flat layers of transparent materials with different indices of refraction. Light is incident from vacuum as indicated in the figure. You may assume all the interfaces are strictly parallel.

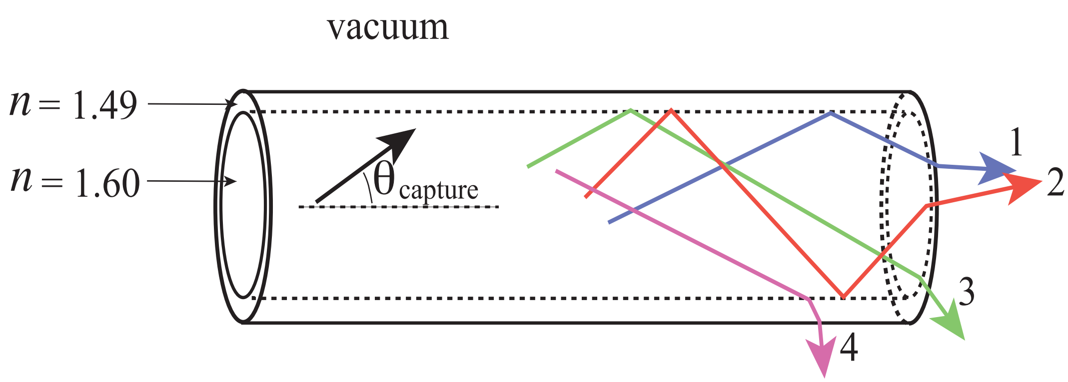


1) What is the angle θ? What happens to the direction of the ray in the third (= bottom) layer if the second layer is removed?

2) Can you choose the index of refraction of the third layer so the ray is internally reflected at P?

Discussion 11-2 Fiber optic cable

A fiber optic cable is illustrated below. The core is made of polystyrene with index of refraction $n = 1.60$, and the cladding (outer layer) is made of poly(methyl methacrylate) (acrylic glass) with $n = 1.49$. The fiber is surrounded by air ($\simeq$ vacuum).



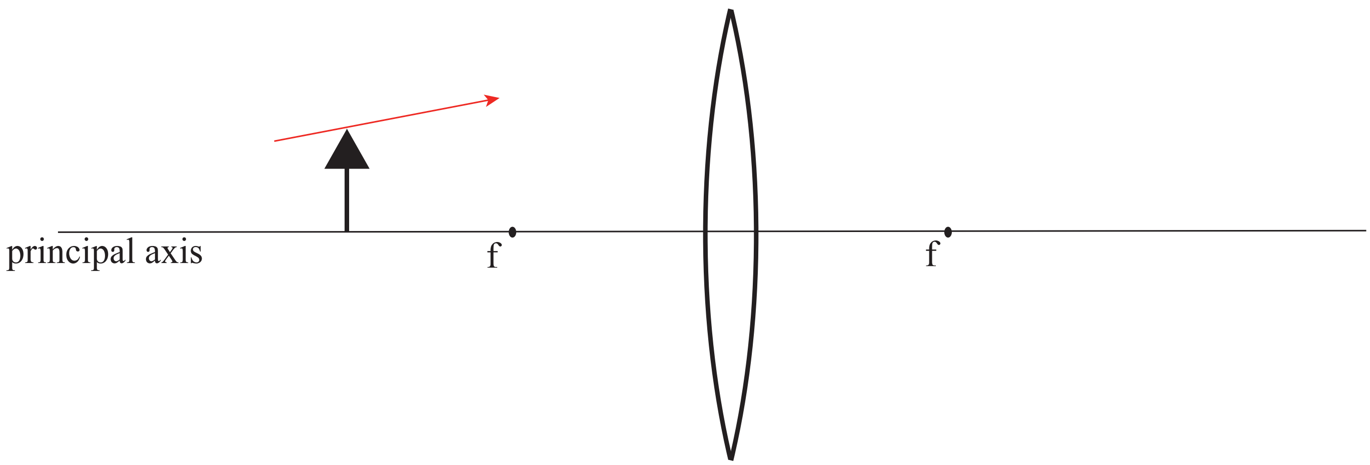
1) There are 4 rays drawn in the above figure. Which is realizable? You must be able to state your reason for your answer.

2) What is the largest capture angle possible?

In the following algebraically identical lens and mirror problems are paired (11-3/11-4, 11-5/11-6, 11-7/11-8).

Discussion 11-3 Convex lens I

In front of a convex lens with focal length 4 cm is an object of height 2cm. It is 7 cm away from the lens.



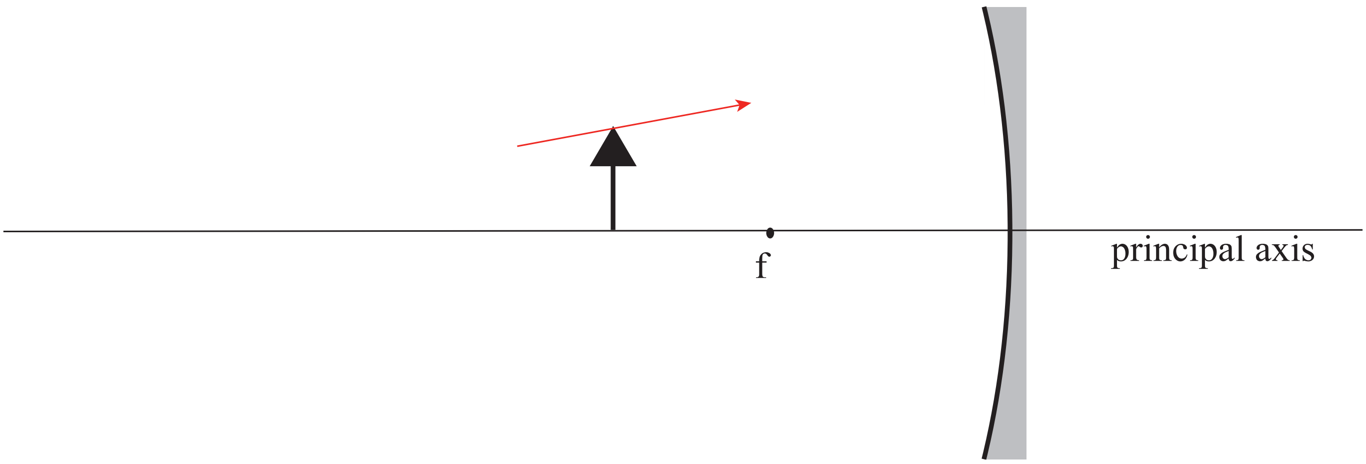
1) Find the location of the image graphically.

2) Continue the red ray beyond the lens.

3) Find the location and the size of the image numerically. Then, describe the characteristics of the image (i.e., whether real/virtual, upright/inverted).

Discussion 11-4 Concave mirror I

In front of a concave mirror with focal length 4 cm is an object of height 2cm. It is 7 cm away from the mirror.



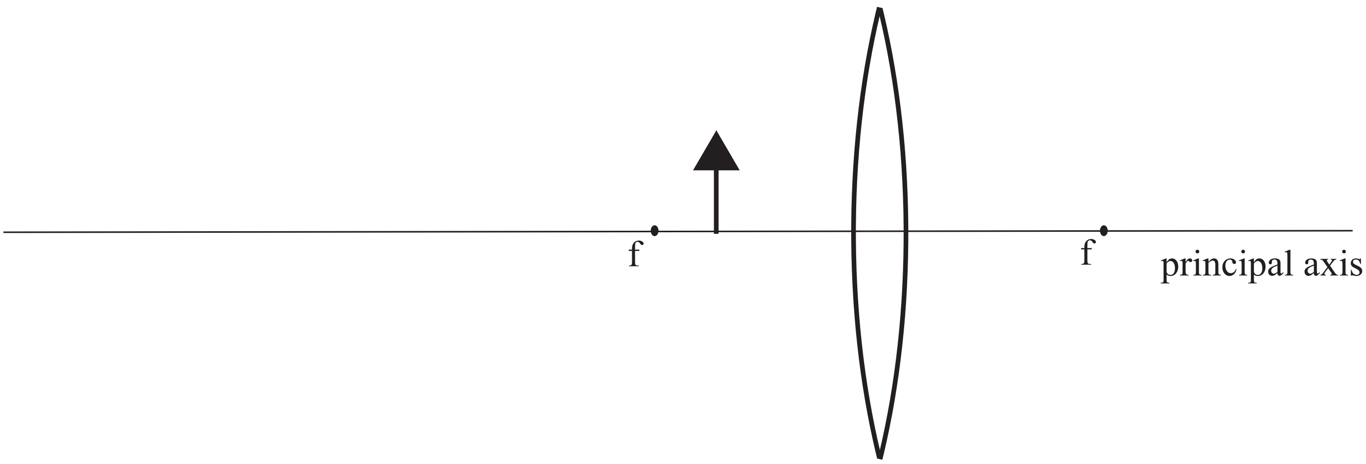
1) Find the location of the image graphically.

2) Continue the red ray after reflected by the mirror.

3) Find the location and the size of the image numerically. Then, describe the characteristics of the image (real/virtual, upright/inverted).

Discussion 11-5 Convex Lens II

In front of a convex lens with focal length 4 cm is an object of height 2cm. It is 3 cm away from the lens.

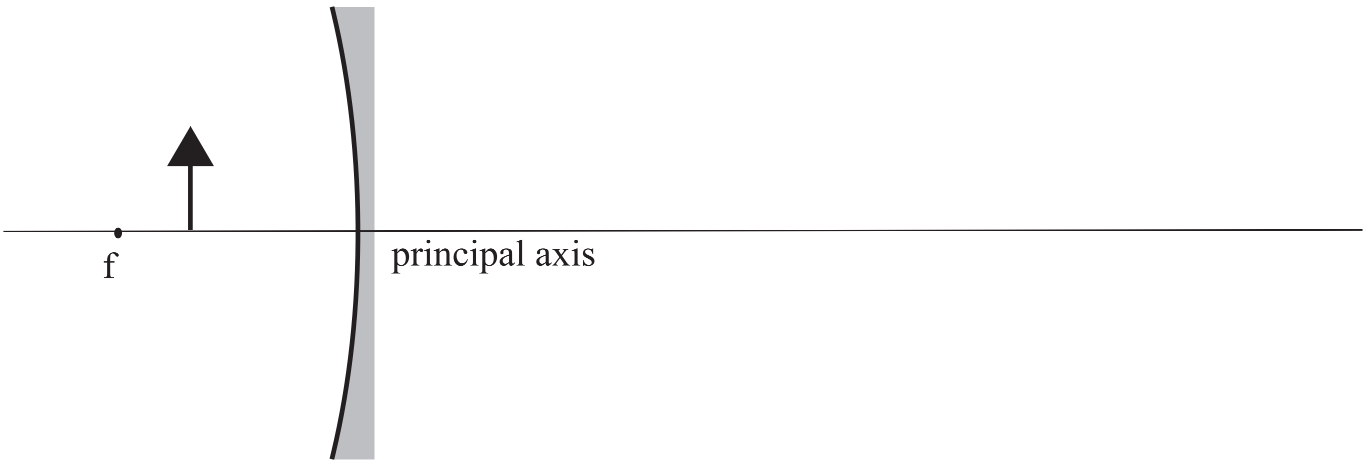


1) Find the location of the image graphically.

2) Find the location and the size of the image numerically. Then, describe the characteristics of the image (real/virtual, upright/inverted).

Discussion 11-6 Concave Mirror II

In front of a concave mirror with focal length 4 cm is an object of height 2cm. It is 3 cm away from the mirror.

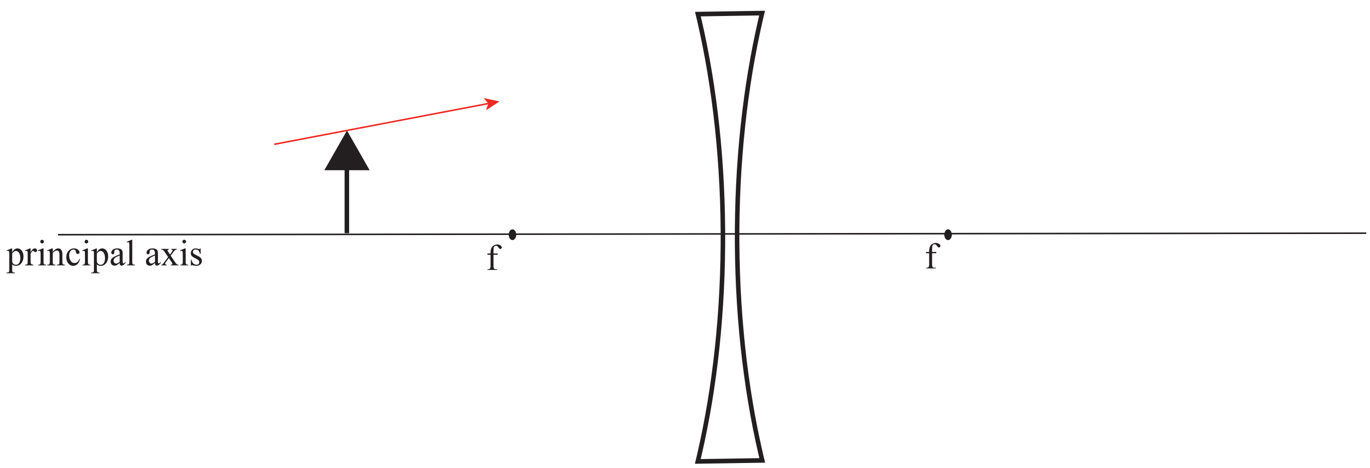


1) Find the location of the image graphically.

2) Find the location and the size of the image numerically. Then, describe the characteristics of the image (real/virtual, upright/inverted).

Discussion 11-7 Concave Lens

In front of a concave lens with focal length -4 cm is an object of height 2cm. It is 7 cm away from the lens.



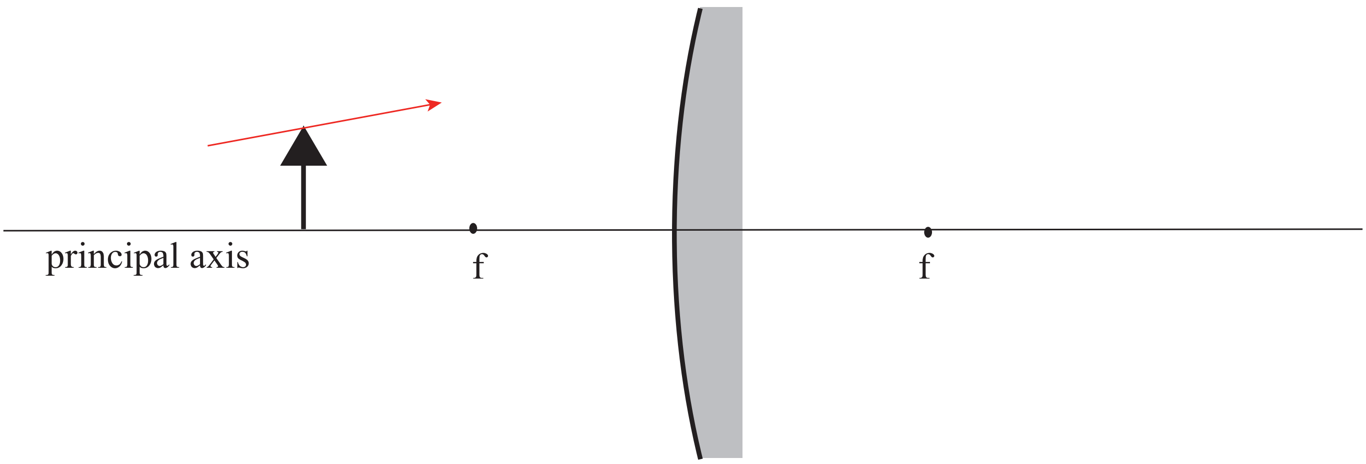
1) Find the location of the image graphically.

2) Continue the red ray after reflected by the mirror.

3) Find the location and the size of the image numerically. Then, describe the characteristics of the image (real/virtual, upright/inverted).

Discussion 11-8 Convex mirror

In front of a convex mirror with focal length −4 cm is an object of height 2cm. It is 7 cm away from the mirror.



1) Find the location of the image graphically.

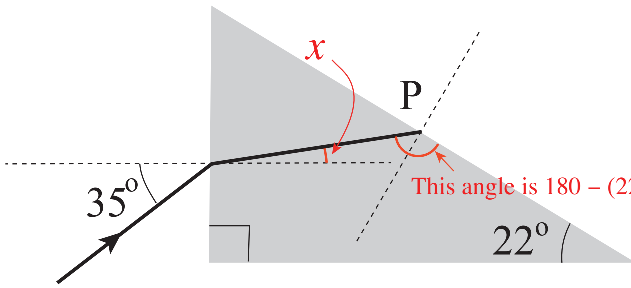
2) Continue the red ray beyond the lens.

3) Find the location and the size of the image numerically. Then, describe the characteristics of the image (real/virtual, upright/inverted).

Physics 102 (Sample Quiz) **Q10s**

Name: ____________________ **Section:** __________ **Score:** __________/**20**

1. Surrounded by air is a triangular prism as sketched in the figure, which is with index of refraction $n = 1.53$. Light of wavelength 585 nm is incident as illustrated in the figure (dotted lines denote normal directions of respective surfaces).



(1) What is the frequency of the light in the prism? [5]

Recall $f = c/\lambda$.

$$f = 3 \times 10^8/(585 \times 10^{-9}) = 0.005128 \times 10^{8+9} = 5.128 \times 10^{14} = 512.8 \text{ THz}$$

(2) Can the light go out from P into air? [5]

Recall: Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Review **D10-3** and **D10-4** for triangles.

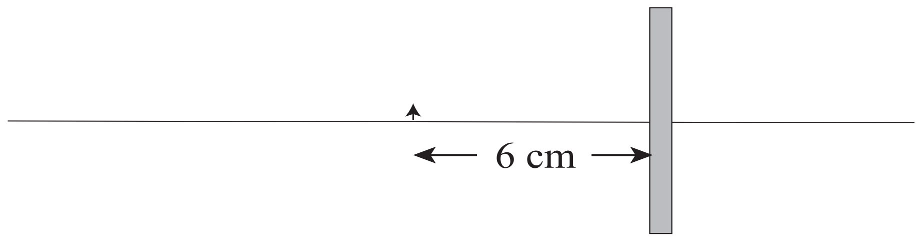
Snell tells us that

$$\sin 35° = 1.53 \sin x \Rightarrow x = 22.33°.$$

Notice that the incidence angle at P is $90 - (22 + 22.3) = 45.7°$.

$1.53 \sin 45.7° = 1.095$, but this (i.e., > 1) is impossible, so there must be total internal reflection at P.

2. 6 cm in front of a lens is a real object of height 2.5 mm whose image is formed 8 cm away from the lens.



(1) What is the (absolute) size of the image? [5]

Recall $m = -d_{\mathrm{i}}/d_{\mathrm{o}}$.

$|m| = 8/6 = 4/3$, so the image size is $2.5 \times (4/3) = 3.33$ mm.

(2) The image is actually upright. What is the focal length of the lens? Is it converging or diverging? [5]

Recall $1/f = 1/d_{\mathrm{o}} + 1/d_{\mathrm{i}}$

The image is upright $\Rightarrow m > 0 \Rightarrow d_{\mathrm{i}} = -8$ cm.

$$1/f = -1/8 + 1/6 = (4-3)/24 = 1/24.$$

That is, $f = 24$ cm, which is positive, so it is a converging (convex) lens.

Physics 102 Discussion Week 12 (Lectures 20 and 21)
Corrective Vision and Optical Instruments

Key Points

Some terminologies:

Accommodation: the process by which the eye changes the focal length of its lens.

Far point: the farthest distance the (uncorrected) eye can see clearly.

Near point: the closest distance on which the (uncorrected) eye can focus.

Farsighted (hyperopic): the condition which occurs when the relaxed eye produces an image behind the retina.

Nearsighted (myopic): the condition which occurs when the focal length of the relaxed eye produces an image in front of the retina.

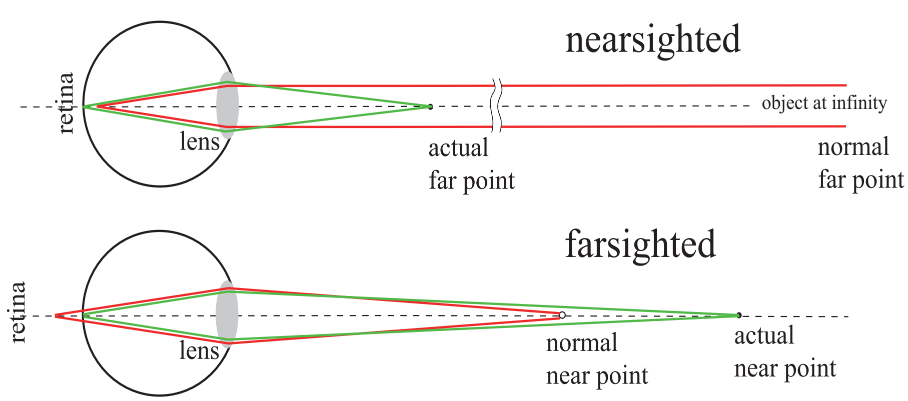
Refractive power: a measure of a lens' ability to focus, measured in D (diopter): $1/f$ with f being measured in m.

Human eye, nearsighted and farsighted

To see an object, the eye must make its real image on the retina. The eye varies the focal length f of its lens by **accommodation**. For a normal eye, the far point is at infinity (>10 m). Rays from a far-away object arrive at the eye parallelly to the principal (optical) axis. The relaxed eye bends these rays to form a real image on the retina.

A **nearsighted eye** with a stronger-than-normal refractive power makes the real image of a far object in front of the retina (see Figure below; red); thus, without correction it can see clearly only as far as the actual far point (green).

A **farsighted eye** with a weaker-than-normal refractive power makes the real image of a near object behind the retina (red); thus, without correction it can see clearly only as near as the actual near point (green).



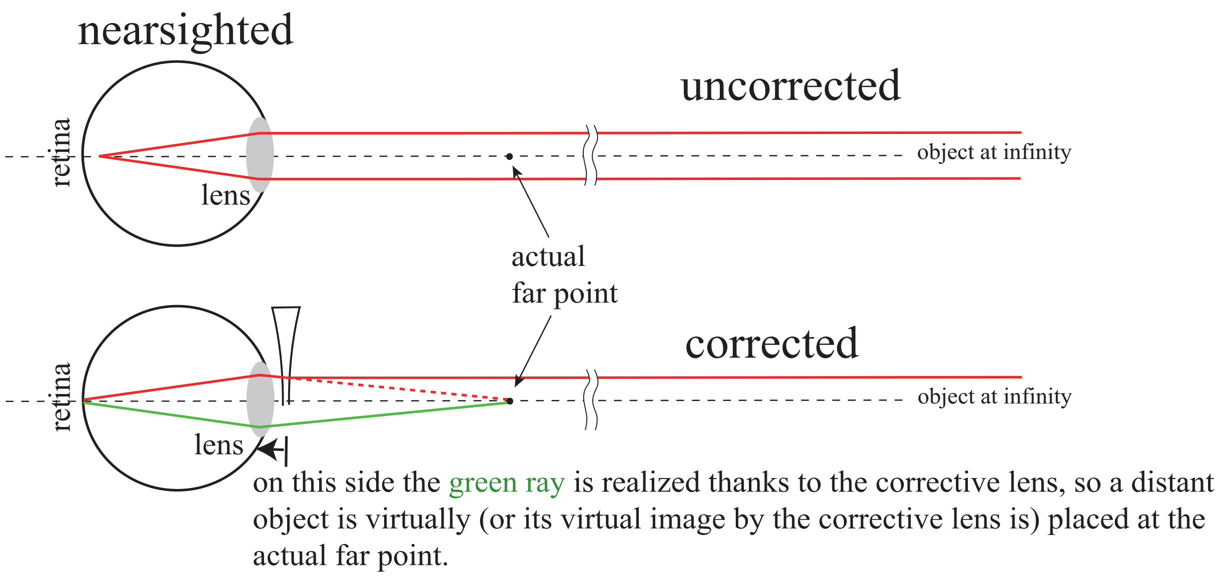
Corrective vision:

Principle of correcting eyes

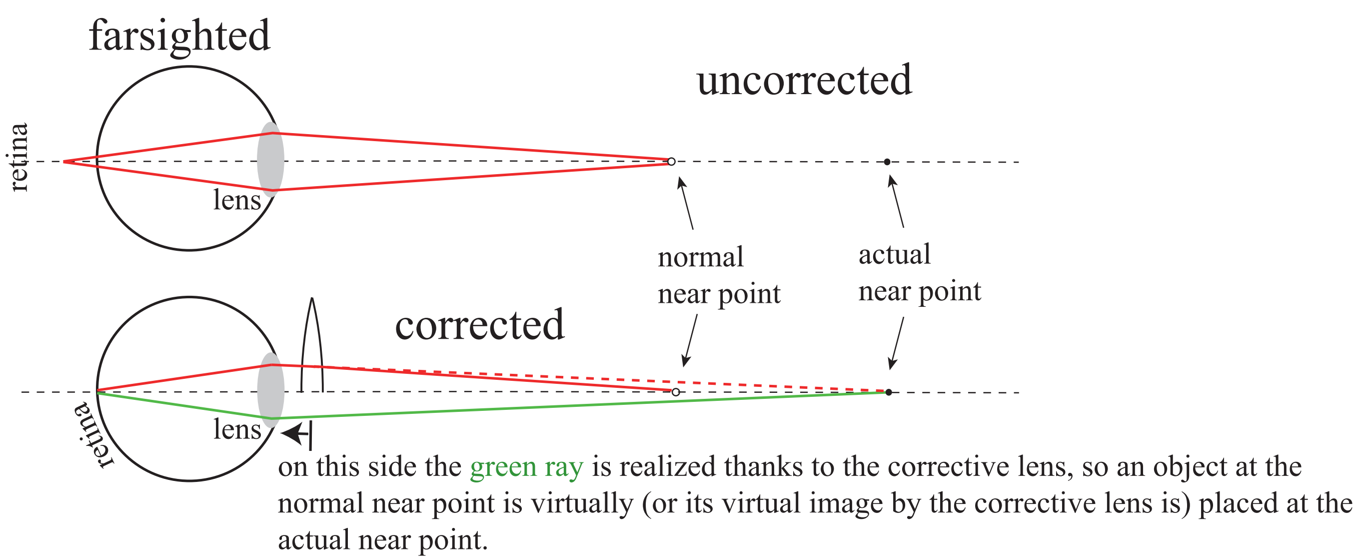
"Make an illusion at the place you can see clearly."

That is, make a virtual image of an object, which you wish to see, at a location where you can see real objects clearly without any corrective means.

Nearsighted eyes: we wish to make the virtual image of objects at infinity at the actual (or uncorrected) far point.



Farsighted eyes: we wish to make the virtual image of objects placed at the normal near point (25 cm from the eyes, usually) at the actual (or uncorrected) near point.



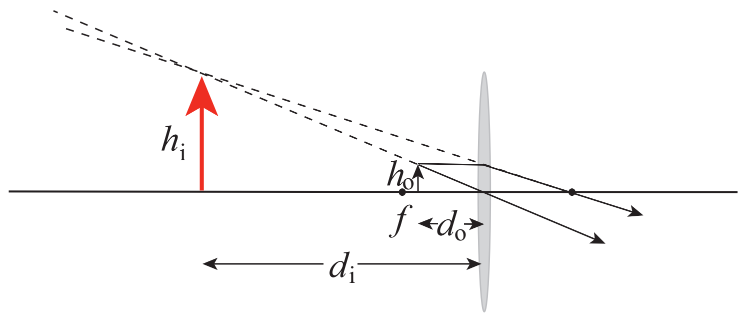
Remark: The prescriptions for glasses and contact lenses are different due to the presence or absence of the distance between the corrective lenses and the eye lenses.

Magnifying glass

We wish to make a magnified virtual image of a real object of height h_o placed at d_o. The position of the virtual image is $|d_i|$ behind the converging lens of focal length f:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} - \frac{1}{|d_i|} \;\Rightarrow\; d_o = \frac{f|d_i|}{f + |d_i|}.$$

We wish to place the image at our near point (distance d_{near} from our eyes). Usually, we place the magnifying glass very close to our eyes, so we may set $d_i = -d_{near}$.



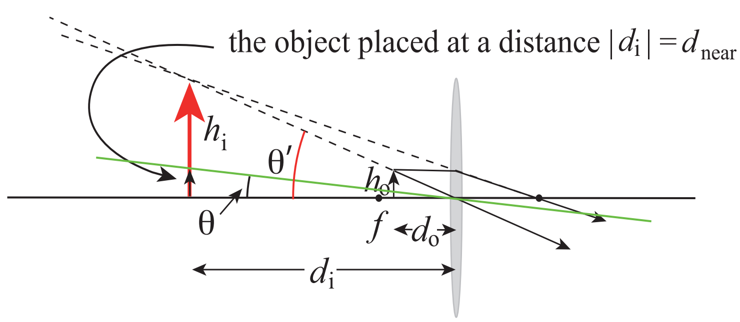
The magnification m, which is written as M in this context, is

$$M = -d_i/d_o = \frac{f + d_{near}}{f}\left(\simeq M = \frac{d_{near}}{f}, \text{if, as usual, } d_{near} \gg f\right).$$

Angular magnification interpretation of *M*

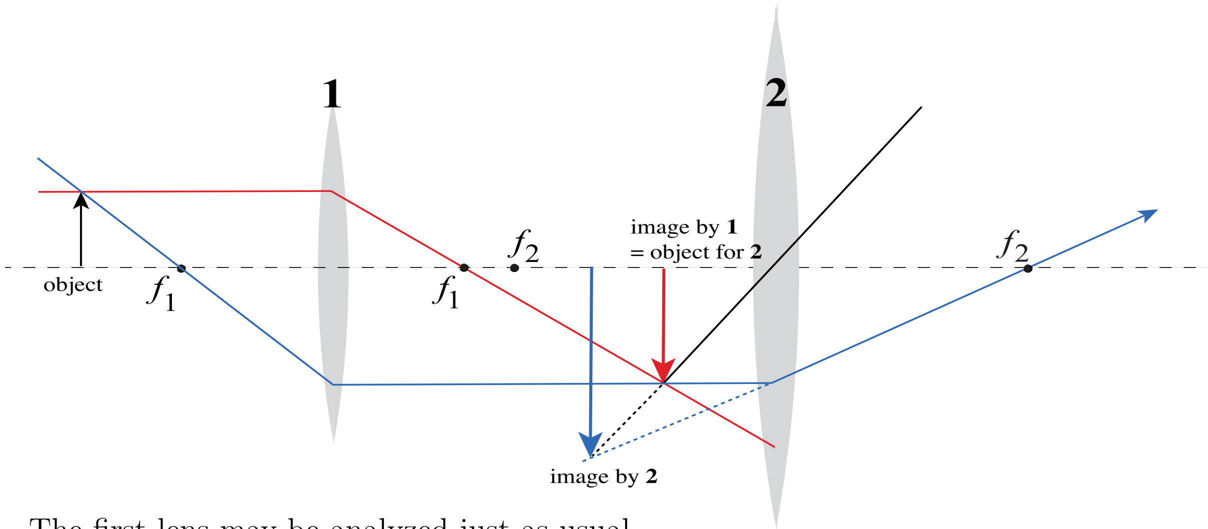
The magnification may be defined as the ratio of the angle we see with and without the magnifier. To see an object with an unaided eye, we must place it at d_{near} (see Figure below), so the angle θ we see is $\theta = h_o/d_{near}$ (in radians; small angle approximation). On the other hand, with the magnifier, the angle we observe is given by $\theta' = h_i/d_{near}$. Hence,

$$M = \frac{\theta'}{\theta} = \frac{h_i/d_{near}}{h_o/d_{near}} = \frac{h_i}{h_o} = \frac{d_{near}}{d_o} \simeq \frac{d_{near}}{f}.$$

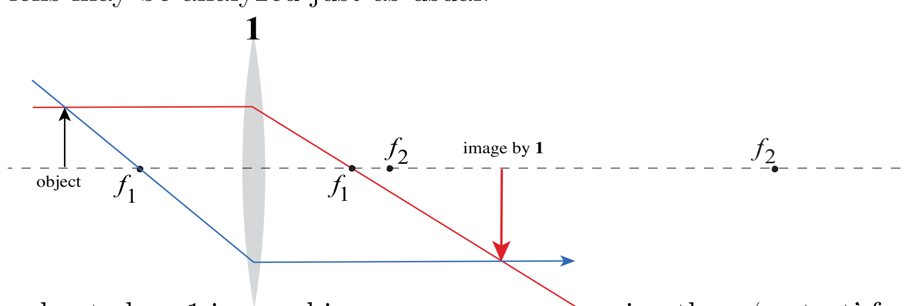


Compound lens system

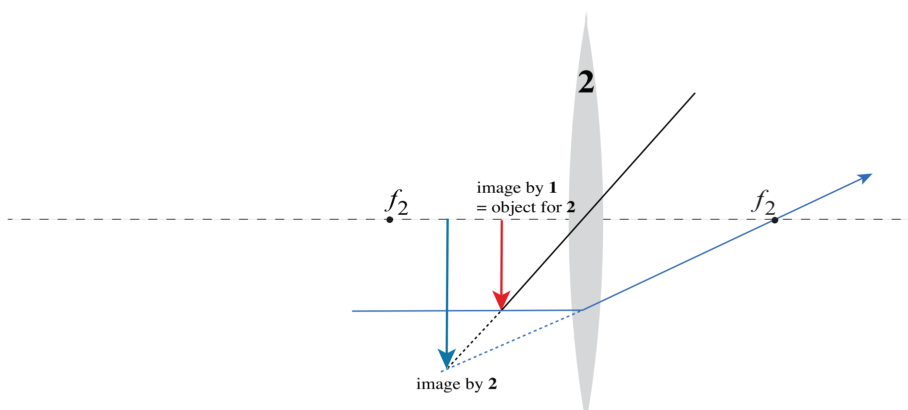
To analyze a system consisting of lenses and mirrors, we have only to study each lens or mirror one by one. This is illustrated with the following example:



The first lens may be analyzed just as usual.



The image due to lens 1 is a real image, so rays converging there 'restart' from this image. Thus, this image acts as the real object for the next lens. The analysis of lens 2 goes just as usual.



When you analyze a compound system numerically, you must recalculate the distances (e.g., d_o) at each step with respect to the lens or mirror being analyzed.

Discussion 12-1 Myopia

A myopic person has her far point at 7 cm from her eyes and the distance between her lens and retina is 27 mm.

1) What is the focal length of her lens when it is relaxed?

2) The lens of the glasses for her to see the scenery clearly is held from her own lens. What should be the power of this collective lens?

3) What is the power of the contact lens for her to see the scenery clearly?

Discussion 12-2 Magnifying glass

A convex lens of focal length 3 cm is to be used as a magnifying glass. An object is placed 2.68 cm in front of the lens.

1) Graphically construct the image in the following figure.

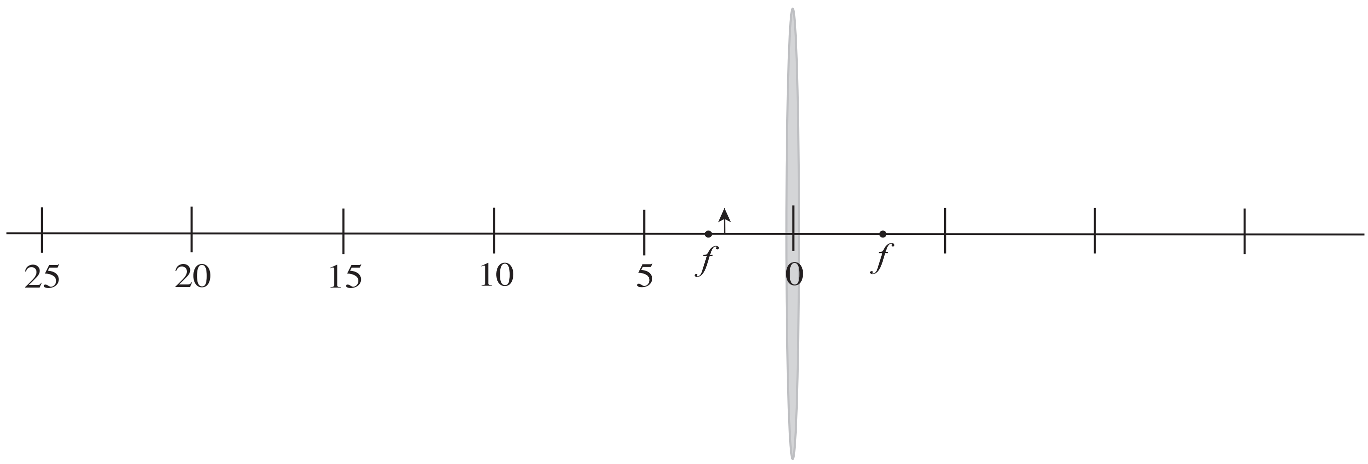


Figure 1:

2) Now, compute the position and the magnification of the image.

Discussion 12-3 Combination of convex and concave lenses

The left lens is a convex lens with a focal length of 30 cm. The right lens is a diverging lens with a focal length of 7 cm.

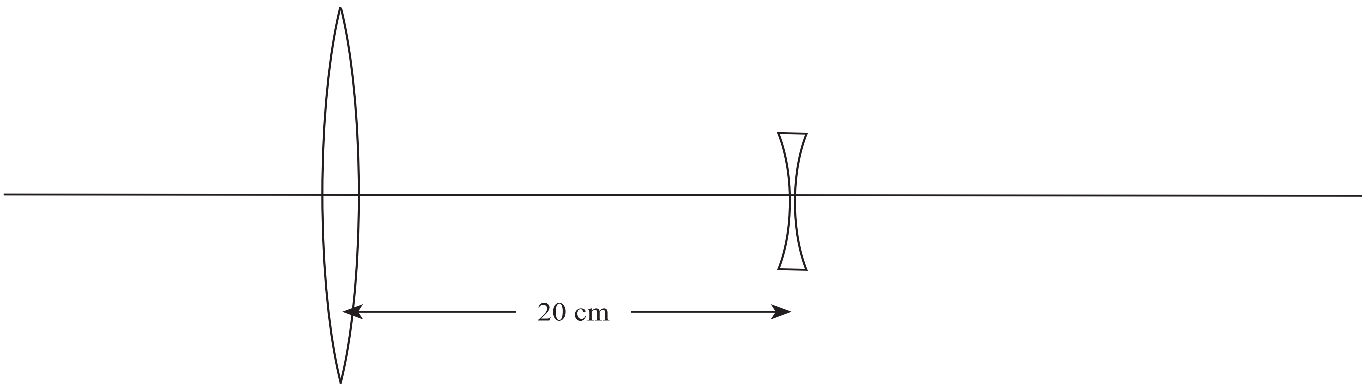


Figure 2:

1) Where is the image of the object at infinity by the converging lens (if the diverging lens were not there)?

2) Discuss why the real image due to the converging lens becomes effectively the virtual object for the diverging lens, and then compute the position of the final image. Is it virtual/real and upright/inverted?

Discussion 12-4 Microscope

A microscope has an objective lens with focal length $f_{\mathrm{o}} = +4$ mm and an eyepiece lens with focal length $f_{\mathrm{e}} = +40$ mm. The two lenses are positioned $L = 118.5$ mm apart in the barrel of the microscope. An object to be viewed is placed 4.2 mm in front of the objective lens.

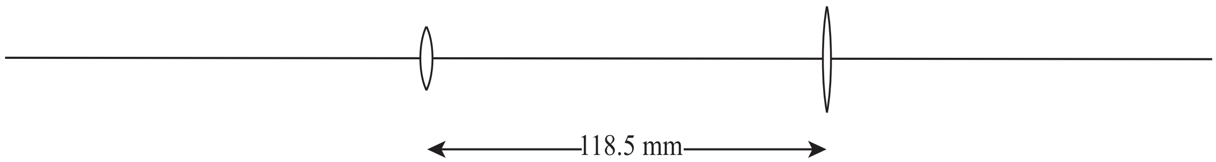


Figure 3:

(1) What is the overall magnification of this microscope (using the formulas in the formula sheet)?

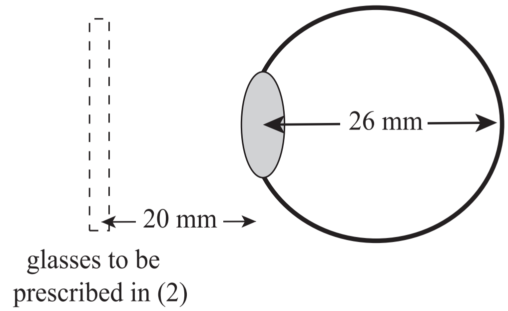
(2) If you use the lens formula, what is the accurate answer?

Physics 102 (Sample Quiz) Q11s

Name: ______________ **Section:** __________ **Score:** __________/20

1. For a myopic student the distance between the middle of her lens and her retina is 26 mm as illustrated below.

(1) Her near point is 5 cm. What is the focal length of her lens when strained? [5]



$1/f = 1/d_o + 1/d_i$

$d_o = 5$ cm, $d_i = 2.6$ cm $\Rightarrow 1/5 + 1/2.7 = 1/1.71$.
Hence, $f = 1.71$ cm.

(2) To read books held 25 cm from her eyes what lenses should be prescribed for her glasses? Give their strength in diopters. Assume that the glasses are held 2 cm from her own lenses. [5]

The principle of eye correction is to put an illusion (= virtual image) at the position of clear vision with uncorrected eyes.

The book is 23 cm = 25 − 2 cm from the glasses: $d_o = 23$ cm.
The virtual image should be at her near point, which is 3 cm in front of the glasses:

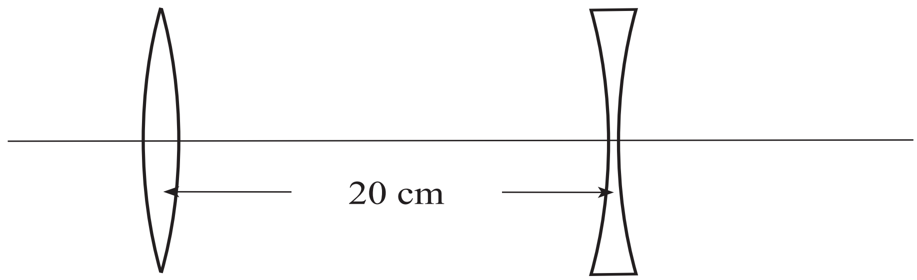
$d_i = -3$ cm (virtual).

Therefore,

$1/f = 1/23 - 1/3 = -1/3.45$ cm.

Thus, $-1/0.0345 = -29$ D.

2. An apparatus consists of a convex lens with focal length $|f_1| = 12$ cm and a concave lens with focal length $|f_2| = 20$ cm. The lenses are separated by 20 cm as illustrated below.



(1) An object of height 2 cm is placed 10 cm to the left to the convex lens. Where is the final image by these two lenses? You must tell whether the image is real or virtual. [You must solve this problem step by step, starting from the left lens.] [5]

The first lens:

$$1/12 = 1/10 + 1/d_{\rm i} \Rightarrow 1/d_{\rm i} = 1/12 - 1/10 = -1/60.$$

That is,
the image is 60 cm left of the convex lens. It is a virtual image, so the rays to the right of the first lens behave as if they emanate from this image. That is, this image acts as the real object for the second lens. Thus,

The second lens:

$$f_2 = -20 \text{ cm}, \; d_{\rm o} = 60 + 20 = 80. \; 1/d_{\rm i} = 1/f - 1/d_{\rm o} = -1/20 - 1/80 = -1/16.$$

Thus, the image is 16 cm to the left of the concave lens; virtual.

(2) What is the size of the image? Is it upright or inverted?[5]

For the first lens: $m_1 = -(-60)/10 = 6$.
For the second lens $m_2 = -(-16)/80 = 0.2$.
Thus, the overall magnification is $m_1 m_2 = 1.2$

Therefore, the image is upright and the size is $2 \times 1.2 = 2.4$ cm.

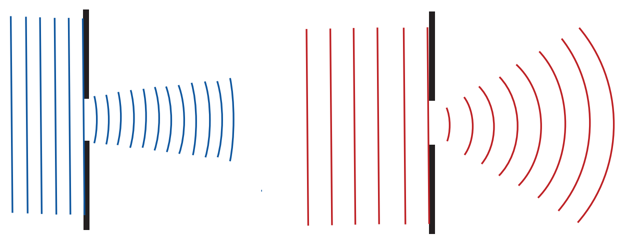
Physics 102 Discussion Week 13 (Lectures 22 and 23)

Interference and Diffraction

Key Points

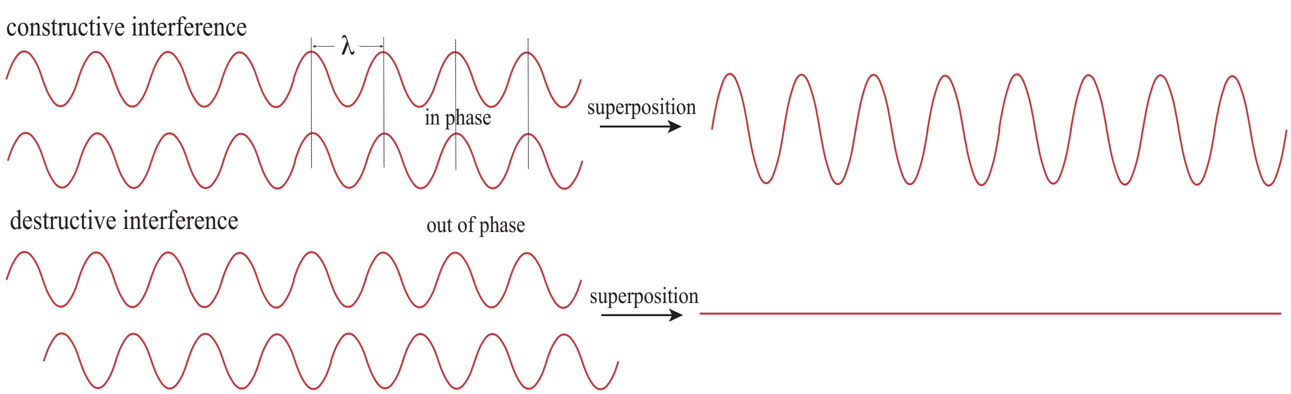
Light is a wave

We know light is a kind of electromagnetic wave, so it is a wave. However, if the wavelength is small enough compared with the beam width, the beam proceeds almost without any spread, so 'particle-like' geometrical-optics behavior can be observed (as we know in geometrical optics). The relation between the spread of a beam and its wavelength is illustrated below.



Wave interference

See Figure below:
If two waves with the same wavelength are **in phase** (i.e., wave peaks align), **constructive** interference occurs between them.
If two waves with the same wavelength are **out of phase** (i.e., not in phase; however, often this means that wave peaks of one wave align with wave troughs of the other wave), destructive interference occurs between them.



If the peak displacement is an integer multiple of the wavelength λ, interference is constructive. If it is a half-odd number multiple of λ, interference is totally destructive.

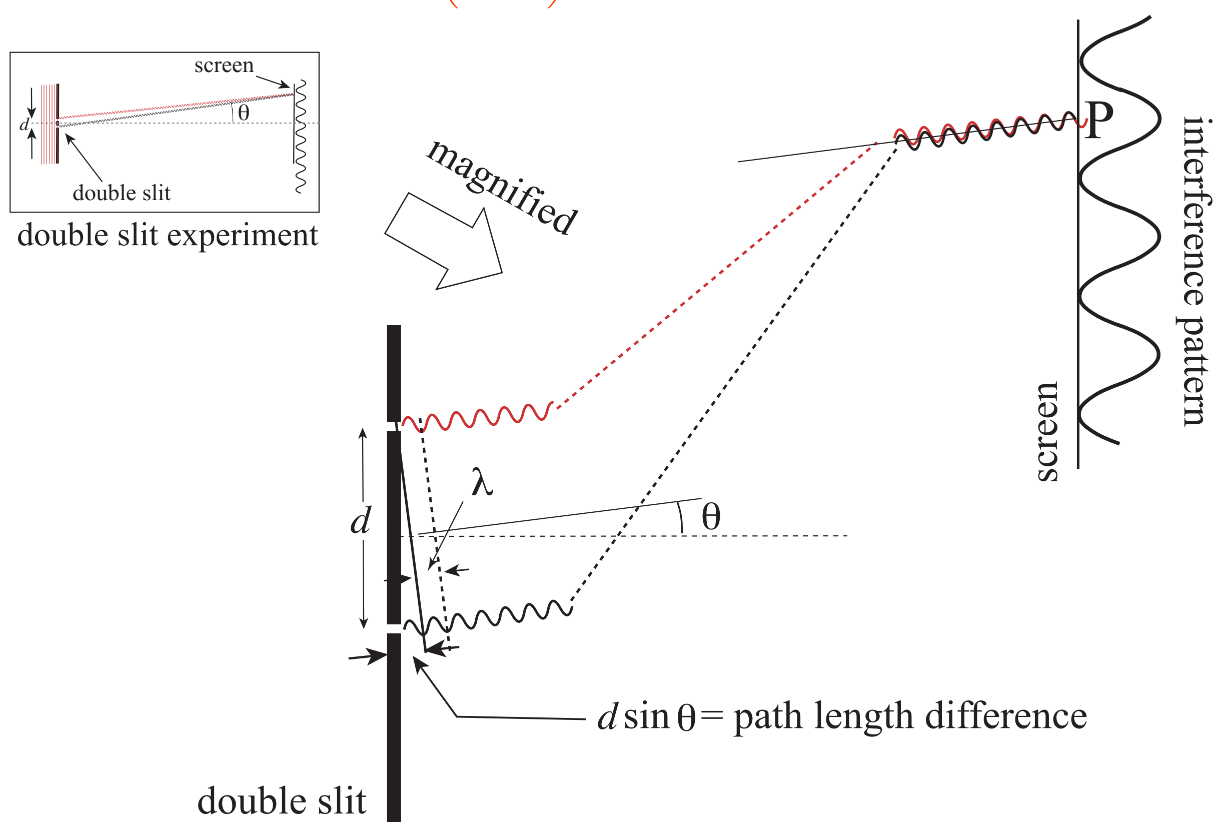
Huygens' principle
Every point on a wave front acts as a source of the wave of the same frequency and phase with the incoming wave.

Double-slit interference
Huygens' principle tells us that two slits become sources of wave in phase. Whether there is a constructive interference or not between these two waves at a point on the screen determines the interference pattern. The optical path length difference between the two beams in the following figure is $d\sin\theta$. Therefore, constructive interference (bright fringes) occurs at P if

$$d\sin\theta = m\lambda \quad \text{for } m = 0, \pm 1, \pm 2, \cdots.$$

Destructive interference (dark fringes) occurs if

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad \text{for } m = 0, \pm 1, \pm 2, \cdots.$$

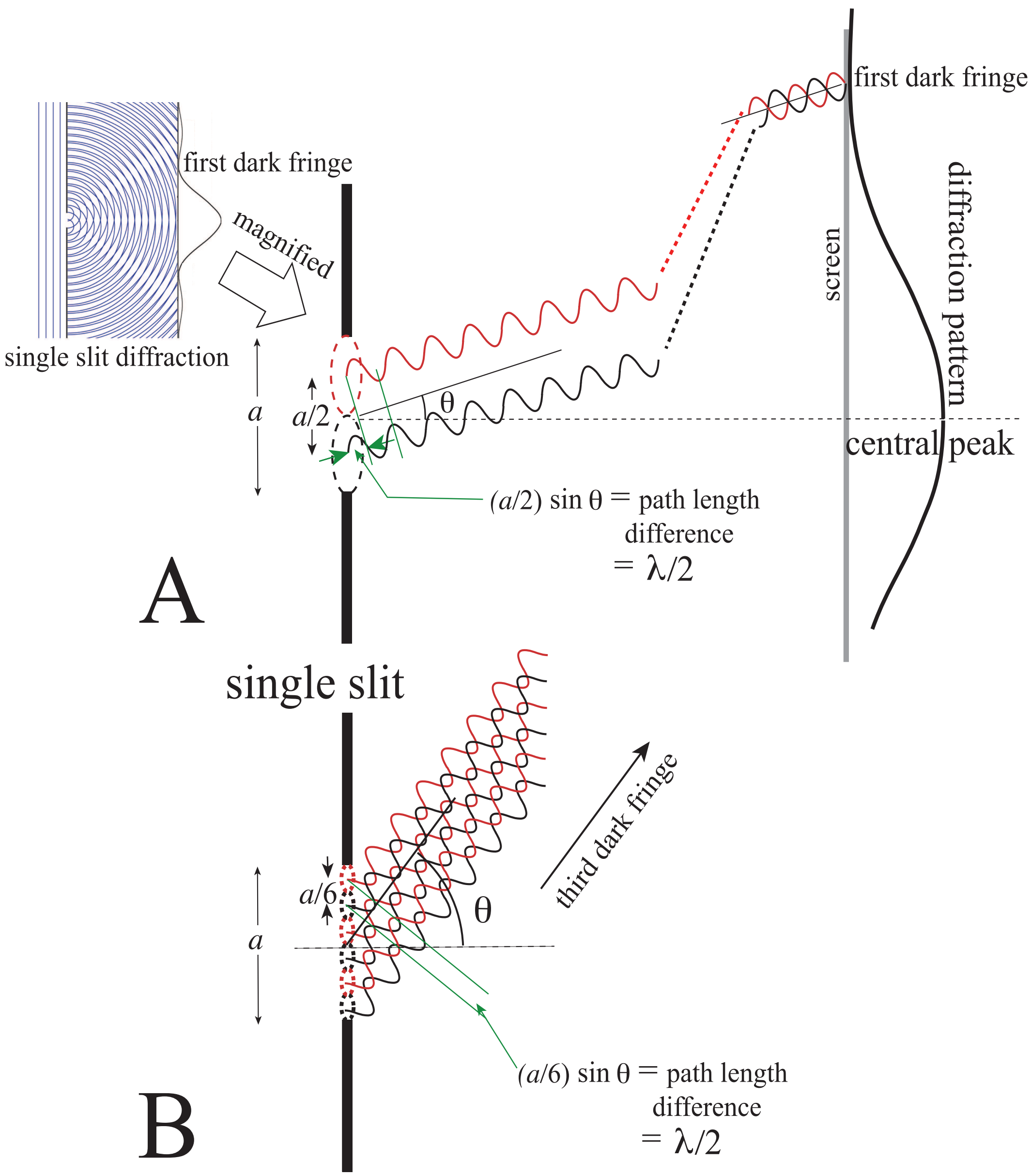


Diffraction grating
Instead of only two, if there are many equally spaced slits, the device is called a **diffraction grating**. Bright fringes can be found at the same positions on the screen just as the double slit case, but bright fringes are much sharper and brighter.

Single slit diffraction

A single slit may be understood as a collection of identical wave sources (thanks to Huygens' principle) (see the following figure for 2 and 6 source interpretations). If the length difference of the paths connecting the adjacent wave sources and the screen is equal to $\lambda/2$, then, these waves interfere totally destructively (see A in the figure). Therefore, if the slit observed from a direction θ may be interpreted as $2m$ ($m = \pm 1, \pm 2, \cdots$) sources (see B in the figure for $m = 3$) and if the differences between the lengths of the paths connecting the adjacent wave sources and the screen are all equal to $\lambda/2$, then the direction θ gives a dark fringe. That is, the dark fringe positions (directions) are given by

$$(a/2m)\sin\theta = \lambda/2 \;\Rightarrow\; a\sin\theta = m\lambda \text{ for } m = \pm 1, \pm 2, \cdots.$$

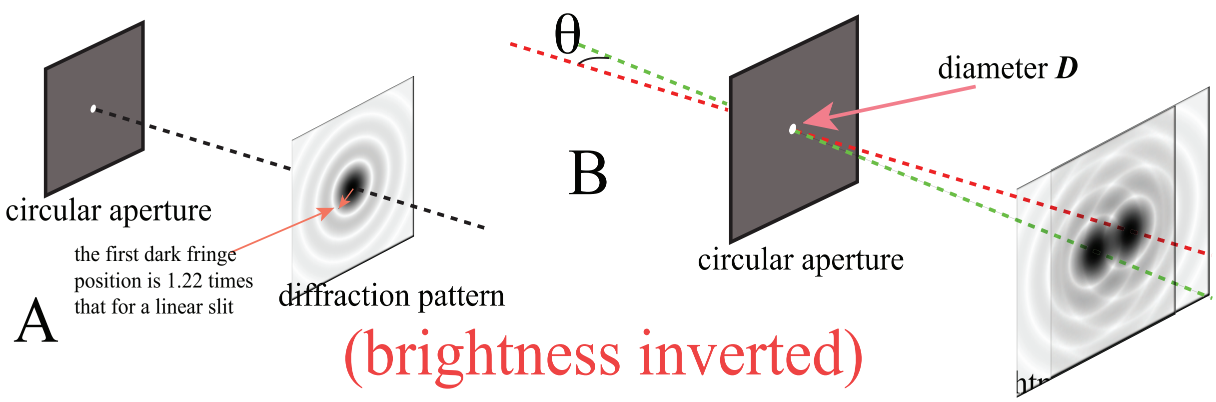


Diffraction by a circular aperture

The diffraction pattern by a circle hole is similar (see A in Figure) to that due to a slit, but some shape-dependent modifications are needed. The first dark fringe position is given by

$$D \sin\theta = 1.22\lambda,$$

if the diameter of the circular aperture is D.



Resolving power or Rayleigh's formula for resolution

Each bright spot makes a diffraction pattern as in A around its principal axis. Therefore, if the directions of the two spots are close (i.e., θ of B in the above Figure is not large enough), then the images of the spots on the screen do not separate well. That is, there is a **diffraction limit** for resolution of images. To recognize two separate spots the distance between the central diffraction peaks for these spots should not be smaller than the angle for the first dark fringe. Thus, the minimum angle discernible by the optical device with aperture D is given by

$$D \sin\theta = 1.22\lambda.$$

This is called Rayleigh's formula.

Small angle approximation (warning)

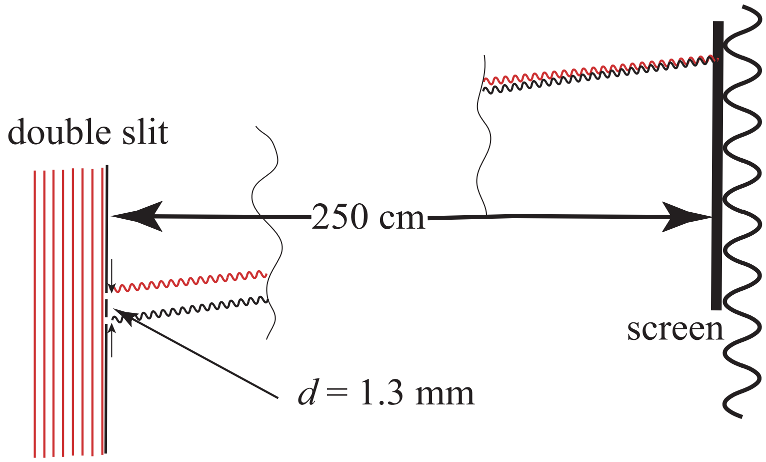
$\sin\theta$ and $\tan\theta$ may be computed approximately easily, if the angle θ is not too large. Practically, if $|\theta| < 0.2$ rad (i.e., $|\theta| \lesssim 10°$), then

$$\sin\theta \simeq \tan\theta \simeq \theta \textbf{ in radians}$$

is accurate. The last approximation (the small angle approximation) is meaningful, **ONLY IF** θ is expressed in radians (see the figure of Key Points of Week 1, Elementary trigonometry on p5).

Discussion 13-1 Double slit

A double slit with slit spacing 0.13 mm is placed 250 cm in front of a screen as illustrated below.

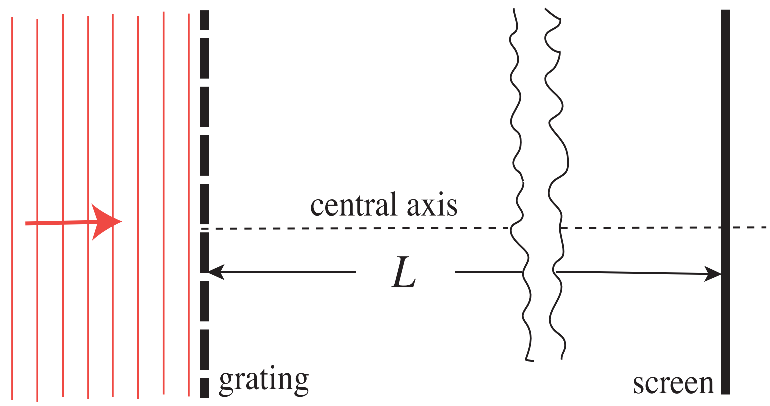


1) Red light ($\lambda = 665$ nm) illuminates the slits. What is the distance on the screen between the central bright fringe and the third order bright fringe?

2) A yellow-green light ($\lambda = 565$ nm) also impinges on the slits. What is the distance between the third order red fringe and the third order yellow-green fringe?

Discussion 13-2 Diffraction grating

We wish to use a diffraction grating to determine the wavelength of a light. We choose $L = 25$ cm.



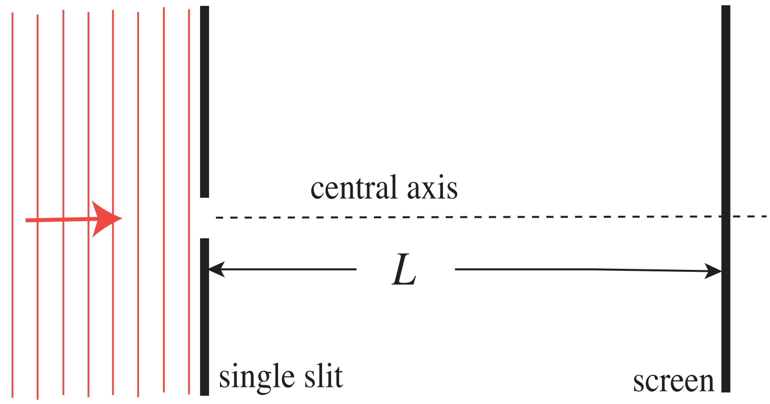
1) First, we determine the spacing of the slits with the aid of a known light from a red laser $\lambda = 680$ nm. The first bright fringe is observed 5 cm below the central axis on the screen. How many lines per cm does this grating have?

2) An unknown light source has a spot exactly 4.18 cm above the central axis. What is its wavelength? Is the answer unique? Why or why not?

3) Where is the second interference maximum for the red (680 nm) light?

Discussion 13-3 Single slit

A single slit is placed $L = 75$ cm in front of a screen as illustrated below.



1) Light of 665 nm is incident on a single slit. Draw the expected intensity pattern above in a qualitative manner. That is, don't expect to draw it to scale. Show at least five brightest peaks.

2) Indicate the width of the central bright fringe (the spacing between the first dark fringes bounding the central peak) on your drawing. How do you estimate its width?

3) If the distance between the two first order dark fringes (i.e., the width of the central peak according to the definition here) is 2.6 cm on the flat screen, what is the width of the slit?

4) If the light had a wavelength of $\lambda = 425$ nm instead, now what would the distance between the two first order dark fringes (i.e., the width of the central peak) be?

Discussion 13-4 Resolution

1) At a clear night you see a UFO flying toward you, which has a purple light of wavelength = 420 nm on each side, 1.5 miles apart.

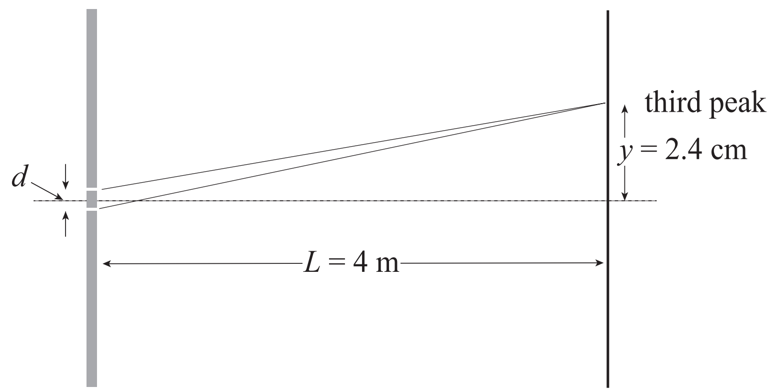
How close must the UFO get to you for you to recognize that the purple light is not coming from a single spot? Assume that you have ideally good vision and that your pupil diameter is 2.5 mm.

2) A radio telescope of diameter 305 m observes the 'hydrogen line' whose frequency is 1420 MHz. What is its angular resolution in degrees?

Physics 102 (Sample Quiz) **Q12s**

Name: ____________________ **Section:** __________ **Score:** __________/20

1. Light of wavelength $\lambda = 560$ nm (greenish yellow) from a dye laser illuminates a double slit. An interference pattern is observed on a screen 4 m away. The following illustration exaggerates the vertical direction.



(1) The third bright fringe is at a position $y = 2.4$ cm on the screen as in the figure. What is the slit separation d? [5]

Recall $d \sin\theta = m\lambda$.

Here, $m = 3$, $\theta = y/L$ (in radian). Since θ is small, $\theta \simeq \sin\theta = y/L$.

$$d = 3L \times \lambda/y = 3 \times 4 \times 560 \times 10^{-9}/2.4 \times 10^{-2} = 2800 \times 10^{-7} = 0.28 \times 10^{-3}.$$

That is, 0.28 mm.

(2) Now, the laser is tuned to 600 nm (orange). Where can you find the third bright fringe on the screen? Get the new y value. [5]

Notice that $\theta = y/L$ is proportional to the wavelength, so new y must be $2.4 \times 600/560 = 2.57$ cm.

Waves with longer wavelengths bend more as illustrated in this week's Key Points.

2. A pinhole camera with a circular aperture is used to make an image of two light sources separated by a distance $\Delta y = 2$ cm and located a distance $L = 75$ m away.

(1) If the light sources emit light of wavelength $\lambda = 620$ nm, what is the minimum aperture diameter D of the 'pinhole' of the camera to resolve the two light sources? [5]

Recall Rayleigh's formula: $D \sin\theta = 1.22\lambda$.

$$\theta = \Delta y/L \Rightarrow D = 1.22\lambda \times L/y$$
$$= 1.22 \times 620 \times 10^{-9} \times 75/2 \times 10^{-2} = 28365 \times 10^{-7} = 2.8 \times 10^{-3} \text{ m}$$

or 2.8 mm.

(2) If you wish to resolve the light sources with the same camera when $\Delta y = 1.8$ cm, what is the longest possible wavelength of light the sources must emit? [5]

The above formula tells us that if θ is made smaller, λ should also be scaled in the same fashion to maintain D. Therefore, $620 \times 1.8/2 = 558$ nm.

Phys 102 Discussion Week 14 (Lectures 24-)

Photons, Matter wave and Bohr atom

Key Points

Photons and photoelectric effect

It was empirically found that when a metal surface is illuminated with a light, light acts in units called **photons**. They may be understood as particles of energy E given by

$$E = hf$$

for light with frequency f, where $h = 6.626 \times 10^{-34}$ J·s is Planck's constant. Thus, light exhibits **wave-particle duality**.

Each photon can produce at most one photoelectron with a kinetic energy whose maximum value is give by

$$E_{\max} = hf - W_0,$$

where W_0 is the **work function** of a given metal. W_0 is the needed energy to liberate an electron from the metal surface under question.

Energy units: The above formula is convenient, if energies are all in eV.

eV is the potential energy of an electron (its charge $= -1.6 \times 10^{-19}$ C) in the electric potential of -1 V. Therefore, 1 eV $= (-1) \times (-1.6 \times 10^{-19}) = 1.6 \times 10^{-19}$ J.

Since $f = c/\lambda$, $hf = hc/\lambda$. The formula sheet tells us that $hc = 1240$ eV·nm. This means that if λ is measured in nm, then the number $1240/\lambda$ is the photon energy in eV, or if the photon energy E is in eV, then its wavelength is $1240/E$ in nm.

For example, if the wavelength of a photon is $\lambda = 540$ nm, then $1240/540 = 2.3$ is the energy of this photon in eV; if a photon has an energy 3.7 eV, its wavelength is $1240/3.7 = 335$ nm.

Quantization is demanded by the existence of atoms

Classical physics does an excellent job, but one of its greatest defects is that it cannot explain why atoms are stable. An atom consists of electrons going around a positively charged nucleus. We know, however, charge undergoing an accelerated motion radiates electromagnetic waves that carry away energy. Consequently, according to classical physics atoms cannot be stable; they always lose energy continuously.

Quantum mechanics solves this most important question by discretizing (= **quantizing**) possible states of an atom. Thus, no continuous loss of energy occurs for the atom. Consequently, atoms are stable.

de Broglie's matter wave

This quantization is based on de Broglie's idea of **wave-particle duality** (or **matter wave**): if light exhibits this duality, why not ordinary particles? With a particle having a momentum p is associated a **matter wave** of wavelength given by

$$\lambda = \frac{h}{p}.$$

This relation is true for photons as well: For photons $p = E/c$, so $h/p = hc/E = hc/hf = c/f = \lambda$.

If there is a standing matter wave of an electron in the atom, that state of an atom is stable. This is the fundamental idea of Bohr's.

Heisenberg's uncertainty relation
The idea of matter wave demands total reconsideration of the concepts of position and momentum of a given particle. They are not well-defined quantities separately, but are complementary with each other: the uncertainty in momentum Δp and the uncertainty in location Δx must satisfy the **uncertainty relation**:

$$\Delta p \Delta x \geq \frac{1}{2}\hbar,$$

where $\hbar = h/2\pi = 1.06 \times 10^{34}$ J·s.

Bohr radius and energy levels
From the condition that the electron matter wave makes a standing wave around the nucleus it was noted that the angular momentum L is quantized as

$$L_n = n\hbar, \text{ where } n = 1, 2, 3 \cdots.$$

This determines stable atomic states specified by the **principal quantum number** n: its radius (**Bohr radius**) is given by

$$r_n = \left(\frac{\hbar^2}{m_e k e^2}\right)\frac{n^2}{Z} = (5.29 \times 10^{-11} \text{ m})\frac{n^2}{Z},$$

and its energy E_n (**Bohr energy level**) by

$$E_n = -\left(\frac{m_e k^2 e^4}{2\hbar^2}\right)\frac{Z^2}{n^2} = (-13.6 \text{ eV})\frac{Z^2}{n^2}.$$

Here, m_e is the electron mass, e is the electron charge, k is the constant appearing in the Coulomb interaction, and Z ($\times$ e) is the nuclear charge.

A transition of state with n to that with n' ($< n$) produces a single photon of energy given by $hf = E_n - E_{n'}$.

Atomic states
A (stable) state of an electron in Bohr's atom is uniquely specified by four quantum numbers n, ℓ, m_ℓ and m_s. Their ranges are as follows:

$n = 1,\ 2,\ \cdots$.
$\ell = 0,\ 1,\ 2,\ \cdots, n-1$ (for an electron with a given n).
$m_\ell = -\ell,\ -\ell+1,\ \cdots -1,\ 0,\ 1,\ \cdots,\ \ell-1,\ \ell$ (for an electron with a given ℓ).
$m_s = \pm\frac{1}{2}$.

n determines the energy E_n of the electron.
ℓ determines the (total) angular momentum L of the electron: $L = \sqrt{\ell(\ell+1)}\hbar$.
m_ℓ determines the z-component L_z of the angular momentum $L_z = m_\ell\hbar$.
m_s determines the z-component S_z of the spin angular momentum $S_z = m_s\hbar$

Pauli's principle: No two electrons can have the same set of quantum numbers (n, ℓ, m_ℓ, m_S) in a given atom.

Discussion 14-1 Photons

1) A laser emits 1.3×10^{18} photons in one second in a beam of light that has a diameter of 2.00 mm and a wavelength 514.5 nm. Determine the average electric field strength of the electromagnetic wave that constitutes the beam. Let us do this step by step.

(i) What is the energy of a single photon of this light (in J and in eV)?

(ii) What is the energy carried by this beam of light? Or, determine the flow rate (energy flux I or S, p106) of week 10, in J/s (= W) of the energy in the beam.

(iii) What is the flow rate of energy per unit area? This is the intensity I of the beam.

(iv) What is the average energy density $\overline{u}$ of this electromagnetic wave?

(v) Finally, determine the average electric field strength.

2) An owl has good night vision because its eyes can detect a light intensity as small as 5.0×10^{-13} W/m^2. What is the minimum number of photons per second that an owl eye can detect if its pupil has a diameter of 8.5 mm and the light has a wavelength of 510 nm?

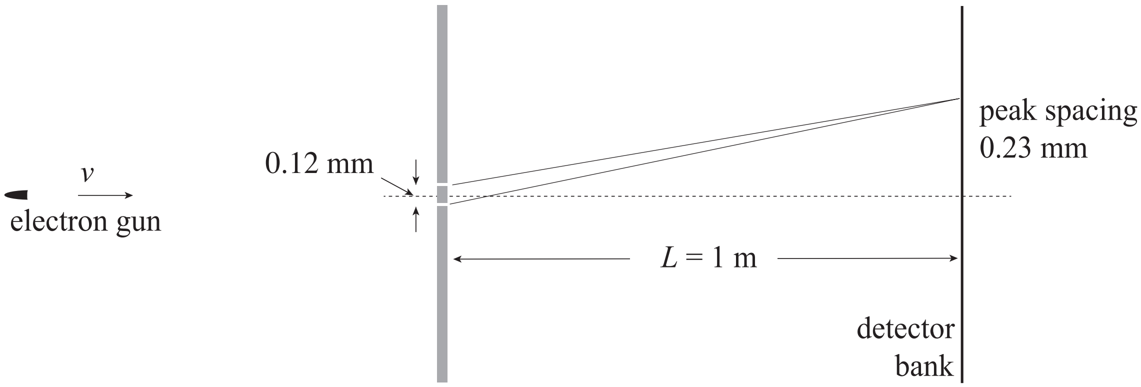
Discussion 14-2 Work function and photoelectric effect

1) The work function for a sodium surface is 2.28 eV. What is the shortest wavelength (in nm) of an electromagnetic wave that cannot eject electrons?

2) What is the maximum kinetic energy of the ejected electrons from a sodium surface if it is illuminated with a light of wavelength 320 nm? What is the fastest possible speed of the electron?

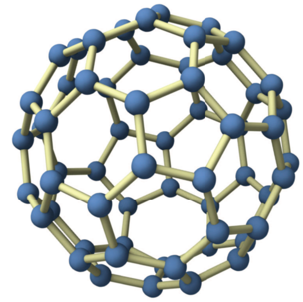
Discussion 14-3 de Broglie wave

1) Electrons are sent one by one with the same speed v from far behind a double slit whose spacing is 0.12 mm (see Figure). 1 m away from the slits is a detector bank on which we observe a bright spot when an electron arrives.



Collecting all the results of numerous electrons, we can observe an interference pattern whose adjacent detection peaks are 0.23 mm apart (cf. watch `https://www.youtube.com/watch?v=_oWRI-LwyC4`; see Figure below for the final outcome by A Tonomura of Hitachi Central Res Lab (1983)). What is the speed v of the electron?

2) Now the slit spacing is 100 nm and electrons are replaced with buckyballs (C_{60}; see Figure) of speed 200 m/s.[13] What is the spacing between the adjacent interference peaks?



[13]M Arndt, O Nairz, J Vos-Andreae, C Keller, G van der Zouw and A Zeilinger, Wave-particle duality of C60 molecules, Nature 401, 680-682 (1999). The figure above is taken from Wikipedia.

Discussion 14-4 Uncertainty principle

1) In a potential well of radius 1 pm is confined a proton. If this radius is considered to be the uncertainty in the position of the proton, what is the minimum uncertainty in its momentum p?

2) This momentum uncertainty implies that the kinetic energy of the proton must be high. What potential energy (< 0, i.e., the depth of the well) do you expect to need to confine the proton in the well?

Discussion 14-5 Bohr atom

1) What is the radius of a ground state hydrogen atom according to the Bohr model?

2) A ground-state hydrogen atom absorbs a photon and is excited to the $n = 3$ level. What is the energy (in eV) of this photon?

3) Calculate the ionization energy of a hydrogen atom with an electron in the $n = 3$ state. If the electron is replaced by a negative muon (mass $= 204m_e$), does this photon have enough energy to ionize this muonic hydrogen atom?

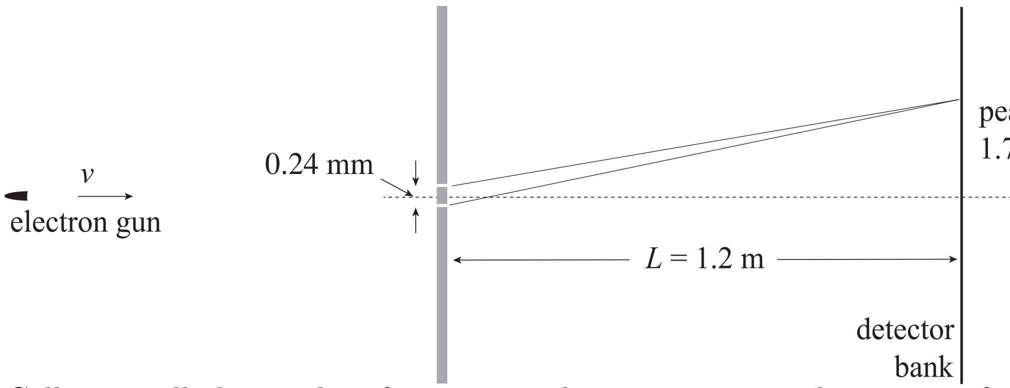
4) What is the wavelength of a photon emitted by the de-excitation of the hydrogen atom in the $n = 6$ state to that in the $n = 5$ state?

5) What is the ionization energy of helium ion He^+ ?

Physics 102 (Sample Quiz) **Q13s**

Name: ____________________ **Section:** __________ **Score:** __________**/20**

1. Electrons are sent one by one with the same speed v from far behind a double slit separated by a 0.24 mm spacing (see the figure). 1.2 m away from the slits is a detector bank on which we observe a bright spot when an electron arrives.



(1) Collecting all the results of numerous electrons, we can observe interference patterns with the spacing of adjacent interference peaks being 1.7 mm. What is the speed v of the electrons? [5]

Recall: $d\sin\theta = m\lambda$ and de Broglie wavelength $\lambda = h/p$, where $p = mv$.

The wavelength may be obtained as

$$\lambda = dy/L = 0.24 \times 10^{-3} \times 1.7 \times 10^{-3}/1.2 = 0.34 \times 10^{-6} \text{ m},$$

so

$$v = h/m\lambda = 6.626 \times 10^{-34}/9.11 \times 10^{-31} \times 3.4 \times 10^{-7} = 0.202 \times 10^{-34+38}$$

That is, $v = 2.02$ km/s.

(2) If the electrons are replaced with muons (whose mass is 200 times as large as that of electrons) with the same kinetic energy as in (1), what is the spacing between adjacent interference peaks? [5]

The kinetic energy reads $K = p^2/2m$, so $p = \sqrt{2mK}$. K is maintained and $m \to 200m$, so the de Broglie wavelength becomes $\lambda \to \lambda/\sqrt{200} = \lambda/14.1$. The spacing is proportional to the wavelength. Therefore, the new spacing would be $1.7/14.1 = 0.12$ mm.

λ becomes smaller, so the wave propagation direction is harder to bend. Recall why sky is blue.

2. Using the photons created by the de-excitation of the $n = 3$ state of Hydrogen to its ground state, we wish to study the photoelectric effect of a metal.

(1) What is the wavelength of the photons? [5]

Use Bohr's formula: $E_n = -13.6/n^2$ eV.

The ground state is with $n = 1$.

$hf - 13.6 + 13.6/9 = 0$, so $hf = 12.09$ eV. This implies $\lambda = 1240/12.09 = 103$ nm.

(2) The work function of the metal is 2.8 eV. What is the maximum speed of the ejected electrons from the metal surface illuminated by the photons discussed in (1)? [5]

Recall $hf = W + K_{\text{max}}$.

$K_{\text{max}} = 12.24 - 2.8 = 9.44 \text{ eV } = 15.1 \times 10^{-19} \text{ J } = (1/2)mv^2$
or
$v = \sqrt{30.2 \times 10^{-19}/9.11 \times 10^{-31}} = \sqrt{3.315 \times 10^{12}} = 1.8 \times 10^6$ m/s.
(This is still not fast enough to observe relativistic effects significantly.)

Physic 102 formula sheet (FA2016)

Kinematics and mechanics

$x = x_0 + v_0 t + \frac{1}{2}at^2$ $\quad v = v_0 + at$ $\quad v^2 = v_0^2 + 2a\Delta x$

$F = ma$ $\quad a_c = \frac{v^2}{r}$

$E_{\text{tot}} = K + U$ $\quad K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ $\quad p = mv$ $\quad W_F = Fd\cos\theta$ $\quad P = Fv\cos\theta$

Electrostatics

$F_{12} = k\frac{q_1 q_2}{r^2}$ $\quad E = \frac{F}{q_0}$ $\quad U_{12} = k\frac{q_1 q_2}{r}$ $\quad V \equiv \frac{U}{q_0}$ $\quad W_E = -\Delta U = -W_{\text{you}}$

Point charge $\quad E = k\frac{q}{r^2}$ $\quad V = k\frac{q}{r}$

Electric dipole $\quad p = qd$ $\quad \tau_{\text{dip}} = pE\sin\theta$ $\quad U_{\text{dip}} = -pE\cos\theta$

Resistance

$R = \frac{V}{I}$ $\quad I = \frac{\Delta q}{\Delta t}$ $\quad$ Physical resistance: $R = \rho\frac{L}{A}$

$P = IV = I^2 R = \frac{V^2}{R}$ $\quad R_{\text{S}} = R_1 + R_2 + \cdots$ $\quad \frac{1}{R_{\text{P}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$

Capacitance

$C = \frac{Q}{V}$ $\quad$ Parallel plate capacitor: $C = \frac{\kappa\epsilon_0 A}{d}$ $\quad E = \frac{Q}{\epsilon_0 A}$ $\quad V = Ed$

$U_C = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$ $\quad C_{\text{P}} = C_1 + C_2 + \cdots$ $\quad \frac{1}{C_{\text{S}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$

Circuits

$\sum \Delta V = 0$ $\quad \sum I_{\text{in}} = \sum I_{\text{out}}$

$q(t) = q_\infty(1 - e^{-t/\tau})$ $\quad q(t) = q_0 e^{-t/\tau}$ $\quad I(t) = I_0 e^{-t/\tau}$ $\quad \tau = RC$

Magnetism

$F = qvB\sin\theta$ $\quad r = \frac{mv}{qB}$ $\quad F_{\text{wire}} = ILB\sin\theta$ $\quad \tau_{\text{loop}} = NIAB\sin\varphi$

Magentic dipole: $\quad \mu = NIA$ $\quad \tau_{\text{dip}} = \mu B\sin\varphi$ $\quad U_{\text{dip}} = -\mu B\cos\varphi$

$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$ $\quad B_{\text{sol}} = \mu_0 nI$

Electromagnetic induction

$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$ $\quad \Phi = BA\cos\varphi$

$|\mathcal{E}_{\text{bar}}| = BLv$ $\quad \mathcal{E}_{\text{gen}} = \mathcal{E}_{\text{max}}\sin\omega t = \omega NAB\sin\omega t$ $\quad \omega = 2\pi f$

$V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}}$ $\quad I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}}$ $\quad \frac{V_{\text{p}}}{V_{\text{s}}} = \frac{I_{\text{s}}}{I_{\text{p}}} = \frac{N_{\text{p}}}{N_{\text{s}}}$

Electromagnetic waves

$\lambda = \frac{c}{f}$ $\quad E = cB$

$u_E = \frac{1}{2}\epsilon_0 E^2$ $\quad u_B = \frac{1}{2\mu_0}B^2$ $\quad \overline{u} = \frac{1}{2}\epsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0}B_{\text{rms}}^2 = \epsilon_0 E_{\text{rms}}^2 = \frac{B_{\text{rms}}^2}{\mu_0}$ $\quad S = I = \overline{u}c = \frac{P}{A}$

$f_0 = f_e\sqrt{\frac{1 + v_{\text{rel}}/c}{1 - v_{\text{rel}}/c}} \approx f_e\left(1 + \frac{v_{\text{rel}}}{c}\right)$ $\quad I = I_0\cos^2\theta$

Reflection and refraction

$\theta_r = \theta_i$ $\quad \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ $\quad f = \pm\frac{R}{2}$ $\quad m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $\quad v = \frac{c}{n}$ $\quad \sin\theta_c = \frac{n_2}{n_1}$ $\quad M = \frac{\theta'}{\theta} \approx \frac{d_{\text{near}}}{f}$

Compound microscope: $\quad m_{\text{obj}} = \frac{L_{\text{tube}}}{f_{\text{obj}}}$ $\quad M_{\text{eye}} = \frac{d_{\text{near}}}{f_{\text{eye}}}$ $\quad M_{\text{tot}} = M_{\text{eye}} m_{\text{obj}}$

Interference and diffraction

Double-slit interference: $\quad d\sin\theta = m\lambda$ $\quad d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$ $\quad m = 0, \pm 1, \pm 2, \cdots$

Single-slit diffraction: $\quad a\sin\theta = m\lambda$ $\quad m = 0, \pm 1, \pm 2, \cdots$
Circular aperture: $\quad D\sin\theta \approx 1.22\lambda$

Quantum mechanics

$E = hf = \frac{hc}{\lambda}$ $\quad \lambda = \frac{h}{p}$ $\quad \Delta p_x \Delta x \geq \frac{\hbar}{2}$ $\quad \hbar = \frac{h}{2\pi}$

Bohr atom: $\quad 2\pi r_n = n\lambda$ $\quad n = 1, 2, 3, \cdots$ $\quad L_n = m_e v_n r_n = n\hbar$

$r_n = \left(\frac{\hbar^2}{m_e k e^2}\right)\frac{n^2}{Z} \approx (5.29 \times 10^{-11}\ \text{m})\frac{n^2}{Z}$ $\quad E_n = -\left(\frac{m_e k^2 e^4}{2\hbar^2}\right)\frac{Z^2}{n^2} \approx -(13.6\ \text{eV})\frac{Z^2}{n^2}$

$\frac{1}{\lambda} \approx (1.097 \times 10^7\ \text{m}^{-1}) Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

Quantum atom: $\quad L = \sqrt{\ell(\ell+1)}\hbar$ $\quad L_z = m_\ell \hbar$ $\quad S_z = m_s \hbar$

Atomic magnetism: $\quad \mu_{e,z} = -\frac{e}{2m_e} L_z$ $\quad \mu_{s,z} = -\frac{ge}{2m_e} S_z,\ \ g \approx 2$ $\quad \mu_B \equiv \frac{e\hbar}{2m_e} \approx 5.8 \times 10^{-5}\ \text{eV/T}$

Nuclear physics and radioactive decay

$A = Z + N$ $\quad r \approx (1.2 \times 10^{-15}\ \text{m}) A^{1/3}$ $\quad E_0 = mc^2$

$m_{\text{nucleus}} = Z m_{\text{proton}} + N m_{\text{neutron}} - \frac{|E_{\text{bind}}|}{c^2}$

$\frac{\Delta N}{\Delta t} = -\lambda N$ $\quad N(t) = N_0 e^{-\lambda t} = N_0 2^{-t/T_{1/2}}$ $\quad T_{1/2} = \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$

Constants and unit conversion

$g = 9.8\ \text{m/s}^2$ $\quad e = 1.60 \times 10^{-19}\ \text{C}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{Nm}^2$ $\quad k \equiv \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ \text{Nm}^2/\text{C}^2$ $\quad \mu_0 = 4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}$

$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8\ \text{m/s}$ $\quad h = 6.626 \times 10^{-34}\ \text{J} \cdot \text{s}$ $\quad hc = 1240\ \text{eV} \cdot \text{nm}$

$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$ $\quad m_{\text{electron}} = 9.11 \times 10^{-31}\ \text{kg} = 511\ \text{keV}/c^2$

$m_{\text{proton}} = 1.673 \times 10^{-27}\ \text{kg} = 938\ \text{MeV}/c^2$ $\quad m_{\text{neutron}} = 1.675 \times 10^{-27}\ \text{kg} = 939.5\ \text{MeV}/c^2$

SI Prefixes

Power	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^0	—	—
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p