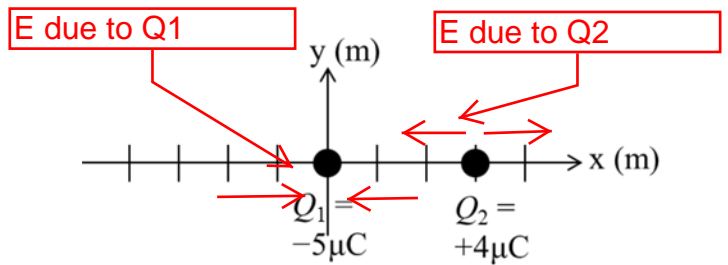


The next three questions pertain to the situation described below.

Two point charges, Q_1 and Q_2 , are placed at (0,0) and (3,0) as shown. The grid spacing is in meters. Assume the charges are glued into place.



This means there is no electric field: $F = qE$

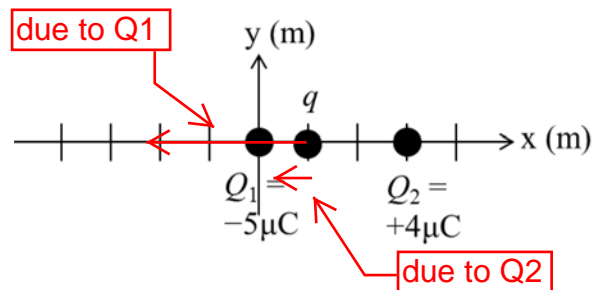
1) There is a point on the x -axis where a small test charge would feel no net force from Q_1 and Q_2 . In which region is this point located?

Recall superposition principle and E due to a single charge

- a. $x < 0$
- b. $x > 3$**
- c. between $x = 0$ and $x = 3$

From the figure (arrows) $0 < x < 3$ is out of question. Since $|Q_1| > |Q_2|$, the magnitude of E due to Q_2 can never be larger than that due to Q_1 for $x < 0$.

2) A positive point charge of $q = 3.4 \times 10^{-6} \text{ C}$ is now placed at (1,0) as shown. What is the magnitude of the electric force on q , due to Q_1 and Q_2 ?



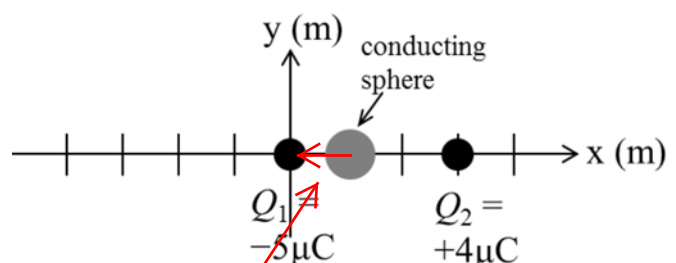
Coulomb force + superposition principle

Sketch the forces due to each pair of charges

- a. $|F_q| = 0.18 \text{ N}$**
- b. $|F_q| = 0.054 \text{ N}$
- c. $|F_q| = 0.12 \text{ N}$
- d. $|F_q| = 0.28 \text{ N}$
- e. $|F_q| = 0 \text{ N}$

due to Q_1 : $kq|Q_1|/1^2$ (to the left)
 due to Q_2 : $kq|Q_2|/2^2$ (to the left)
 Therefore,
 $|F_q| = kq(|Q_1| + |Q_2|/4) = 9 \times 10^9 \times 3.4 \times 10^{-6} (5 \times 10^{-6} + 1 \times 10^{-6})$
 $= 183.6 \times 10^{9-6-6} = 183.6 \times 10^{-3} = 0.18 \text{ N}$

3) Charge q is now removed and replaced with an uncharged conducting sphere at the same location, as shown. The sphere is free to move and is insulated from the ground.



In which direction do you expect the sphere to move?

induced charges

- a. To the left**
- b. To the right
- c. The sphere will not move

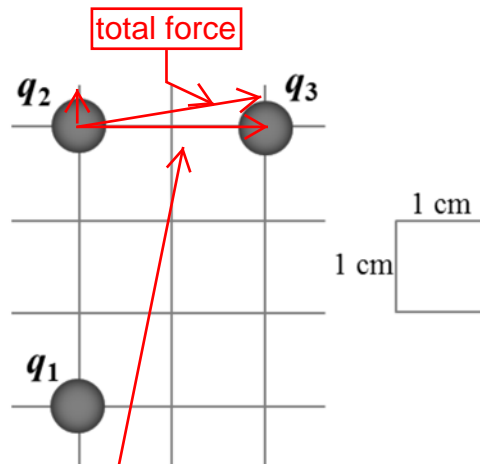
more + here than - on the other side

The next two questions pertain to the situation described below.

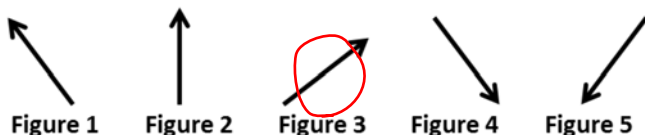
Consider the configuration of point charges shown:

$q_1 = -1 \text{ nC}$, $q_2 = -4 \text{ nC}$, and $q_3 = +2 \text{ nC}$.

The grid is 1 cm on a side.



4) Which of the following vectors best represents the direction of the total force $F_{2,tot}$ on charge q_2 due to q_1 and q_3 ?



- a. Figure 4
- b. Figure 5
- c. Figure 1
- d. Figure 3**
- e. Figure 2

5) Calculate the magnitude of the total force $|F_{2,tot}|$ on charge q_2 due to q_1 and q_3 .

- a. $|F_{2,tot}| = 184 \mu\text{N}$**
- b. $|F_{2,tot}| = 31.8 \mu\text{N}$
- c. $|F_{2,tot}| = 2700 \mu\text{N}$
- d. $|F_{2,tot}| = 114 \mu\text{N}$
- e. $|F_{2,tot}| = 773 \mu\text{N}$

Magnitudes are (we know the directions)

$$\text{due to } q_1: k|q_1q_2|/(3 \times 10^{-2})^2 = 9 \times 10^9 \times 4 \times 10^{-18} / 9 \times 10^{-4} \\ = 4 \times 10^{13-18} = 4 \times 10^{-5} \text{ N}$$

$$\text{due to } q_3: k|q_3q_2|/(2 \times 10^{-2})^2 = 9 \times 10^9 \times 8 \times 10^{-18} / 4 \times 10^{-4} \\ = 18 \times 10^{13-18} = 18 \times 10^{-5} \text{ N.}$$

Therefore, Pythagoras tells us

$$|F_{2,tot}| = \sqrt{4^2 + 18^2} \times 10^{-5} = 18.4 \times 10^{-5} \text{ N}$$

6) Electrons are placed on an initially uncharged conducting disk. Which of the following diagrams (showing the top-view of the disk) best represents how the charges could be distributed?

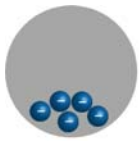


Figure 1

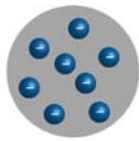


Figure 2

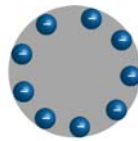


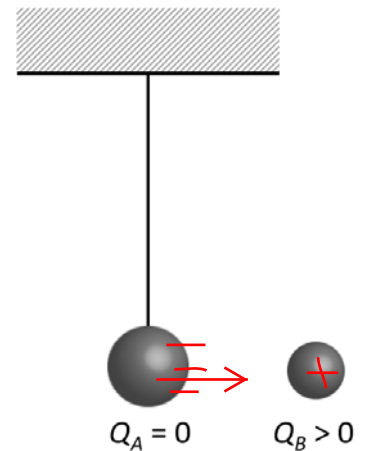
Figure 3

Same charges wish to be apart as much as possible.

- a. Figure 1
- b. Figure 2
- c. Figure 3

The next two questions pertain to the situation described below.

An uncharged conducting sphere *A* hangs from the ceiling by a non-conducting string. As shown in the figure, a positively charged conducting sphere *B* is brought close to the hanging sphere but **does not** touch it.



electrostatic induction

7) What happens to the hanging sphere *A*?

- a. It moves away from sphere *B*.
- b. It moves toward sphere *B*.
- c. It does not move.

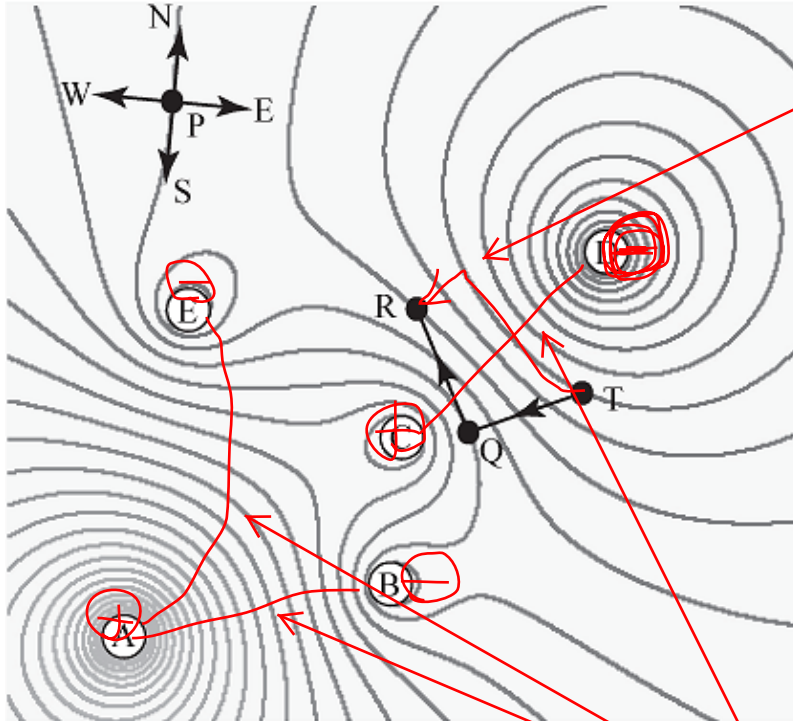
8) Now the two spheres are **touched** briefly and then separated by a small distance. What happens to the hanging sphere *A*?

- a. It moves toward sphere *B*.
- b. It does not move.
- c. It moves away from sphere *B*.

both are charged positively, since same charges wish to spread as much as possible

The next four questions pertain to the situation described below.

There are only five charges A - E on the plane in this experimental system as shown below. The equipotential contours are also described in the figure.



convenient path to calculate the needed work from T to R.

9) Suppose charge D is negative. Choose the correct statement about the positive charges in the figure from below.

- a. There are only two positive charges, A and C.
- b. There are only three positive charges, B, C and E.
- c. There are only two positive charges, B and E.
- d. C is the only positive charge in the figure.
- e. There are only three positive charges, A, B and E.

Try to draw force lines connecting charges. If you can easily connect them they have opposite signs.

10) At point P choose the correct direction of the electric field.

- a. S
- b. E
- c. N

E points the steepest descending direction of the potential landscape.

Thus, N and S are out of question.

E must be perpendicular to the equipotential lines.

11) The spacing between the adjacent equipotential contours is 5 V. A charge of 0.2 C is initially at T. To bring this charge to R, first you drag the charge to Q and then from there to R as described by line segments with arrows in the figure. What is the total work you have to do? Assume that all the dots denoting the points are exactly on the contours described in the figure above.

- a. 2 J
- b. 0 J
- c. 3 J
- d. 1 J
- e. 4 J

Needed work does not depend on the path but only the end points.
Try to draw the most convenient path connecting the same end points.

$$W = q \Delta V$$

See the above figure. We must cross 2 contour spacings = 10V = ΔV .
Therefore, $W = 0.2 \times 10 = 2 \text{ J}$.

12) A charge of 0.3 C is placed initially at Q. You drag the charge to P and you have to do work W. Now, a charge of -0.3 C is initially at P. You wish to drag the charge to Q. What is the work you have to do?

- a. none of the other answers
- b. $-W$
- c. W

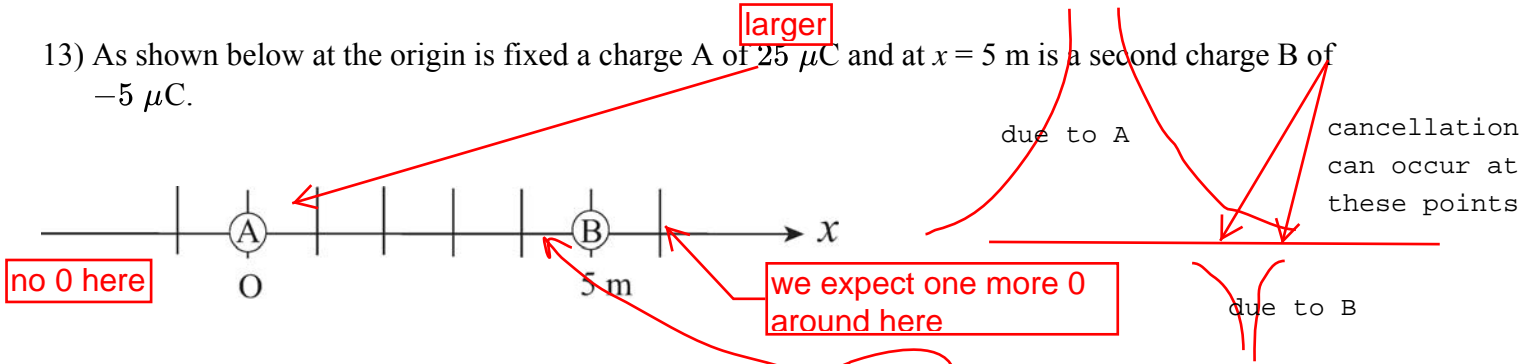
To answer this question, actually no figure is needed.

$$W = 0.3x(VP - VQ).$$

What is $(-0.3)x(VP - VQ)$?

This is just W.

13) As shown below at the origin is fixed a charge A of $25 \mu\text{C}$ and at $x = 5 \text{ m}$ is a second charge B of $-5 \mu\text{C}$.



We know the electric potential of this system vanishes at $x = 25/6 \text{ m}$, if we adopt the convention that the electric potential is zero at infinity. There is one more point along the x -axis where the electric potential vanishes.

Give its x -coordinate.

- a. $50/7 \text{ m}$
- b. $25/3 \text{ m}$
- c. $50/9 \text{ m}$
- d. $25/4 \text{ m}$
- e. 10 m

due to A: kQ_A/x

due to B: $kQ_B/(x-5)$

Using the superposition principle

$$kQ_A/x + kQ_B/(x-5) = 0 \text{ or } 25/x - 5/(x-5) = 0 \text{ or } x = 5(x-5)$$

That is, $4x = 25$.

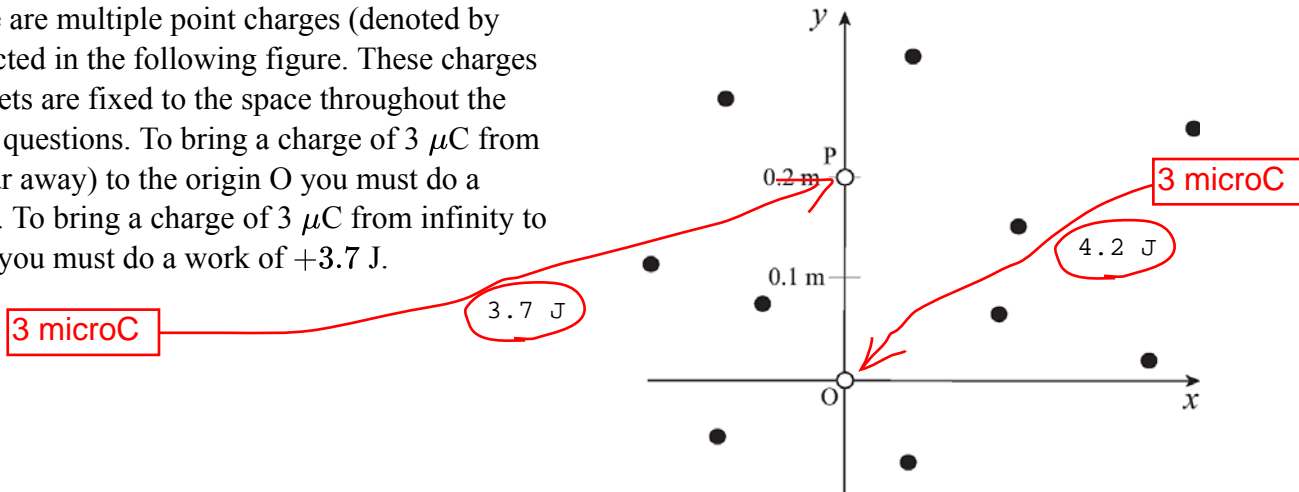
Slightly advanced comment:

$$kQ_A/|x| + kQ_B/|x-5| = 0, \text{ so } 25/|x| - 5/|x-5| = 0 \text{ or } |x| = 5|x-5|$$

This implies $x = +5(x-5)$ or $x = -5(x-5)$. The former gives $25/4$ and the latter gives $25/6$, the already given location.

The next three questions pertain to the situation described below.

On the xy -plane are multiple point charges (denoted by bullets) as depicted in the following figure. These charges denoted by bullets are fixed to the space throughout the following three questions. To bring a charge of $3 \mu\text{C}$ from infinity (very far away) to the origin O you must do a work of $+4.2 \text{ J}$. To bring a charge of $3 \mu\text{C}$ from infinity to point P instead you must do a work of $+3.7 \text{ J}$.



14) What work do you have to do to bring a charge of $-6 \mu\text{C}$ from infinity to the origin?

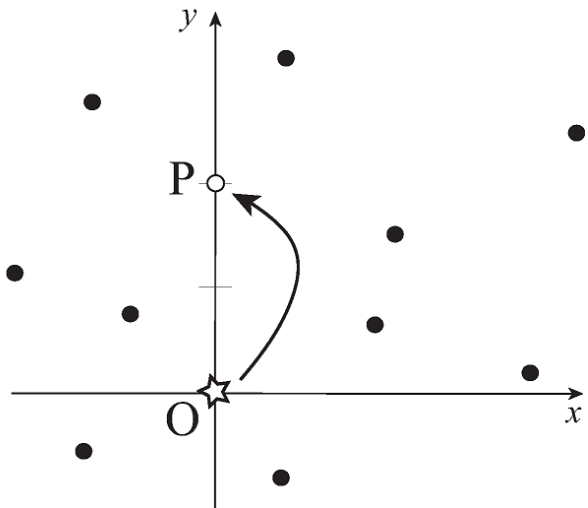
- a. $W = -8.4 \text{ J}$
- b. $W = 4.2 \text{ J}$
- c. $W = 8.4 \text{ J}$

$$W = q \Delta V = q (V_{\text{final}} - V_{\text{init}})$$

From infinity, $V_{\text{init}} = 0$. $W = q V_O$.

This is 4.2 J if $q = 3 \text{ micro C}$. Then, the answer is obviously -2 times this = -8.4 J .

15) What work do you have to do to bring a $3 \mu\text{C}$ charge (star) located originally at the origin to point P?

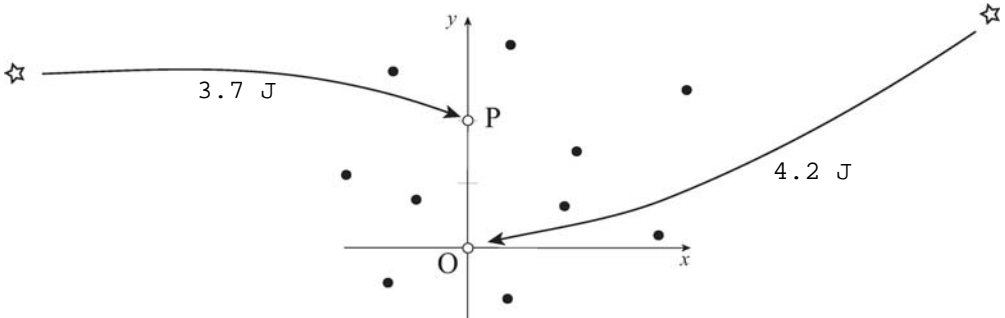


$$W = q(V_{\text{final}} - V_{\text{init}})$$

$$W = q(V_P - V_O) = 3.7 - 4.2 = -0.5 \text{ J.}$$

- a. $W = 0 \text{ J}$
- b. $W = -3.7 \text{ J}$
- c. $W = -0.5 \text{ J}$
- d. $W = 3.7 \text{ J}$
- e. $W = 0.5 \text{ J}$

16) Now, you wish to bring simultaneously one $3 \mu\text{C}$ charge to the origin O and one charge of the same $3 \mu\text{C}$ to point P when O and P are empty. The distance between O and P is 0.2 m. What work do you have to do? Assume that initially the charges to be brought in are far away from the origin and are far apart from each other as shown in the figure.



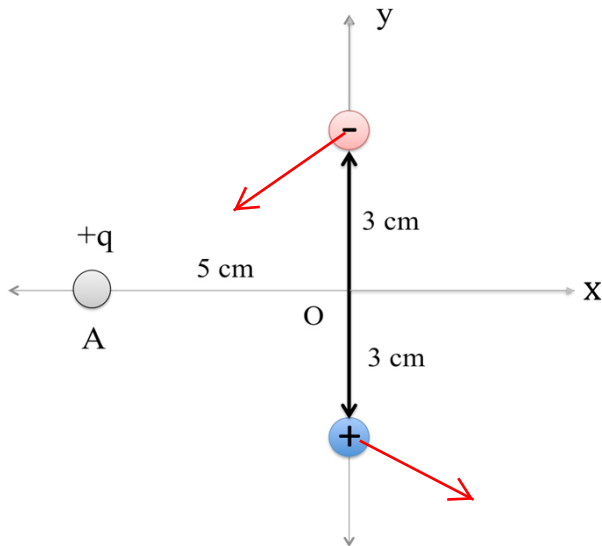
The stored potential energy is the sum of all the pairwise potential energies.

- a. $W = 7.5 \text{ J}$
- b. $W = 7.9 \text{ J}$
- c. $W = 3.7 \text{ J}$
- d. $W = -0.5 \text{ J}$
- e. $W = 8.3 \text{ J}$

If you assume there is no interaction between the charges located at P and O, then the work you have to do is just the sum $3.7 + 4.2 = 7.9 \text{ J}$. However, you must take into account the newly formed pairwise interaction:
 $kq^2/0.2 = 9 \times 10^9 (3 \times 10^{-6})^2 / 0.2 = 405 \times 10^{-12} = 0.405 \text{ J}$.
 Thus, the total work you have to do is $7.9 + 0.4 = 8.3 \text{ J}$.

The next two questions pertain to the situation described below.

An isolated electric dipole (two equal and oppositely charged objects connected by an insulating rigid rod) is at rest, initially aligned along the y-axis. The negative and positive charges of the dipole are equidistant from the origin O (each at 3cm distance from the origin). The center of the dipole is attached to the origin of the coordinate system so that the dipole can pivot in the plane around the origin. Initially no other charges are present.



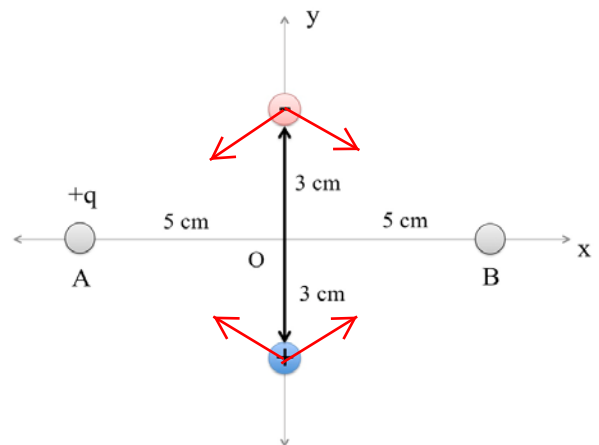
17) A positively charged object is placed at position A (5cm away from O) as shown in the Figure, while the dipole is kept fixed. Then the dipole is released gently so that it can rotate freely around the origin. In which direction will the dipole begin to turn ?

Sketch the pairwise Coulomb forces.

- a. Initially it will rotate clockwise in the diagram
- b. It won't move at all
- c. Initially it will rotate counterclockwise in the diagram.

18) Now the dipole is returned to its original orientation and kept fixed. Then, in addition to the charge at A, a fourth charge is placed at B (5 cm away from O to the side opposite of A). What should be the charge at B such that the dipole does not rotate at all when it is released gently to pivot around the origin in this case ?

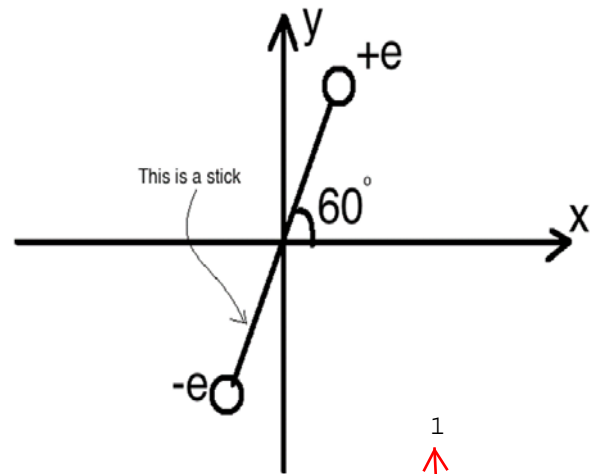
You must cancel the torque (so you must cancel the forces).



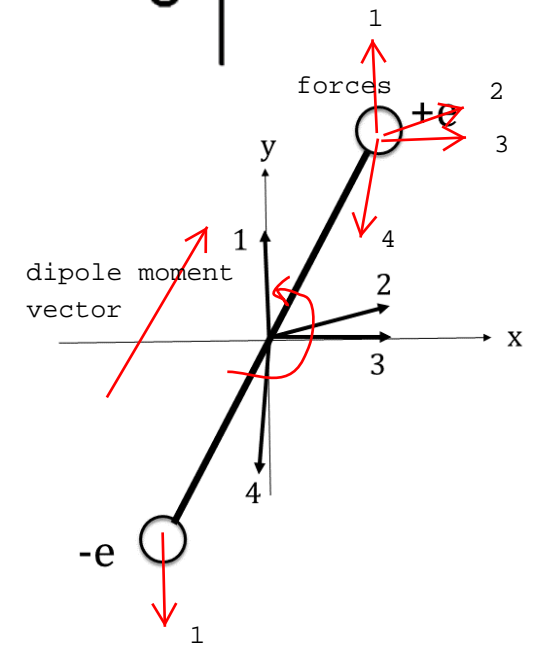
- a. +q
- b. Zero charge
- c. -q

The next two questions pertain to the situation described below.

Consider a rigid, insulating stick with two opposite charges at its two ends, as shown in the following figure. The stick is confined in the x,y plane, and the z -axis is pointing out the page. Initially the angle between the stick and x -axis is 60 degrees. The length of the stick is 2 meters. The magnitude of both charges are $e = 1.602 \times 10^{-19}$ C.



19) A homogeneous external electric field is turned on, which points in one of the directions shown below. In response to the field, the dipole starts to rotate counterclockwise in the xy plane. In which of the following directions does the electric field point (see figure below)?



Sketch the force. Then the answer is obvious.

- a. out of the page
- b. In direction 1
- c. In direction 4
- d. In direction 3
- e. In direction 2

20) If the added electric field E points in the x direction and has magnitude $E = 1$ V/m, how much total work will it have done on the dipole system during the time from when E was first switched on until when the dipole has settled into its new equilibrium position?

space needed here

$$U = -pE \cos \theta$$

- a. 10J
- b. 2 J
- c. -2 J
- d. None of the other answers
- e. 0 J

Initially, $U = -pE/2$.

After settling down $\theta = 0$, so $U = -pE$.

Therefore, the dipole must get rid of $pE/2$.

$$p = 1.6 \times 10^{-19} \times 2 = 3.2 \times 10^{-19}$$

Thus, obviously the energy is almost 0.

Of course, this is obvious without any calculation!

21) You are designing a particle accelerator that is required to accelerate electrons with initial velocity $v_i = 0$ m/s up to a final velocity of $v_f = 10^5$ m/s using a constant (homogeneous) electric field.

Remember that electrons have a mass of $m_e = 9.1 \times 10^{-31}$ kg and a charge of $q_e = -1.602 \times 10^{-19}$ C.

What should the magnitude of the electric field be if the accelerator is $L = 0.25$ m long?

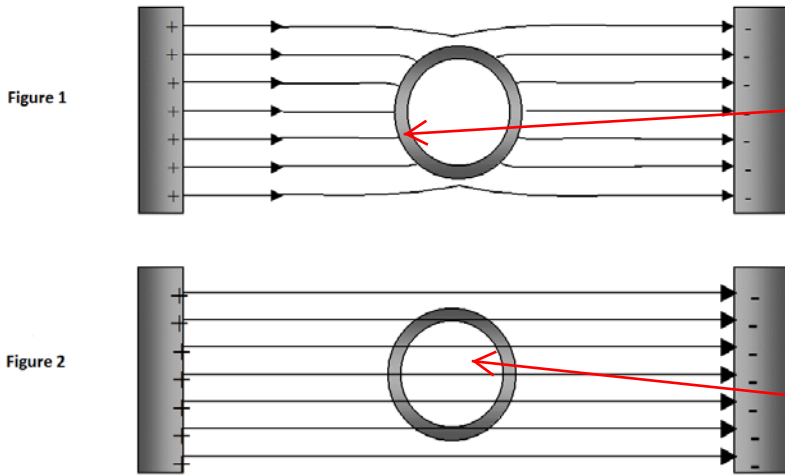
Conservation of energy

- a. $|\vec{E}| = 0.03$ N/C
- b. $|\vec{E}| = 0.23$ N/C
- c. $|\vec{E}| = 0.061$ N/C
- d. $|\vec{E}| = 0.11$ N/C
- e. $|\vec{E}| = 0.46$ N/C

$(1/2)mv^2 = ELq$, because EL is the electric potential difference.

Therefore, $E = mv^2/2qL = 9.1 \times 10^{-31} \times (10^5)^2 / 2(1.602 \times 10^{-19}) \times 0.25$
 $= 11.36 \times 10^{-31+10+19} = 11.36 \times 10^{-2}$ V/m (or N/C)

22) There is an uncharged conducting sphere in the middle of two charged, parallel conducting plates as shown below. Shown is a cross section through the middle of the sphere. Which schematic best represents the electric field line distribution inside the sphere in this situation?



The force lines must be perpendicular to the ring, so the figure is actually wrong.

No field should exist inside a metal sphere.

- a. Figure 2
- b. Figure 1
- c. Neither

23) As to the relation between electric field lines and equipotential curves, choose the correct statement from below:

see Problem 10.

- a. At a given point there is no general relation between the direction of the electric field line and the direction of the equipotential curve through the point
- b. At a given point the electric field line is perpendicular to the equipotential curve through the point.
- c. At a given point the electric field line is parallel to the equipotential curve through the point.

24) Which of the schematic figures shown best represents the electric field lines due to two point charges with $+10 \mu\text{C}$ of charge each?

Imagine water is coming out of faucets.

Figure 1

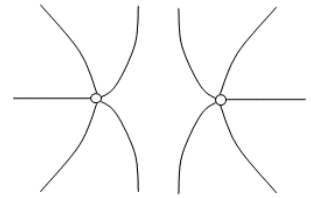


Figure 2

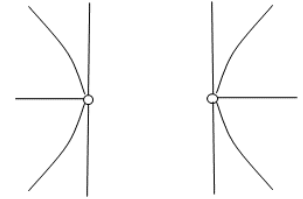
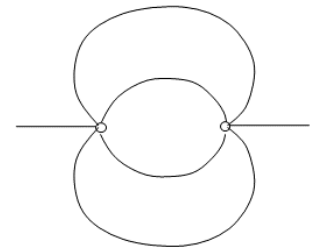


Figure 3



- a. Figure 1
- b. Figure 2
- c. Figure 3