

This version is almost version A but not exactly version A. In 26 and 27 the alphabetical positions of the correct answers are swapped in the actual version A.
Pay attention to the answers and not to the alphabets.

Discussion Section: _____ Discussion 1A Name: _____

Turn off your cell phone and put it out of sight.

Keep your calculator on your own desk. Calculators cannot be shared.

This is a closed book exam. You have ninety (90) minutes to complete it.

1. Use a #2 pencil. Do not use a mechanical pencil or pen. Darken each circle completely, but stay within the boundary. If you decide to change an answer, erase vigorously; the scanner sometimes registers incompletely erased marks as intended answers; this can adversely affect your grade. Light marks or marks extending outside the circle may be read improperly by the scanner. Be especially careful that your mark covers the **center** of its circle.
2. You may find the version of **this Exam Booklet at the top of page 2**. Mark the version circle in the TEST FORM box near the middle of your answer sheet. **DO THIS NOW!**
3. Print your **NETWORK ID** in the designated spaces at the *right* side of the answer sheet, starting in the left most column, then **mark the corresponding circle** below each character. If there is a letter "o" in your NetID, be sure to mark the "o" circle and not the circle for the digit zero. If and only if there is a hyphen "-" in your NetID, mark the hyphen circle at the bottom of the column. When you have finished marking the circles corresponding to your NetID, check particularly that you have not marked two circles in any one of the columns.
4. Print **YOUR LAST NAME** in the designated spaces at the *left* side of the answer sheet, then mark the corresponding circle below each letter. Do the same for your **FIRST NAME INITIAL**.
5. Print your UIN# in the STUDENT NUMBER designated spaces and mark the corresponding circles. You need not write in or mark the circles in the SECTION box.
6. Sign your name (**DO NOT PRINT**) on the **STUDENT SIGNATURE line**.
7. On the **SECTION line**, print your **DISCUSSION SECTION**. You need not fill in the COURSE or INSTRUCTOR lines.

*Before starting work, check to make sure that your test booklet is complete. You should have 10 **numbered pages** plus three (3) Formula Sheets following these instructions.*

Academic Integrity—Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including dismissal from the University.

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Exam Grading Policy—

The exam is worth a total of **129** points, composed of three types of questions.

MC5: *multiple-choice-five-answer questions, each worth 6 points.*

Partial credit will be granted as follows.

- (a) If you mark only one answer and it is the correct answer, you earn **6** points.
- (b) If you mark *two* answers, one of which is the correct answer, you earn **3** points.
- (c) If you mark *three* answers, one of which is the correct answer, you earn **2** points.
- (d) If you mark no answers, or more than *three*, you earn **0** points.

MC3: *multiple-choice-three-answer questions, each worth 3 points.*

No partial credit.

- (a) If you mark only one answer and it is the correct answer, you earn **3** points.
- (b) If you mark a wrong answer or no answers, you earn **0** points.

MC2: *multiple-choice-two-answer questions, each worth 2 points.*

No partial credit.

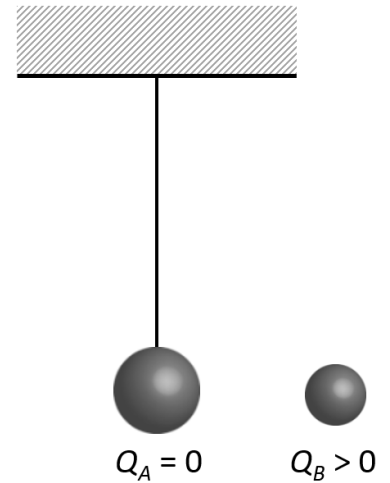
- (a) If you mark only one answer and it is the correct answer, you earn **2** points.
- (b) If you mark the wrong answer or neither answer, you earn **0** points.

Some helpful information:

- A reminder about prefixes: p (pico) = 10^{-12} ; n (nano) = 10^{-9} ; μ (micro) = 10^{-6} ; m (milli) = 10^{-3} ; k (kilo) = 10^{+3} ; M or Meg (mega) = 10^{+6} ; G or Gig (giga) = 10^{+9} .

The next two questions pertain to the situation described below.

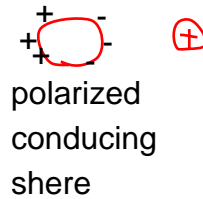
An uncharged conducting sphere A hangs from the ceiling by a non-conducting string. As shown in the figure below, a positively charged conducting sphere B is brought close to the hanging sphere but does not touch it.



induced charges, induced dipole

1) What happens to the hanging sphere A?

- a. It moves toward sphere B.
- b. It moves away from sphere B.
- c. It does not move.



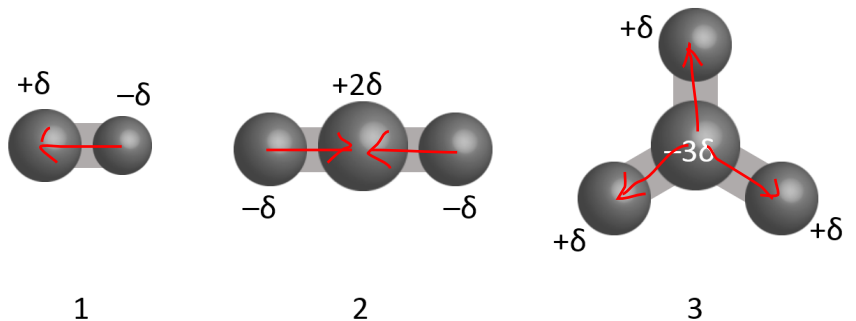
2) Now the two spheres are touched briefly and then separated to the same distance as before. What happens to the hanging sphere A?

- a. It does not move.
- b. It moves away from sphere B.
- c. It moves toward sphere B.

Since the same charges 'hate each other', when B touches A, + charges spread over A and B, charging A positively.



3) Consider the three molecules to the right. Each atom has a partial charge indicated in the figure. The bond lengths and δ are the same in all three molecules.



Which of these molecules has the largest electric dipole moment?

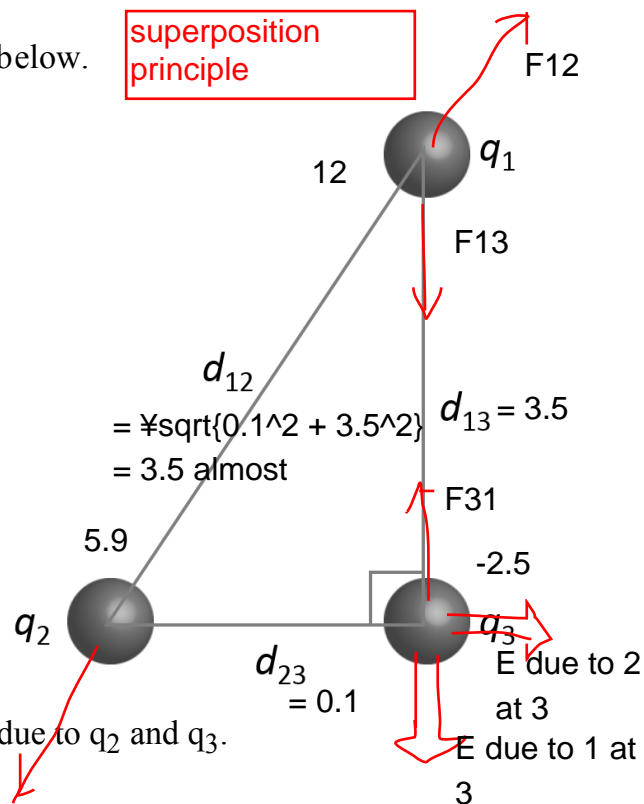
In 2 and 3, all the partial dipoles cancel.

- a. molecule 1
- b. molecule 3
- c. molecule 2

The next three questions pertain to the situation described below.

Consider the following configuration of three point charges $q_1 = 12 \mu\text{C}$, $q_2 = 5.9 \mu\text{C}$, and $q_3 = -2.5 \mu\text{C}$ arranged in a right triangle with sides $d_{23} = 0.1 \text{ m}$ and $d_{13} = 3.5 \text{ m}$. The point charges q_2 and q_3 are both on the x axis of the coordinate system.

Draw force vectors of interest.



4) Calculate the x-component $F_{1\text{tot},x}$ of the force on charge q_1 due to q_2 and q_3 .

- a. $F_{1\text{tot},x} = -0.0022 \text{ N}$
- b. $F_{1\text{tot},x} = 0.003 \text{ N}$
- c. $F_{1\text{tot},x} = 0.0015 \text{ N}$**
- d. $F_{1\text{tot},x} = -0.0018 \text{ N}$
- e. $F_{1\text{tot},x} = 6.8 \times 10^{-4} \text{ N}$

As you can see from the figure, the x component is contributed only from q_2 . The magnitude of the force is $kq_1q_2/(d_{12}^2)$

$$= (9 \times 10^9) (12 \times 10^{-6}) (5.9 \times 10^{-6}) / 3.5^2 = 52.01 \times 10^{9-6-6} = 5.2 \times 10^{-2} \text{ (N)}$$

The x component is this times $0.1/3.5$, i.e.

$$5.2 \times 10^{-2} \times 0.1/3.5 = 1.48 \times 10^{-3} \text{ N.}$$

5) If q_1 were replaced with a charge of the same magnitude but opposite sign, $-12 \mu\text{C}$, how would the magnitude $|F_{1\text{tot}}|$ of the total force on charge q_1 due to q_2 and q_3 change?

- a. It would increase.
- b. It would remain the same.**
- c. It would decrease.

Both forces in the figure flip their directions by 180 deg, so the magnitudes cannot change.

6) Calculate the magnitude of the electric field, E_{tot} , at the position of charge q_3 due to charge q_1 and q_2 .

Sketch!

- a. $E_{\text{tot}} = 0 \text{ kV/m}$
- b. $E_{\text{tot}} = 9100 \text{ kV/m}$
- c. $E_{\text{tot}} = 7200 \text{ kV/m}$
- d. $E_{\text{tot}} = 5300 \text{ kV/m}$**
- e. $E_{\text{tot}} = 1.1 \times 10^4 \text{ kV/m}$

As you can see from the figure we compute the magnitudes and $\sqrt{\text{their square sum}}$ will give the answer.

$$|E_1 \text{ at } 3| = kQ_1/d_{13}^2 = (9 \times 10^9) (12 \times 10^{-6}) / (3.5^2) = 8.816 \times 10^3 \text{ V/m}$$

$$|E_2 \text{ at } 3| = kQ_2/d_{23}^2 = (9 \times 10^9) (5.9 \times 10^{-6}) / (0.1^2) = 53.1 \times 10^{9-6+2} = 5.31 \times 10^6$$

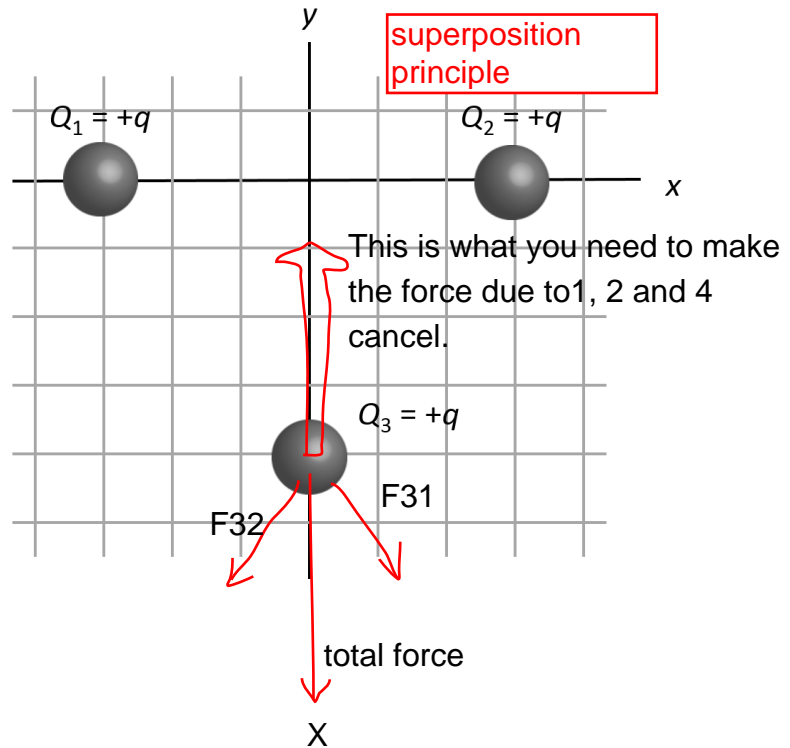
This implies that E_2 dominates the scene.

7) Consider the following arrangement of three charges on the coordinate axes. All of the charges are positive and have the same magnitude, i.e., $Q_1 = Q_2 = Q_3 = +q$. Note that the grid spacing is the same in x and y.

Where should you place a fourth charge $Q_4 = +q$ such that the total force on Q_3 is zero, i.e. $F_{3\text{tot}} = 0$?

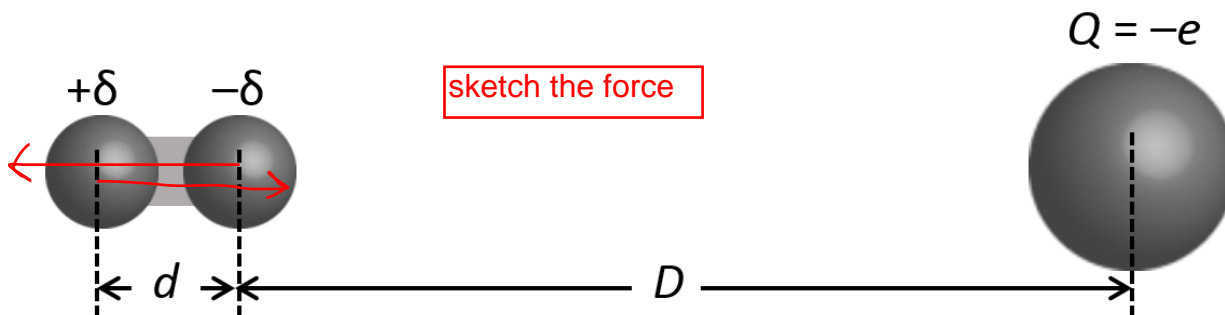
Sketch the forces

To produce with a + charge, it must be at X



- a. on the y axis below Q_3 .
- b. on the y axis above Q_3 but below the origin.
- c. on the x axis, to the left of Q_1 .
- d. on the x axis, to the right of Q_2 .
- e. at the origin.

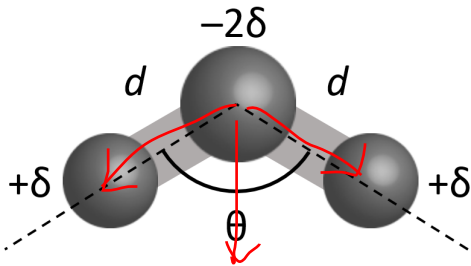
8) A molecule with an electric dipole moment is placed a distance $D = 9.9$ nm from an ion of charge $Q = -e$ and is oriented as shown in the following figure. The molecule has a bond length $d = 0.1$ nm and partial charges $\delta = +0.3e$.



Calculate the magnitude of the total force on the dipole due to the ion.

- a. $F_{\text{tot}} = 1.4 \times 10^{-14}$ N Force on -: $k e \delta / D^2 = -(9 \times 10^9) 0.3 (1.6 \times 10^{-19})^2 / (9.9 \times 10^{-9})^2 = -0.0705 \times 10^{-11}$
- b. $F_{\text{tot}} = 9.3 \times 10^{-15}$ N Force on +: $k e \delta / (D-d)^2 = (9 \times 10^9) 0.3 (1.6 \times 10^{-19})^2 / (10 \times 10^{-9})^2 = 0.0691 \times 10^{-11}$
- c. $F_{\text{tot}} = 4 \times 10^{-15}$ N
- d. $F_{\text{tot}} = 4.5 \times 10^{-14}$ N
- e. $F_{\text{tot}} = 3.1 \times 10^{-14}$ N The total force = $-0.00138 \times 10^{-11} = -1.4 \times 10^{-14}$ N

9) A molecule consists of three atoms arranged at an angle $\theta = 120^\circ$ as shown in the figure below. The atoms have partial charges, with $\delta = +0.1e$ and the same bond length $d = 1.9 \times 10^{-10}$ m.



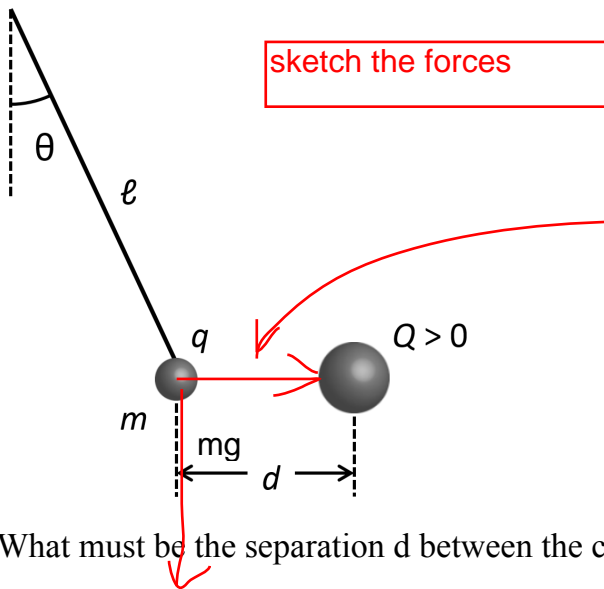
Magnitudes of the three arrows in the figure are the same.

$$\delta \times d = 0.1 (1.6 \times 10^{-19}) \times (1.9 \times 10^{-10}) \\ = 3.04 \times 10^{-1-19-10} = 3.04 \times 10^{-30}$$

Determine the net electric dipole moment p of the molecule.

- a. $p = 3 \times 10^{-30}$ C·m
- b. $p = 2 \times 10^{-30}$ C·m
- c. $p = 0$ C·m
- d. $p = 6.6 \times 10^{-30}$ C·m
- e. $p = 9.6 \times 10^{-30}$ C·m

10) Consider a point charge of mass $m = 19$ g and charge $q = -6.2$ nC suspended from the ceiling by a massless and non-conducting string of length $l = 7$ cm. When another point charge of magnitude $Q = 8$ nC approaches, the hanging charge swings and comes to rest at an angle of $\theta = 20^\circ$, as shown in the figure. Assume that the point charge q comes to rest at the same height as the point charge Q .



$$k \frac{qQ}{d^2}$$

The force balance implies

$$mg \tan \theta = k \frac{|qQ|}{d^2}$$

Therefore,

$$d^2 = \frac{k|qQ|}{mg \tan \theta} \\ = \frac{(9 \times 10^9) (8 \times 10^{-9}) (6.2 \times 10^{-9})}{(0.019 \times 9.8) \tan(20)} \\ = 6587 \times 10^{-9} = 6.587 \times 10^{-6}$$

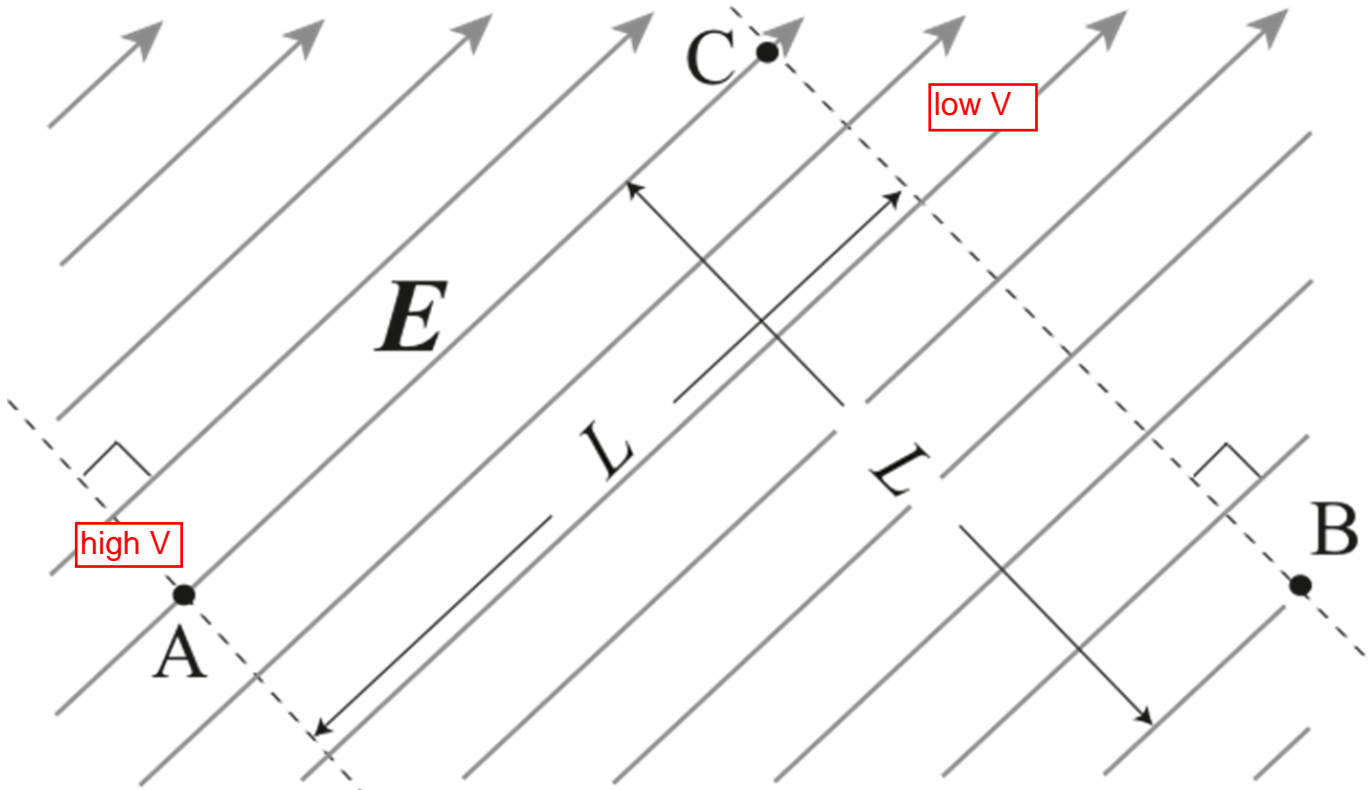
What must be the separation d between the charges?

$$\text{That is: } d = 2.567 \times 10^{-3} \text{ m}$$

- a. $d = 0.87$ mm
- b. $d = 2.6$ mm
- c. $d = 4.4$ mm
- d. $d = 1.3$ mm
- e. $d = 9.6$ mm

The next two questions pertain to the situation described below.

The following figure describes a uniform electric field E whose magnitude is E . The dashed lines denote planes perpendicular to the field.



11) What is the electric potential difference $\Delta V = V_B - V_A$ between points B and A in the figure?

- a. $\Delta V = \sqrt{2}EL$
- b. $\Delta V = 0$
- c. $\Delta V = -\sqrt{2}EL$
- d. $\Delta V = -EL$**
- e. $\Delta V = EL$

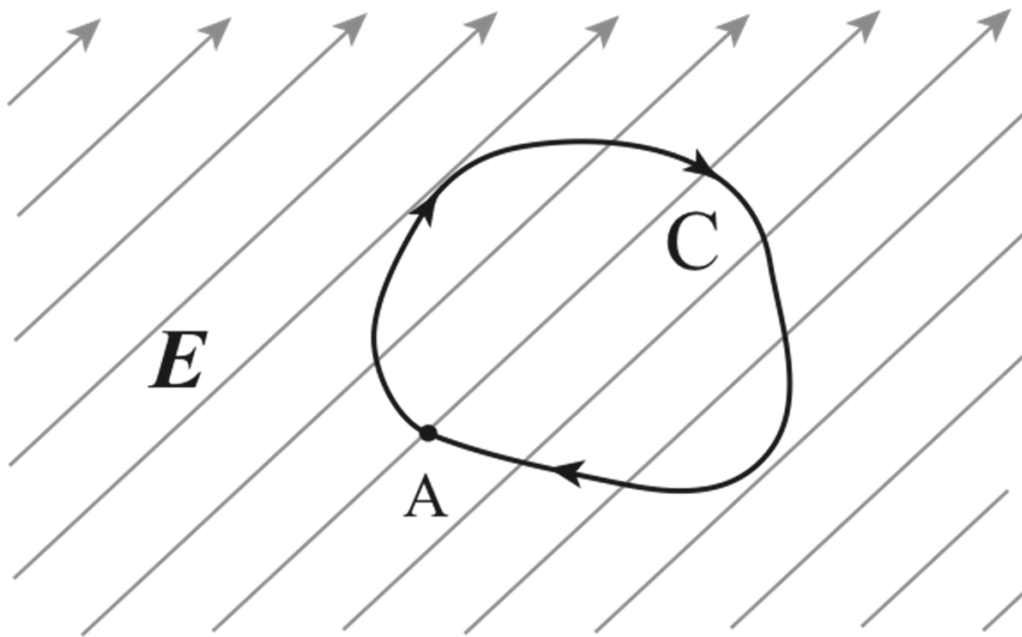
Notice that A and B have the same electric potential. Therefore, we have only to consider $V_C - V_A = -EL$. Clearly recognize $V_A > V_C$, because E points downhill.

12) A charge Q is placed initially at B. You drag the charge to point C. What is the work you must do?

Since $V_A = V_B$, the work needed is zero.

- a. $W = 0$**
- b. $W = -EL$
- c. $W = EL$

13) What work W do you have to supply to drag charge Q from point A along the curve C back to the same point A in the uniform electric field E ?



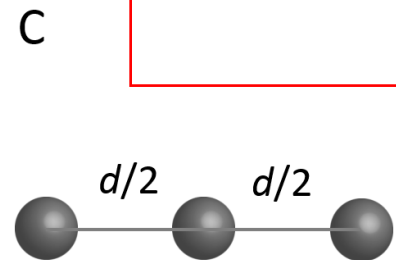
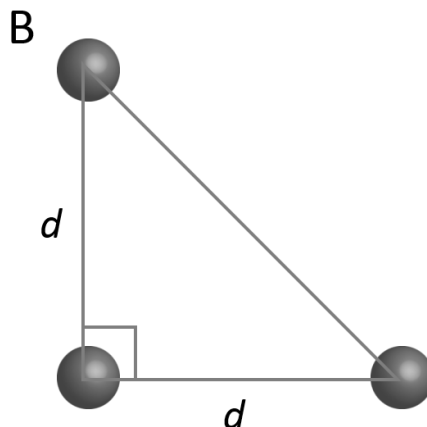
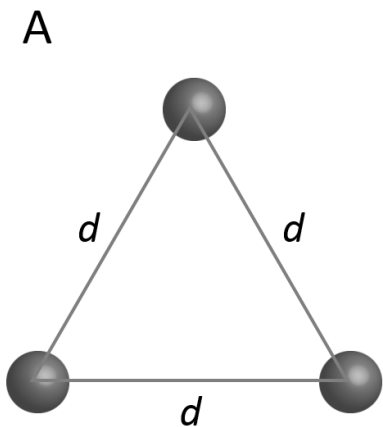
No work!

- a. $W < 0$
- b. $W = 0$
- c. $W > 0$

let us say it is Q .

14) Which of the following arrangements of equal positive charges has the highest potential energy?

Superposition principle



$$V_A = kQ^2(1/d + 1/d + 1/d) = 3kQ^2/d.$$

$$V_B = kQ^2(1/d + 1/d + 1/\sqrt{2}d) < V_A$$

$$V_C = kQ^2(2/d + 2/d + 1/d) = 5kQ^2/d > V_A$$

That is,

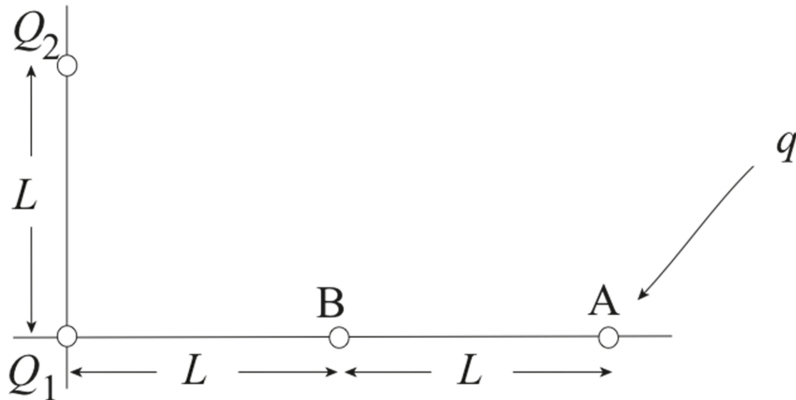
$$V_B < V_A < V_C$$

Without calculation you should have recognized this.

- a. B
- b. C
- c. A

The next two questions pertain to the situation described below.

On the y axis are two charges $Q_1 = 1 \mu\text{C}$ and $Q_2 = 3 \mu\text{C}$ separated by a distance $L = 0.4 \text{ m}$. Now the charge $q = 4 \mu\text{C}$ is brought to the position A from infinity. (Assume there are no other charges.)



$$U = kQq/d$$

Superposition principle

15) What is the required work by you to bring the charge q to A from infinity?

The initial potential when q is at infinity = 0

When q is at A, its potential energy is

- a. $W = 0.44 \text{ J}$
- b. $W = 0.17 \text{ J}$
- c. $W = 0.15 \text{ J}$
- d. $W = 0.22 \text{ J}$
- e. $W = 0.65 \text{ J}$

$$U_{\text{after}} = kq\left[\frac{Q_1}{2L} + \frac{Q_2}{\sqrt{5}L}\right]$$

$$= (9 \times 10^9) (4 \times 10^{-6}) \left[\frac{1 \times 10^{-6}}{0.8} + \frac{3 \times 10^{-6}}{(\sqrt{5}) \times 0.4}\right]$$

$$= (9 \times 10^9) (4 \times 10^{-6}) (4.64 \times 10^{-6}) = 167 \times 10^{9-6-6}$$

$$= 167 \times 10^{-3} = 0.167 \text{ J}$$

This increase of U is due to your work.

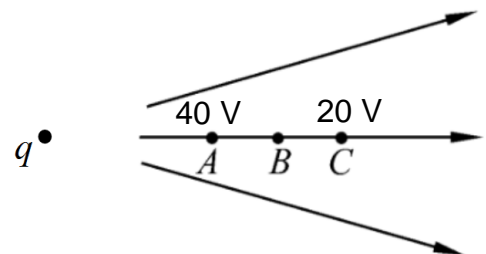
16) Suppose the work needed by you to bring q from A to B is W . If we double both the charges Q_1 and Q_2 the needed work will become

- a. $W/2$
- b. $2W$
- c. $4W$

The work scales with Q , so it will be doubled.

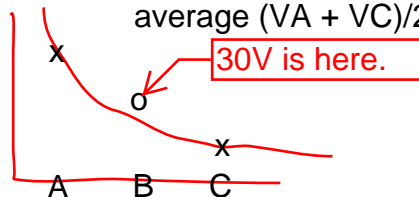
(q is NOT scaled.)

17) The arrows to the right indicate the electric field emanating from the charge q . No other charges are present. Point B is in the middle between Point A and Point C. We know Point A has an electric potential of 40 V and Point C has an electric potential of 20 V. What statement about the electric potential V_B at Point B is correct?

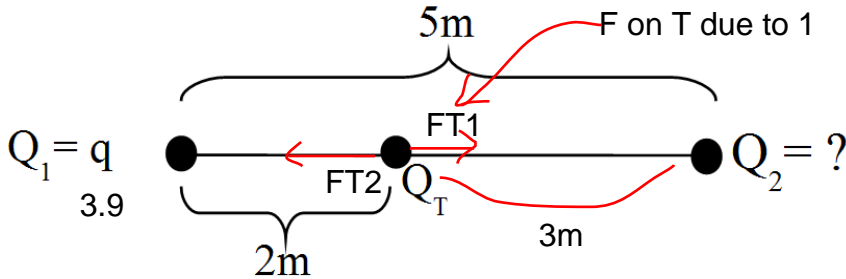


- a. $V_B < 30 \text{ V}$
- b. $V_B = 30 \text{ V}$
- c. $V_B > 30 \text{ V}$

$U \propto 1/r$, so the potential decrease from A to B must be larger than that from B to C, so V_B must be smaller than the average $(V_A + V_C)/2 = 30 \text{ V}$



18) Two charges Q_1 and Q_2 are placed on the x-axis with no other charges around. The charge Q_1 is glued to the origin and has a charge of $q = 3.9 \mu\text{C}$. The other charge Q_2 is firmly glued to the x-axis at $x = 5 \text{ m}$. A small positive test charge Q_T is brought into the position as shown in the figure and gently released. The test particle does not start to move along the x-axis. What is the value of Q_2 ?



Superposition principle

Sketch the forces.

- a. $Q_2 = 9.8 \mu\text{C}$
- b. $Q_2 = 24 \mu\text{C}$
- c. $Q_2 = 1.7 \mu\text{C}$
- d. $Q_2 = 5.8 \mu\text{C}$
- e. $Q_2 = 8.8 \mu\text{C}$**

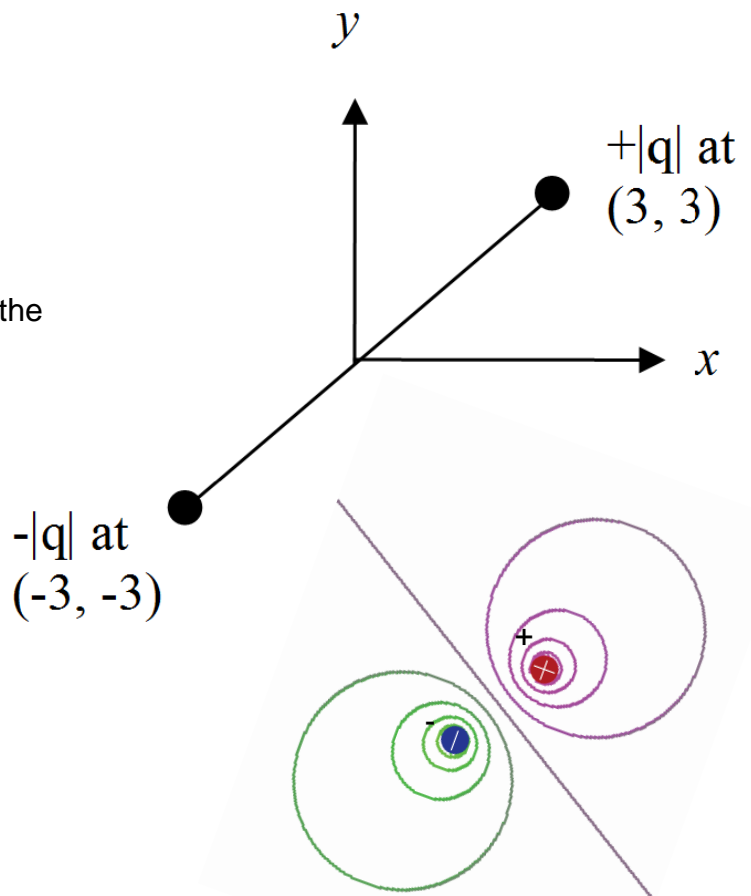
$$|F_{T1}| = kQ_1Q_T/2^2$$

$$|F_{T2}| = kQ_2Q_T/9$$

They agree: so $Q_1/4 = Q_2/9$ or $Q_2 = 9Q_1/4 = 8.775 \mu\text{C}$

19) A charge of $+|q|$ is placed at the point $(3, 3)$ and a charge of $-|q|$ is placed at the point $(-3, -3)$. There are no other charges. Which of the following points has a potential of 0 V ?

Look at the figure in the right lower corner. This tells $V = 0$ equipotential surface bisect the segment connecting $+$ and $-$ charges. That is, any point on the line $y = -x$ satisfies 0 V ,



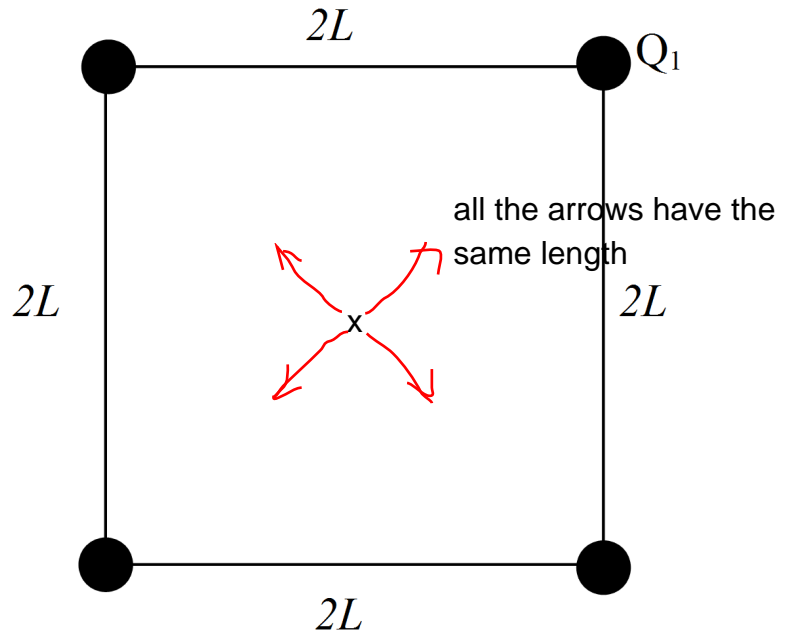
- a. $(4, -4)$**
- b. $(3, 2)$
- c. $(4, -5)$
- d. $(-5, 4)$
- e. $(-1, -1)$

The next three questions pertain to the situation described below.

Four identical negative point charges, each with charge $-Q$, are placed at the corners of a square as shown below. The edge length of the square is $2L$.

Superposition principle

Sketch the fields.



20) What is the magnitude of the electric field E at the center of the square?

- a. $E = -2\sqrt{2}k|Q|/L$
- b. $E = 2\sqrt{2}k|Q|/L^2$
- c. $E = 0$
- d. $E = -2\sqrt{2}k|Q|/L^2$
- e. $E = 2\sqrt{2}k|Q|/L$

21) What is the absolute value of the electric potential at the center of the square?

- a. $V = 2\sqrt{2}k|Q|/L$
- b. $V = 8k|Q|/L$
- c. $V = k|Q|/L$
- d. $V = 4k|Q|/L^2$
- e. $V = 0$

Every charge contributes the same amount:

$$k|Q|/\sqrt{2}L$$

Therefore,

$$V = 4 \times k|Q|/\sqrt{2}L = 2\sqrt{2}k|Q|/L$$

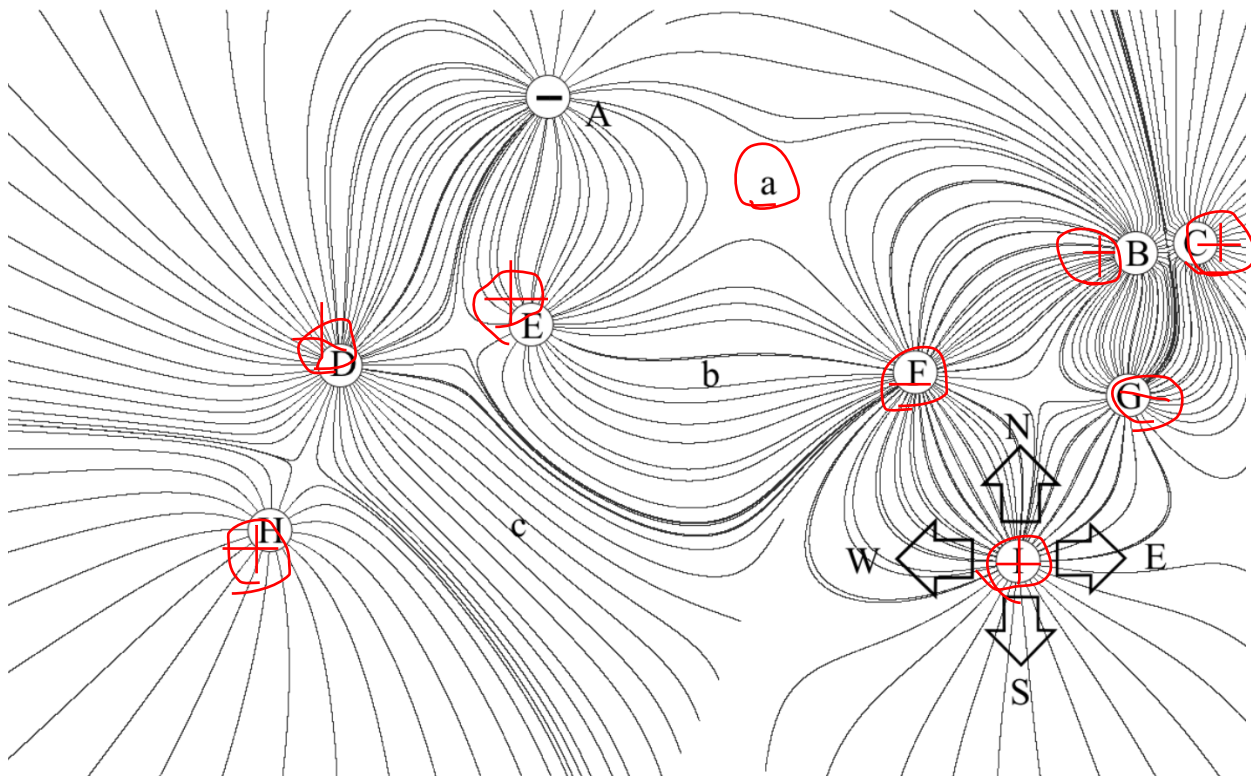
22) Now we move the charge Q_1 at the upper right corner of the square to somewhere infinitely far away while keeping the other charges fixed in place. Which of the following statements about the work done by you on the charge Q_1 is true?

- a. The work you need to do to move the charge is infinitely large.
- b. The work you need to do to move the charge from the corner of the square to infinity depends on the path we take.
- c. The work you need to do to move the charge is negative.
- d. The work you need to do to move the charge is zero.
- e. The work you need to do to move the charge is positive.

All the charges are negative, so Q_1 is repelled, so you need negative work to move Q_1 .

The next three questions pertain to the situation described below.

On a plane there are many point charges. In the following figure eight (8) charges, A, B, C, D, E, F, G, H, and I are depicted with field lines on the plane. The charge A is known to be negative.



23) Choose the correct statement from the following three statements.

- a. B, C, and E are negatively charged.
- b. D, E, and F are positively charged.
- c. F and G are negatively charged.

24) The electric field vanishes at a point very close to

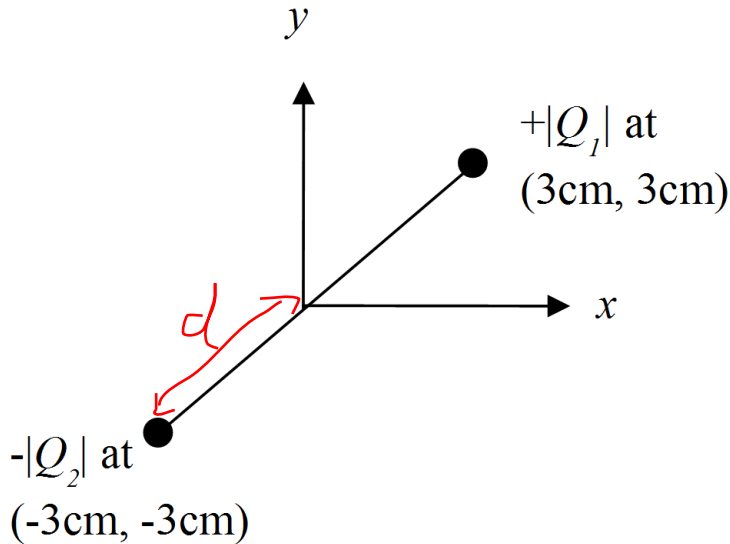
- a. a
- b. c
- c. b

25) Suppose the charge I is allowed to move, but the rest are all fixed. In which direction does charge I start to move?

- a. Roughly in the E direction.
- b. Roughly in the W direction.
- c. Roughly in the S direction.
- d. Roughly in the N direction.
- e. None of N, S, W, E.

The next two questions pertain to the situation described below.

A positive charge $Q_1 = 6 \mu\text{C}$ is located at position $(x, y) = (3 \text{ cm}, 3 \text{ cm})$ and a negative charge $Q_2 = -2 \mu\text{C}$ is located at position $(x, y) = (-3 \text{ cm}, -3 \text{ cm})$.



Superposition principle

26) What is the potential at the origin of the coordinate system $(x, y) = (0, 0)$ due to these two charges? No other charges are present.

- a. $V = 340 \text{ kV}$
- b. $V = 1700 \text{ kV}$
- c. $V = 420 \text{ kV}$
- d. $V = 850 \text{ kV}$ NOTE THAT THIS WAS LISTED AS ANSWER b IN THE PRINTED EXAM**
- e. $V = 2100 \text{ kV}$

$$V = k(Q_1/d + Q_2/d) = (Q_1 + Q_2)/d$$

$$= (9 \times 10^9) (4 \times 10^{-6}) / (\sqrt{18} \times 10^{-2})$$

$$= 8.48 \times 10^{9-6+2} = 8.48 \times 10^5 \text{ V}$$

27) How much work (absolute value) was done by you to assemble the charge configuration as shown above?

- a. $W = 0.52 \text{ J}$
- b. $W = 1.3 \text{ J}$ LISTED AS e**
- c. $W = 3.2 \text{ J}$
- d. $W = 0.65 \text{ J}$
- e. $W = 2.6 \text{ J}$

$$V = k Q_1 Q_2 / (2d)$$

$$= (9 \times 10^9) (6 \times 10^{-6}) (-2 \times 10^{-6}) / (\sqrt{72} \times 10^{-2})$$

$$= -12.727 \times 10^{9-6-6+2} = -1.27 \text{ J}$$

Physic 102 formula sheet (SP2015)

Kinematics and mechanics

$$x = x_0 + v_0t + \frac{1}{2}at^2 \quad v = v_0 + at \quad v^2 = v_0^2 + 2a\Delta x$$

$$F = ma \quad a_c = \frac{v^2}{r}$$

$$E_{\text{tot}} = K + U \quad K = \frac{1}{2}mv^2 = \frac{p^2}{2m} \quad p = mv \quad W_F = Fd \cos \theta$$

Electrostatics

$$F_{12} = k \frac{q_1 q_2}{r^2} \quad E = \frac{F}{q_0} \quad U_{12} = k \frac{q_1 q_2}{r} \quad V \equiv \frac{U}{q_0} \quad W_E = -\Delta U = -W_{\text{you}}$$

$$\text{Point charge} \quad E = k \frac{q}{r^2} \quad V = k \frac{q}{r}$$

$$\text{Electric dipole} \quad p = qd \quad \tau_{\text{dip}} = pE \sin \theta \quad U_{\text{dip}} = -pE \cos \theta$$

Resistance

$$R = \frac{V}{I} \quad I = \frac{\Delta q}{\Delta t} \quad \text{Physical resistance: } R = \rho \frac{L}{A}$$

$$P = IV = I^2 R = \frac{V^2}{R} \quad R_S = R_1 + R_2 + \dots \quad \frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

Capacitance

$$C = \frac{Q}{V} \quad \text{Parallel plate capacitor: } C = \rho \frac{\kappa \epsilon_0 A}{d} \quad E = \frac{Q}{\epsilon_0 A} \quad V = Ed$$

$$U_C = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} \quad C_P = C_1 + C_2 + \dots \quad \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

Circuits

$$\sum \Delta V = 0 \quad \sum I_{\text{in}} = \sum I_{\text{out}}$$

$$q(t) = q_{\infty}(1 - e^{-t/\tau}) \quad q(t) = q_0 e^{-t/\tau} \quad I(t) = I_0 e^{-t/\tau} \quad \tau = RC$$

Magnetism

$$F = qvB \sin \theta \quad r = \frac{mv}{qB} \quad F_{\text{wire}} = ILB \sin \theta \quad \tau_{\text{loop}} = NIAB \sin \varphi$$

$$\text{Magnetic dipole:} \quad \mu = NIA \quad \tau_{\text{dip}} = \mu B \sin \varphi \quad U_{\text{dip}} = -\mu B \cos \varphi$$

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \quad B_{\text{sol}} = \mu_0 nI$$

Electromagnetic induction

$$\mathcal{E} = -N \frac{\Delta \Phi}{\Delta t} \quad \Phi = BA \cos \varphi$$

$$|\mathcal{E}_{\text{bar}}| = BLv \quad \mathcal{E}_{\text{gen}} = \mathcal{E}_{\text{max}} \sin \omega t = \omega NAB \sin \omega t \quad \omega = 2\pi f$$

$$V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}} \quad I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} \quad \frac{V_p}{V_s} = \frac{I_s}{I_p} = \frac{N_p}{N_s}$$

Electromagnetic waves

$$\lambda = \frac{c}{f} \quad E = cB$$

$$u_E = \frac{1}{2} \epsilon_0 E^2 \quad u_B = \frac{1}{2\mu_0} B^2 \quad \bar{u} = \frac{1}{2} \epsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0} B_{\text{rms}}^2 = \epsilon_0 E_{\text{rms}}^2 = \frac{B_{\text{rms}}^2}{\mu_0} \quad S = I = \bar{u}c = \frac{P}{A}$$

$$f_0 = f_e \sqrt{\frac{1 + v_{\text{rel}}/c}{1 - v_{\text{rel}}/c}} \approx f_e \left(1 + \frac{v_{\text{rel}}}{c}\right) \quad I = I_0 \cos^2 \theta$$

Reflection and refraction

$$\begin{aligned} \theta_r &= \theta_i & \frac{1}{d_o} + \frac{1}{d_1} &= \frac{1}{f} & f &= \pm \frac{R}{2} & m &= \frac{h_i}{h_o} = -\frac{d_i}{d_o} \\ n_1 \sin \theta_1 &= n_2 \sin \theta_2 & v &= \frac{c}{n} & \sin \theta_c &= \frac{n_2}{n_1} & M &= \frac{\theta'}{\theta} \approx \frac{d_{\text{near}}}{f} \\ \text{Compound microscope:} & m_{\text{obj}} &= \frac{L_{\text{tube}}}{f_{\text{obj}}} & & M_{\text{eye}} &= \frac{d_{\text{near}}}{f_{\text{eye}}} & M_{\text{tot}} &= M_{\text{eye}} m_{\text{obj}} \end{aligned}$$

Interference and diffraction

$$\begin{aligned} \text{Double-slit interference:} & d \sin \theta = m\lambda & d \sin \theta &= \left(m + \frac{1}{2}\right)\lambda & m &= 0, \pm 1, \pm 2, \dots \\ \text{Single-slit diffraction:} & a \sin \theta = m\lambda & & & m &= 0, \pm 1, \pm 2, \dots \\ \text{Circular aperture:} & a \sin \theta \approx 1.22\lambda & & & & \end{aligned}$$

Quantum mechanics

$$\begin{aligned} E = hf &= \frac{hc}{\lambda} & \lambda &= \frac{h}{p} & \Delta p_x \Delta x &\geq \frac{\hbar}{2} & \hbar &= \frac{h}{2\pi} \\ \text{Bohr atom:} & 2\pi r_n = n\lambda & n &= 1, 2, 3, \dots & L_n &= m_e v_n r_n = n\hbar \\ r_n &= \left(\frac{\hbar^2}{m_e k e^2}\right) \frac{n^2}{Z} \approx (5.29 \times 10^{-11} \text{ m}) \frac{n^2}{Z} & E_n &= -\left(\frac{m_e k^2 e^4}{2\hbar^2}\right) \frac{Z^2}{n^2} \approx -(13.6 \text{ eV}) \frac{Z^2}{n^2} \\ \frac{1}{\lambda} &\approx (1.097 \times 10^7 \text{ m}^{-1}) Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \\ \text{Quantum atom:} & L = \sqrt{\ell(\ell+1)}\hbar & L_Z &= m_{\ell}\hbar & S_z &= m_s\hbar \\ \text{Atomic magnetism:} & \mu_{e,z} = -\frac{e}{2m_e} L_z & \mu_{s,z} &= -\frac{ge}{2m_e} S_z, g \approx 2 & \mu_B &\equiv \frac{e\hbar}{2m_e} \approx 5.8 \times 10^{-5} \text{ eV/T} \end{aligned}$$

Nuclear physics and radioactive decay

$$\begin{aligned} A &= Z + N & r &\approx (1.2 \times 10^{-15} \text{ m}) A^{1/3} & E_0 &= mc^2 \\ m_{\text{nucleus}} &= Zm_{\text{proton}} + Nm_{\text{neutron}} - \frac{|E_{\text{bind}}|}{c^2} \\ \frac{\Delta N}{\Delta t} &= \lambda N & N(t) &= N_0 e^{-\lambda t} = N_0 2^{-t/T_{1/2}} & T_{1/2} &= \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda} \end{aligned}$$

Constants and unit conversion

$$\begin{aligned} g &= 9.8 \text{ m/s}^2 & e &= 1.60 \times 10^{-19} \text{ C} \\ \epsilon_0 &= 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 & k &\equiv \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2 & \mu_0 &= 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \\ c &= \frac{1}{\sqrt{\epsilon_0\mu_0}} = 3 \times 10^8 \text{ m/s} & h &= 6.626 \times 10^{-34} \text{ J} \cdot \text{s} & hc &= 1240 \text{ eV} \cdot \text{nm} \\ 1 \text{ eV} &= 1.60 \times 10^{-19} \text{ J} & m_{\text{electron}} &= 9.11 \times 10^{-31} \text{ kg} = 511 \text{ keV}/c^2 \\ m_{\text{proton}} &= 1.673 \times 10^{-27} \text{ kg} = 938 \text{ MeV}/c^2 & m_{\text{neutron}} &= 1.675 \times 10^{-27} \text{ kg} = 939.5 \text{ MeV}/c^2 \end{aligned}$$

SI Prefixes

Power	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^0	—	—
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p