

Please check the problem 14 (and 22) with X.

Phys

Fall 2014

Last

Network-ID _____

Discussion Section: _____ Discussion TA Name: _____

**Turn off your cell phone and put it out of sight.
Keep your calculator on your own desk. Calculators cannot be shared.
This is a closed book exam. You have ninety (90) minutes to complete it.**

1. Use a #2 pencil. Do not use a mechanical pencil or pen. Darken each circle completely, but stay within the boundary. If you decide to change an answer, erase vigorously; the scanner sometimes registers incompletely erased marks as intended answers; this can adversely affect your grade. Light marks or marks extending outside the circle may be read improperly by the scanner. Be especially careful that your mark covers the **center** of its circle.
2. You may find the version of **this Exam Booklet at the top of page 2**. Mark the version circle in the TEST FORM box near the middle of your answer sheet. **DO THIS NOW!**
3. Print your **NETWORK ID** in the designated spaces at the *right* side of the answer sheet, starting in the left most column, then **mark the corresponding circle** below each character. If there is a letter "o" in your NetID, be sure to mark the "o" circle and not the circle for the digit zero. If and only if there is a hyphen "-" in your NetID, mark the hyphen circle at the bottom of the column. When you have finished marking the circles corresponding to your NetID, check particularly that you have not marked two circles in any one of the columns.
4. Print **YOUR LAST NAME** in the designated spaces at the *left* side of the answer sheet, then mark the corresponding circle below each letter. Do the same for your **FIRST NAME INITIAL**.
5. Print your UIN# in the STUDENT NUMBER designated spaces and mark the corresponding circles. You need not write in or mark the circles in the SECTION box.
6. Sign your name (**DO NOT PRINT**) on the **STUDENT SIGNATURE line**.
7. On the **SECTION line**, print your **DISCUSSION SECTION**. You need not fill in the COURSE or INSTRUCTOR lines.

*Before starting work, check to make sure that your test booklet is complete. You should have 10 **numbered pages** plus three (3) Formula Sheets following these instructions.*

Academic Integrity—Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including dismissal from the University.

This Exam Booklet is Version A. Mark the **A** circle in the **TEST FORM** box near the middle of your answer sheet. **DO THIS NOW!**

Exam Grading Policy—

The exam is worth a total of **106** points, composed of three types of questions.

MC5: *multiple-choice-five-answer questions, each worth 6 points.*

Partial credit will be granted as follows.

- (a) If you mark only one answer and it is the correct answer, you earn **6** points.
- (b) If you mark *two* answers, one of which is the correct answer, you earn **3** points.
- (c) If you mark *three* answers, one of which is the correct answer, you earn **2** points.
- (d) If you mark no answers, or more than *three*, you earn **0** points.

MC3: *multiple-choice-three-answer questions, each worth 3 points.*

No partial credit.

- (a) If you mark only one answer and it is the correct answer, you earn **3** points.
- (b) If you mark a wrong answer or no answers, you earn **0** points.

MC2: *multiple-choice-two-answer questions, each worth 2 points.*

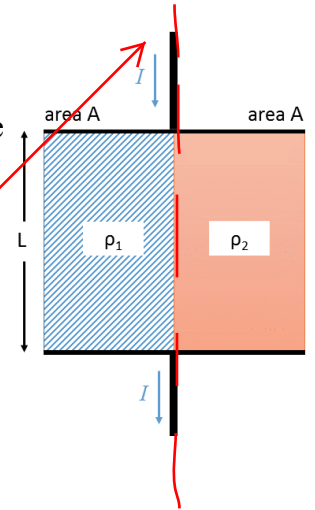
No partial credit.

- (a) If you mark only one answer and it is the correct answer, you earn **2** points.
- (b) If you mark the wrong answer or neither answer, you earn **0** points.

Some helpful information:

- A reminder about prefixes: p (pico) = 10^{-12} ; n (nano) = 10^{-9} ; μ (micro) = 10^{-6} ; m (milli) = 10^{-3} ; k (kilo) = 10^{+3} ; M or Meg (mega) = 10^{+6} ; G or Gig (giga) = 10^{+9} .

1) A single resistor is made by attaching two blocks of different materials to two conducting plates, as shown in the figure. The two blocks have identical cross sectional areas $A = 1.5 \times 10^{-4} \text{ m}^2$ and identical lengths $L = 0.1 \text{ m}$. The materials have different resistivities: $\rho_1 = 3 \times 10^{-8} \Omega \cdot \text{m}$ and $\rho_2 = 5 \times 10^{-8} \Omega \cdot \text{m}$. What is the total resistance R of the combined element?

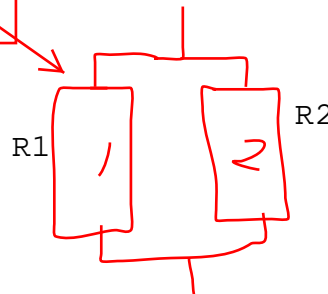


Even if you split this system along the red line, nothing changes, because both sides have the same voltage at every point of contact.

We can split and solve this as an equivalent resistive system consisting of two resistors.

We need $R = \rho L/A$.

- a. $R = 42 \mu\Omega$
- b. $R = 13 \mu\Omega$**
- c. $R = 53 \mu\Omega$
- d. $R = 0.83 \mu\Omega$
- e. $R = 21 \mu\Omega$



$$R1 = \rho_1 L/A = (3 \times 10^{-8}) \times 0.1 / (1.5 \times 10^{-4}) = 0.2 \times 10^{-4} = 20 \text{ micro ohms}$$

$$R2 = (5 \times 10^{-8}) \times \text{the same as } R1 = 100/3 \text{ micro ohms}$$

Since $R1$ and $R2$ are in parallel

$$R = R1R2 / (R1 + R2)$$

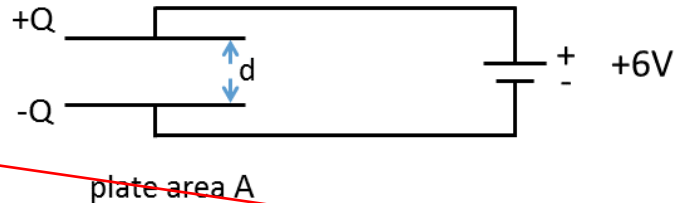
Therefore,

$$R = (20 \times 100/3) / (20 + 100/3) = 2000/160 = 12.5 \text{ micro ohms}$$

It is convenient to remember this formula

The next two questions pertain to the situation described below.

The parallel plate capacitor shown in the figure is charged by a 6 V battery. The plate separation is $d = 5 \text{ cm}$, and the surface area of each plate is $A = 500 \text{ cm}^2$.



- 2) What is the resulting magnitude of the charge Q stored on each plate of the capacitor?
- a. $Q = 0.0015 \text{ nC}$
 b. $Q = 1.3 \text{ nC}$
 c. $Q = 0.11 \text{ nC}$
 d. $Q = 0.053 \text{ nC}$
 e. $Q = 0.003 \text{ nC}$
- $C = \epsilon_0 \times A/d$ $Q = CV$
- $C = (8.85 \times 10^{-12}) (500 \times 10^{-4}) / (0.05)$
 $= 88500 \times 10^{-16} = 8.85 \times 10^{-12} \text{ F}$
- Therefore, $Q = 6C = 53.1 \times 10^{-12} \text{ C}$
 $= 53.1/1000 \times 10^{-9} \text{ C}$

- 3) If the capacitor had included a dielectric material with dielectric strength κ placed between the two plates, how would the energy stored in the capacitor change?

- a. The energy stored would increase by a factor of κ .
 b. The energy stored would decrease by a factor of κ .
 c. The energy stored would be the same.

$C = \kappa \times \epsilon_0 \times A/d$

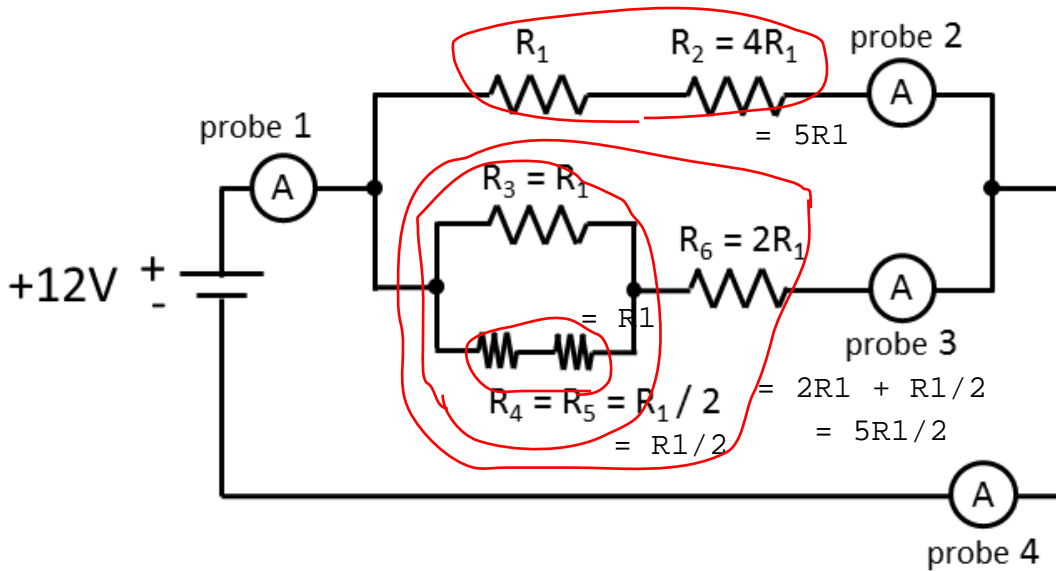
$U = CV^2/2$ (note that V is constant)

Therefore, $C \rightarrow \kappa \times C > C$

What happens is: more charge flows into the capacitor from the battery, and larger $+Q$ and $-Q$ can store larger potential energy.

The next three questions pertain to the situation described below.

A group of resistors are connected to a 12 V battery in the configuration shown in the figure, with ammeter probes placed at several locations as well. The resistance of the first element is $R_1 = 2 \Omega$.



in series
 $R = R_1 + R_2$
 in parallel
 $R = R_1 R_2 / (R_1 + R_2)$

Remember, ultimately, these rules are due to conservation of charges.

4) What is the equivalent resistance R_{EQ} of the total configuration?

- a. $R_{EQ} = 4.44 \Omega$
- b. $R_{EQ} = 20 \Omega$
- c. $R_{EQ} = 3.33 \Omega$
- d. $R_{EQ} = 0.3 \Omega$
- e. $R_{EQ} = 15 \Omega$

Proceed step by step from the smallest (or local) structures.

As you see from the above figure, one branch is $5R_1$ and the other is $5R_1/2$. They are in parallel, so
 $R_{total} = (25R_1^2/2) / (15R_1/2) = 5R_1/3 = 10/3 \text{ ohms.}$

5) Which statement best describes the measurements of current made at the locations of ammeter probes 1 and 4?

- a. The current at point 1 would be larger than at point 4.
- b. The current at point 4 would be larger than at point 1.
- c. The currents would be equal.

These two ammeters measure the same current flowing a single branch.

6) Which statement best describes the measurements of current made at the locations of ammeter probes 2 and 3?

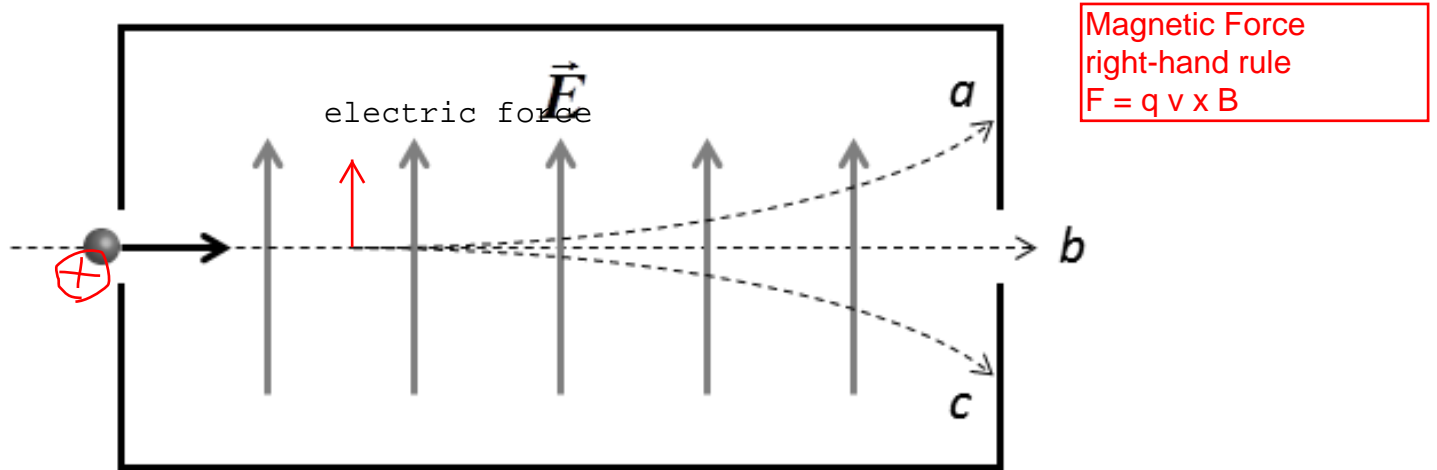
- a. The current at point 3 would be larger than at point 2.
- b. The currents would be equal.
- c. The current at point 2 would be larger than at point 3.

Since the voltage drops across these branches are identical, the currents are inversely proportional to the resistances.

Probe 3 is in the branch with a smaller resistance than Probe 2, so as to the currents
 $3 > 2.$

The next three questions pertain to the situation described below.

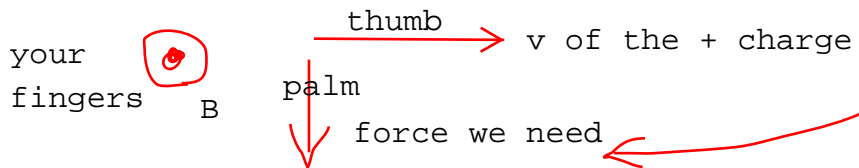
Mass spectrometers often contain a device called a velocity selector, which consists of a chamber with perpendicular \vec{E} and \vec{B} fields. The magnitudes of the fields are such that only particles with speed $v_b = 1 \times 10^5 \text{ m/s}$ travel along a straight line trajectory b through the opening at the far end of the chamber.



7) Given the \vec{E} field pointing up, which direction of the \vec{B} field would give the observed trajectory b ? You may assume that the particle charge Q is positive.

- a. out of the page
- b. to the right
- c. to the left
- d. into the page
- e. the B field is zero

Since the electric field is in the direction \uparrow , we must cancel this with the Magnetic Force \downarrow .



8) Does your answer to the previous problem change if the charge Q is negative?

- a. No
- b. Yes

Both forces are proportional to q , there should not be any change.

This is the best logic, but you should explicitly repeat the above logic with a negative charge.

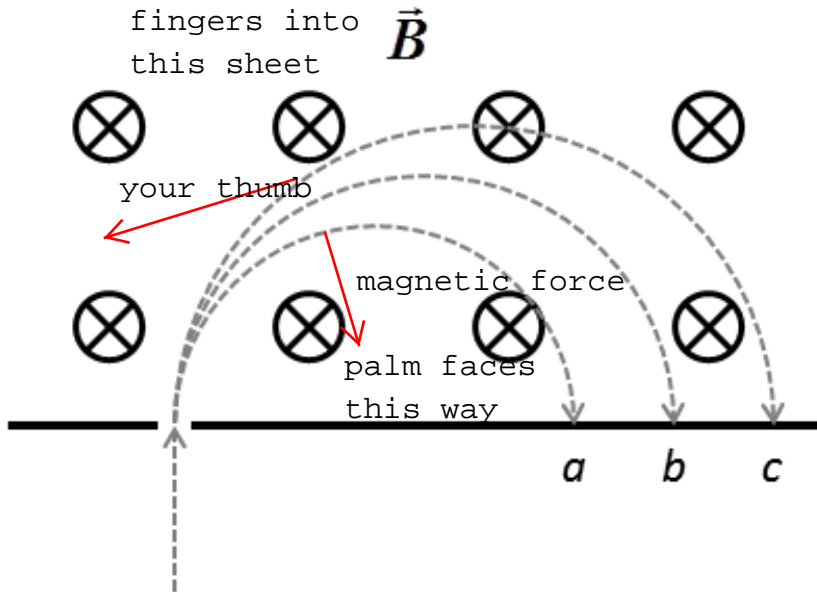
9) As shown in the figure, two other particles travel along the dotted trajectories a and c . Which of the following statements must be true? Again assume Q is positive.

- a. $v_a > v_b > v_c$
- b. $v_a = v_b = v_c$
- c. $v_a < v_b < v_c$

The electric force does not depend on the velocity. The Magnetic force that bends the particle trajectory is stronger if faster.

The next two questions pertain to the situation described below.

Consider a beam of identical particles with the same charge Q and mass m travelling along the dotted trajectories as they enter a region containing a uniform \vec{B} field pointing into the page.



The magnetic force is always perpendicular to the velocity

The direction of the right-hand thumb is the direction of the CURRENT.

In the present case, the current direction and the charge velocity are in the opposite direction, the charge must be negative.

10) What is the sign of the charge of the particles?

- a. *negative*
- b. *the sign cannot be determined*
- c. *positive*

11) Particles moving along which trajectory have the largest speed?

- a. *trajectory c*
- b. *trajectory a*
- c. *trajectory b*

$F = qv \times B$
 $a_c = v^2/r$ is the radial acceleration.

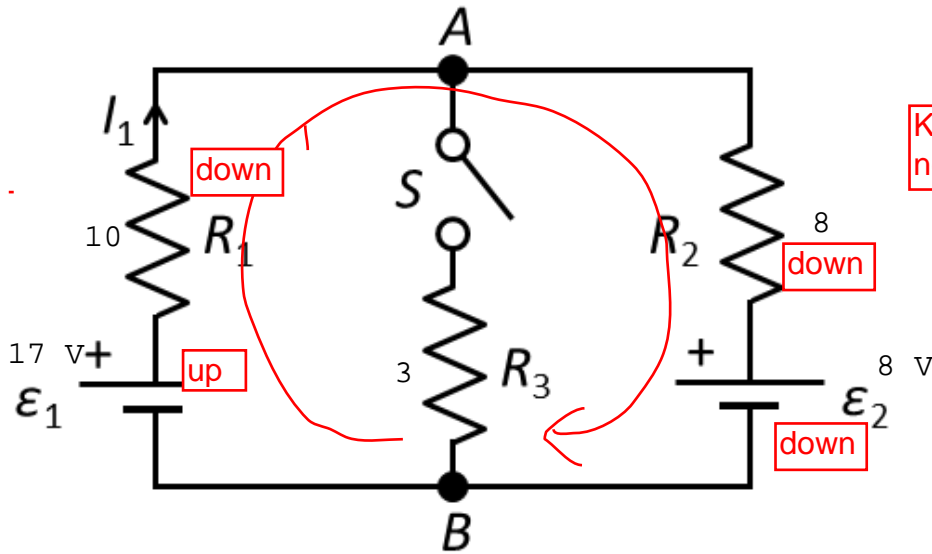
Newton teaches us that

$$mv^2/r = qvB \rightarrow r = mv/qB.$$

Larger v implies larger r .

The next three questions pertain to the situation described below.

Consider the following circuit: $R_1 = 10 \Omega$, $R_2 = 8 \Omega$, $R_3 = 3 \Omega$, $\epsilon_1 = 17 V$ and $\epsilon_2 = 8 V$. Initially the switch is open.



Kirchhoff's voltage or loop rule is needed, since the current is not zero.

12) You connect a voltmeter at points A and B in the circuit. What is the electric potential difference, $\Delta V_{AB} = V_A - V_B$, measured between those points?

- a. $\Delta V_{AB} = 12 V$
- b. $\Delta V_{AB} = 17 V$
- c. $\Delta V_{AB} = 9 V$

$$\epsilon_1 - I_1 R_1 - I_1 R_2 - \epsilon_2 = 0$$

or

$$17 - I_1(18) - 8 = 0 \rightarrow I_1 = 0.5 A$$

If $B = 0$, then A is $17 - 0.5 \times 10 = 12 V$

13) Now the switch is closed. Using the same voltmeter as above, you measure the electric potential difference $\Delta V_{AB} = V_A - V_B = 3.33 V$. In which direction does the current flow through resistor R_3 ?

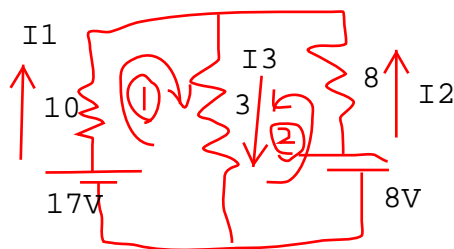
- a. no current flows
- b. up
- c. down

Obviously, a positive current flows from A to B .

This problem is a bit hard (algebraically), AND there is NO CORRECT solution to choose!!!

14) What is the current I_1 through resistor R_1 after the switch is closed?

- a. $I_1 = 0.8 A$
- b. $I_1 = 1.7 A$
- c. $I_1 = 1.4 A$



$$I_1 + I_2 = I_3 \text{ (junction rule)}$$

Loop1

$$17 - 10I_1 - 3(I_1 + I_2) = 0$$

or $17 = 13I_1 + 3I_2$,

Loop2

$$8 - 8I_2 - 3(I_1 + I_2) = 0$$

or $8 = 3I_1 + 11I_2$.

We must solve this set:

$$17 \times 11 = 13 \times 11 I_1 + 3 \times 11 I_2$$

$$8 \times 3 = 3 \times 3 I_1 + 3 \times 11 I_2$$

thus

$$17 \times 11 - 8 \times 3 = (13 \times 11 - 3 \times 3) I_1$$

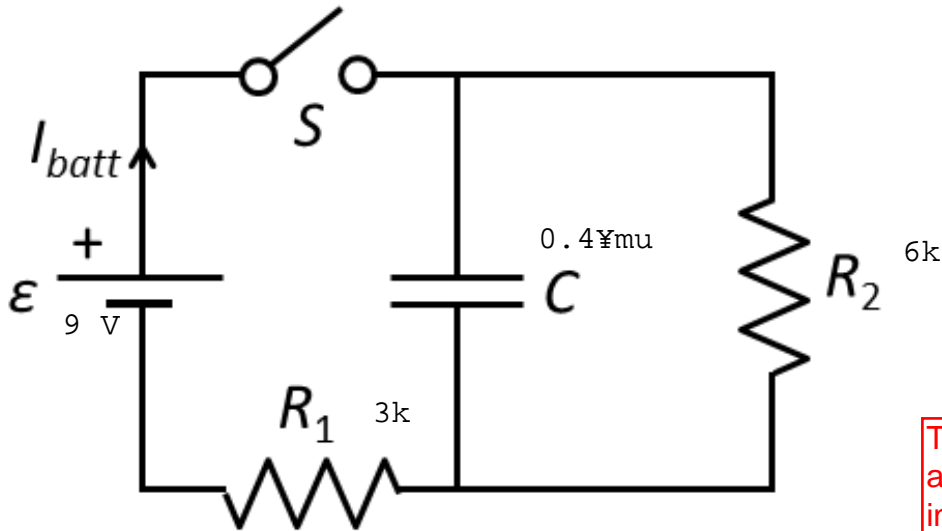
or $I_1 = 163/134 = 1.2164 A$.

$$I_2 = (17 - 13 \times 1.2164) / 3 = 0.3956 A$$

Check: from I_1 : $V_a - V_b = 17 - 10 \times 1.2164 = 4.836 V$
 from I_2 : $V_a - V_b = 8 - 8 \times 0.3956 = 4.835 V$
 from I_3 : $V_a - V_b = 3(1.2164 + 0.3956) = 4.836 V$

The next three questions pertain to the situation described below.

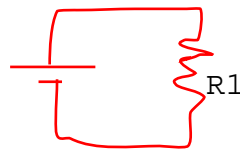
Consider the following RC circuit: $R_1 = 3\text{ k}\Omega$, $R_2 = 6\text{ k}\Omega$, $C = 0.4\text{ }\mu\text{F}$, and $\varepsilon = 9\text{ V}$. Initially the capacitor is uncharged. At some time, the switch is closed.



The charge or the voltage across C cannot change instantaneously.

15) What is the current out of the battery, I_{batt} , immediately after the switch is closed?

- a. $I_{batt} = 1\text{ mA}$
- b. $I_{batt} = 3\text{ mA}$**
- c. $I_{batt} = 1.5\text{ mA}$
- d. $I_{batt} = 0\text{ mA}$
- e. $I_{batt} = 22\text{ mA}$

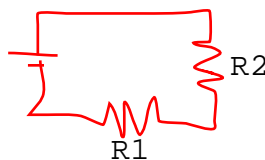


This is the equivalent circuit:
 $I = 9/3000 = 3\text{ mA}$.

16) What is the current out of the battery, I_{batt} , a long time after the switch is closed?

- a. $I_{batt} = 1.5\text{ mA}$
- b. $I_{batt} = 22\text{ mA}$
- c. $I_{batt} = 0\text{ mA}$
- d. $I_{batt} = 3\text{ mA}$
- e. $I_{batt} = 1\text{ mA}$**

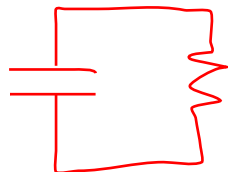
If C is charged full, there is no current through it, so you can totally ignore it.



$$I = 9/(3000+6000) = 1\text{ mA}.$$

17) How much time does it take for the charge Q to decrease to 50% of its initial value after the switch is re-opened?

- a. $t_{50\%} = 1.7\text{ ms}$**
- b. $t_{50\%} = 0.83\text{ ms}$
- c. $t_{50\%} = 2.5\text{ ms}$



$$\tau = CR = 6000 \times 0.4 \times 10^{-6} = 2.4\text{ m sec}.$$

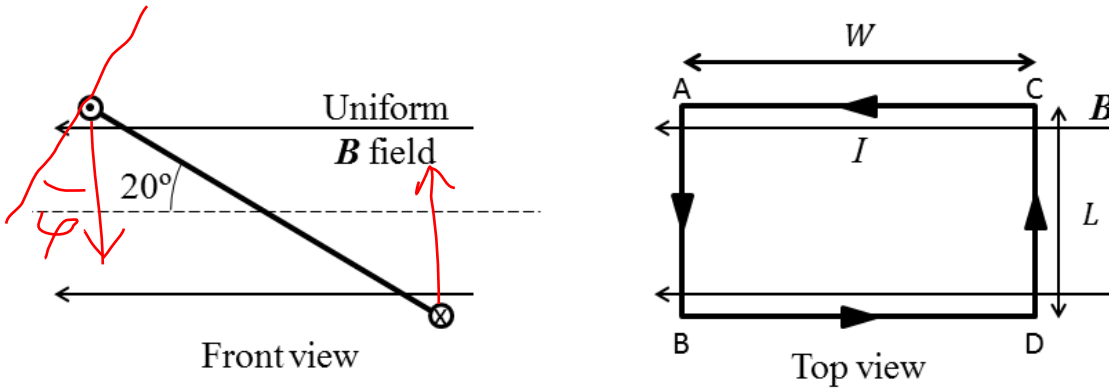
The equivalent circuit is as Left
 $Q(t) = Q(0) \exp(-t/\tau)$

$$Q(t)/Q(0) = 1/2 = \exp(-t/\tau)$$

or

$$t = \tau \ln 2 = 2.4 \times 0.693 = 1.66\text{ m sec}$$

The next three questions pertain to the situation described below.



A rectangular loop of length $L = 0.445 \text{ m}$ and width $W = 0.285 \text{ m}$ carries a current $I = 3.9 \text{ A}$ is exposed to a uniform magnetic field of magnitude $B = 5.5 \text{ T}$, as shown in the figure.

18) What is the magnitude of the force experienced on wire segment **AB**?

- a. $F_{AB} = 2.09 \text{ N}$
- b. $F_{AB} = 3.26 \text{ N}$
- c. $F_{AB} = 9.55 \text{ N}$**
- d. $F_{AB} = 8.97 \text{ N}$
- e. $F_{AB} = 5.74 \text{ N}$

$$F = ILB \sin \theta$$

Notice that $\theta = 90 \text{ deg.}$

$$F = 3.9 \times 0.445 \times 5.5 = 9.545 \text{ N}$$

19) What is the magnitude of the torque exerted on the loop?

Sketch the forces as above.

- a. 2.56 Nm**
- b. 1.06 Nm
- c. 0.93 Nm

$$\begin{aligned} \text{torque} &= F \times W \cos \phi = 9.545 \times 0.285 \times \cos 20 \\ &= 2.556 \text{ Nm} \end{aligned}$$

Or $\text{torque} = NIAB \sin \phi$

$$\begin{aligned} \text{torque} &= IWL B \cos(90 - 20) \\ &= 1 \times 3.9 \times 0.445 \times 0.285 \times 5.5 \sin 70 = 2.556 \text{ Nm} \end{aligned}$$

Here, ϕ is the angle from 'perp.'
Do not confuse definitions of angles.

20) As seen from the front, in which direction will the loop rotate?

- a. Counterclockwise**
- b. Clockwise

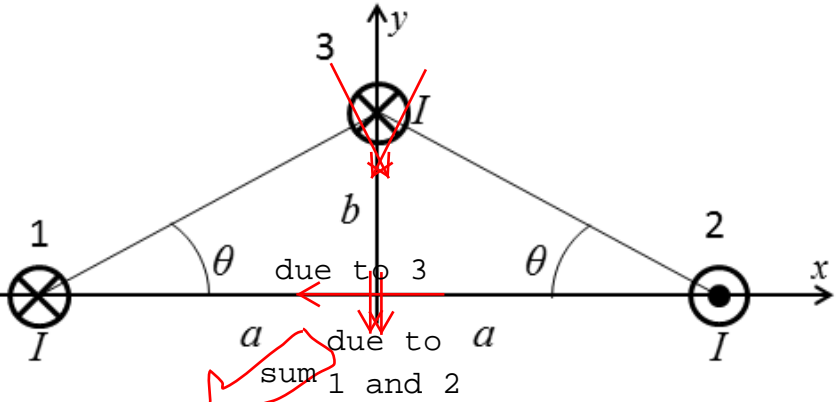
See the figure.

The next three questions pertain to the situation described below.

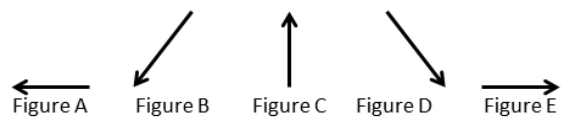
Three long, straight wires, are arranged as shown in the figure:

$a = 3.5 \text{ m}$, $b = 2 \text{ m}$ and $\theta = 30^\circ$. Each wire carries a current $I = 8.8 \text{ A}$

B generated by a wire.
 $|B| = \mu_0 I / 2\pi r$
 Right-screw rule



21) Which vector best represents the direction of the total magnetic field at the origin due to the three wires?



- a. Figure B
- b. Figure C
- c. Figure D
- d. Figure E
- e. Figure A

however, this is not quite correct.



22) What is the magnitude of the total magnetic field at the origin, B_{total} , due to the three wires?
 B due to 1 and 2 are identical and in the -y direction.

Magnitude = $\mu_0 I / (2\pi \times a)$
 $= 4\pi \times 10^{-7} \times 8.8 / (2\pi \times 3.5)$
 $= 2 \times 10^{-7} \times 8.8 / 3.5 = 5.028 \times 10^{-7} \text{ T}$

Therefore, the y-component of B is $-10.056 \times 10^{-7} \text{ T}$.
 B due to 3 is in the -x direction.

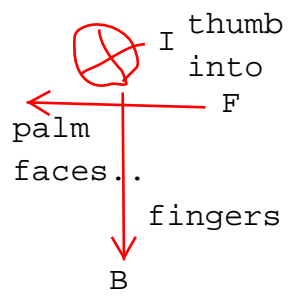
Magnitude = $4\pi \times 10^{-7} \times 8.8 / (2\pi \times 2) = 8.8 \times 10^{-7} \text{ T}$
 Hence, the magnitude of the total B is
 $B_{total} = \sqrt{8.8^2 + 10.056^2} \times 10^{-7} = 13.75 \times 10^{-7}$

23) What is the direction of the net force on one meter of Wire 3 due to the other two wires?

- a. +x-direction
- b. 0
- c. +y-direction
- d. -y-direction
- e. -x-direction

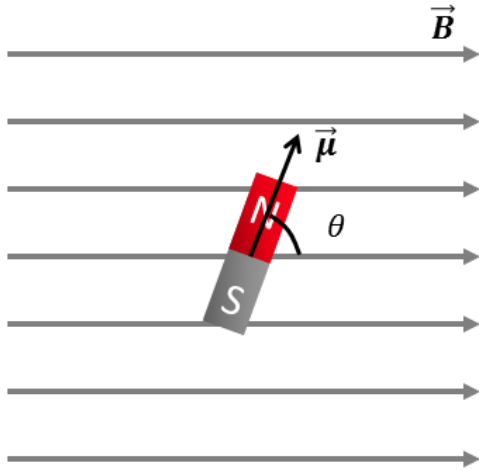
As seen from the figure, the B field is in the -y direction.

Right-hand rule



The next two questions pertain to the situation described below.

A bar magnet sits at an angle $\theta = 50^\circ$ in a magnetic field as shown. Assume the bar magnet is a magnetic dipole.



$$U = -\mu B \cos \theta$$

$$\tau = \mu B \sin \theta$$

24) When the magnet is aligned with the magnetic field $U_{dip} = -1.5 J$. What is the torque τ experienced by the magnet in the figure above.

$$\text{Minimum } U \text{ implies } \mu B = 1.5 \text{ Nm.}$$

$$\tau = 1.5 \sin 50 = 1.149 \text{ N m}$$

a. $\tau = 1.79 \text{ N m}$

b. $\tau = 0.964 \text{ N m}$

c. $\tau = 1.15 \text{ N m}$

25) If the magnetic field has strength $|\vec{B}| = 0.8 T$, what is the magnetic moment of the bar magnet?

a. $\mu = 1.88 \text{ A} \cdot \text{m}^2$

b. $\mu = 0.919 \text{ A} \cdot \text{m}^2$

c. $\mu = 1.2 \text{ A} \cdot \text{m}^2$

d. $\mu = 1.44 \text{ A} \cdot \text{m}^2$

e. $\mu = 0.771 \text{ A} \cdot \text{m}^2$

$$\mu = |U_{\min}| / B = 1.5 / 0.8 = 1.875$$

Kinematics and mechanics:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$F = ma$$

$$a_c = \frac{v^2}{r}$$

$$E_{tot} = K.E. + P.E.$$

$$K.E. = \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

$$p = mv$$

$$W_F = Fd \cos \theta$$

Electrostatics:

$$F_{12} = \frac{kq_1 q_2}{r^2}$$

$$E \equiv \frac{F}{q_0}$$

$$U_{12} = \frac{kq_1 q_2}{r}$$

$$V \equiv \frac{U}{q_0}$$

$$W_E = -\Delta U = -W_{you}$$

Point charge:

$$E = \frac{kq}{r^2}$$

$$V = \frac{kq}{r}$$

Electric dipole:

$$p \equiv qd$$

$$\tau_{dip} = pE \sin \theta$$

$$U_{dip} = -pE \cos \theta$$

Resistance:

$$R \equiv \frac{V}{I}$$

$$I = \frac{\Delta q}{\Delta t}$$

Physical resistance: $R = \rho \frac{L}{A}$

$$P = IV = I^2 R = \frac{V^2}{R}$$

$$R_S = R_1 + R_2 + \dots$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

Capacitance:

$$C \equiv \frac{Q}{V}$$

Parallel plate capacitor: $C = \frac{\kappa \epsilon_0 A}{d}$, $E = \frac{Q}{\epsilon_0 A}$, $V = Ed$

$$U_C = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

$$C_P = C_1 + C_2 + \dots$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

Circuits:

$$\sum \Delta V = 0$$

$$\sum I_{in} = \sum I_{out}$$

$$q(t) = q_\infty (1 - e^{-t/\tau})$$

$$q(t) = q_0 e^{-t/\tau}$$

$$I(t) = I_0 e^{-t/\tau}$$

$$\tau = RC$$

Magnetism:

$$F = qvB \sin \theta$$

$$r = \frac{mv}{qB}$$

$$F_{wire} = ILB \sin \theta$$

$$\tau_{loop} = NIAB \sin \phi$$

Magnetic dipole:

$$\mu \equiv NIA$$

$$\tau_{dip} = \mu B \sin \phi$$

$$U_{dip} = -\mu B \cos \phi$$

$$B_{wire} = \frac{\mu_0 I}{2\pi r}$$

$$B_{sol} = \mu_0 nI$$

Electromagnetic induction:

$$\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$$

$$|\varepsilon_{bar}| = BLv$$

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$$\Phi = BA \cos \varphi$$

$$\varepsilon_{gen} = \varepsilon_{max} \sin \omega t = \omega NAB \sin \omega t$$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

$$\omega = 2\pi f$$

$$\frac{V_p}{V_s} = \frac{I_s}{I_p} = \frac{N_p}{N_s}$$

Electromagnetic waves:

$$\lambda = \frac{c}{f}$$

$$E = cB$$

$$u_E = \frac{1}{2} \varepsilon_0 E^2$$

$$u_B = \frac{1}{2\mu_0} B^2$$

$$\bar{u} = \frac{1}{2} \varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0} B_{rms}^2 = \varepsilon_0 E_{rms}^2 = \frac{B_{rms}^2}{\mu_0}$$

$$S = I = \bar{u}c$$

$$f' = f \left(1 \pm \frac{u}{c} \right)$$

$$I = I_0 \cos^2 \theta$$

Reflection and refraction:

$$\theta_r = \theta_i$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$f = \pm \frac{R}{2}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$v = \frac{c}{n}$$

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$M = \frac{\theta'}{\theta} \approx \frac{d_{near}}{f}$$

Interference and diffraction:

Double slit interference:

$$d \sin \theta = m\lambda$$

$$d \sin \theta = (m + \frac{1}{2})\lambda$$

$$m = 0, \pm 1, \pm 2, \dots$$

Single-slit diffraction:

$$a \sin \theta = m\lambda$$

$$m = \pm 1, \pm 2, \dots$$

Circular aperture:

$$a \sin \theta \approx 1.22\lambda$$

Thin film: $\delta_1 = (0 \text{ or } \frac{1}{2})\lambda$

$$\delta_2 = (0 \text{ or } \frac{1}{2})\lambda + 2t \frac{n_{film}}{\lambda_0}$$

$$|\delta_2 - \delta_1| = (m \text{ or } m + \frac{1}{2})\lambda$$

$$m = 0, 1, 2, \dots$$

Quantum mechanics:

$$E = hf = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{p}$$

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

$$\hbar \equiv \frac{h}{2\pi}$$

Bohr atom: $2\pi r_n = n\lambda$ $n = 1, 2, 3, \dots$

$$L_n = m v_n r_n = n\hbar$$

$$r_n = \left(\frac{\hbar^2}{mke^2} \right) \frac{n^2}{Z} \approx (5.29 \times 10^{-11} \text{ m}) \frac{n^2}{Z}$$

$$E_n = - \left(\frac{mk^2 e^4}{2\hbar^2} \right) \frac{Z^2}{n^2} \approx -(13.6 \text{ eV}) \frac{Z^2}{n^2}$$

$$\frac{1}{\lambda} \approx (1.097 \times 10^7 \text{ m}^{-1}) Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Quantum atom: $L = \sqrt{\ell(\ell+1)}\hbar$

$$L_z = m_\ell \hbar$$

Nuclear physics and radioactive decay:

$$A = Z + N$$

$$r \approx (1.2 \times 10^{-15} \text{ m}) A^{1/3}$$

$$E_0 = mc^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-t/T_{1/2}}$$

$$T_{1/2} \equiv \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$$

Constants and unit conversions:

$$g = 9.8 \text{ m/s}^2$$

$$e = 1.60 \times 10^{-19} \text{ C}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2$$

$$k \equiv \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ Nm}^2 / \text{C}^2$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/s}$$

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$$

$$hc = 1240 \text{ nm} \cdot \text{eV}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

$$m_{\text{proton}} = 1.67 \times 10^{-27} \text{ kg} = 938 \text{ MeV}$$

$$m_{\text{electron}} = 9.11 \times 10^{-31} \text{ kg} = 511 \text{ keV}$$

SI Prefixes		
Power	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^0	—	—
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p