

Soln with explanations.
Always check the principles stated clearly in the comments.

____Network-ID____

not be shared.

This is a closed book exam. You have ninety (90) minutes to complete it.

When the problems are with some problems, YO criticizes them.
Students as independent human beings should always be prepared to criticize the course and the instructors for their accuracy/correctness.

circle may be read improperly by the scanner. Be especially careful that your mark covers the **center** of its circle.

2. You may find the version of **this Exam Booklet at the top of page 2**. Mark the version circle in the TEST FORM box near the middle of your answer sheet. **DO THIS NOW!**
3. Print your **NETWORK ID** in the designated spaces at the *right* side of the answer sheet, starting in the left most column, then **mark the corresponding circle** below each character. If there is a letter "o" in your NetID, be sure to mark the "o" circle and not the circle for the digit zero. If and only if there is a hyphen "-" in your NetID, mark the hyphen circle at the bottom of the column. When you have finished marking the circles corresponding to your NetID, check particularly that you have not marked two circles in any one of the columns.
4. Print **YOUR LAST NAME** in the designated spaces at the *left* side of the answer sheet, then mark the corresponding circle below each letter. Do the same for your **FIRST NAME INITIAL**.
5. Print your UIN# in the STUDENT NUMBER designated spaces and mark the corresponding circles. You need not write in or mark the circles in the SECTION box.
6. Sign your name (**DO NOT PRINT**) on the **STUDENT SIGNATURE line**.
7. On the **SECTION line**, print your **DISCUSSION SECTION**. You need not fill in the COURSE or INSTRUCTOR lines.

*Before starting work, check to make sure that your test booklet is complete. You should have 10 **numbered pages** plus three (3) Formula Sheets following these instructions.*

Academic Integrity—Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including dismissal from the University.

This Exam Booklet is Version A. Mark the **A** circle in the **TEST FORM** box near the middle of your answer sheet. **DO THIS NOW!**

Exam Grading Policy—

The exam is worth a total of **116** points, composed of three types of questions.

MC5: *multiple-choice-five-answer questions, each worth 6 points.*

Partial credit will be granted as follows.

- (a) If you mark only one answer and it is the correct answer, you earn **6** points.
- (b) If you mark *two* answers, one of which is the correct answer, you earn **3** points.
- (c) If you mark *three* answers, one of which is the correct answer, you earn **2** points.
- (d) If you mark no answers, or more than *three*, you earn **0** points.

MC3: *multiple-choice-three-answer questions, each worth 3 points.*

No partial credit.

- (a) If you mark only one answer and it is the correct answer, you earn **3** points.
- (b) If you mark a wrong answer or no answers, you earn **0** points.

MC2: *multiple-choice-two-answer questions, each worth 2 points.*

No partial credit.

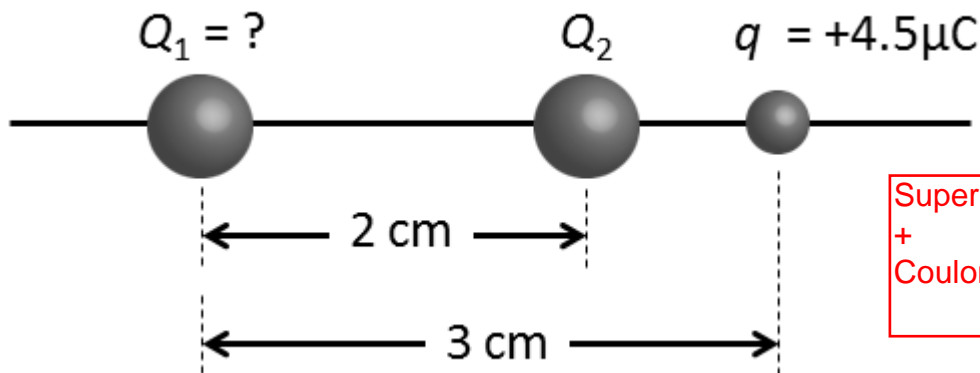
- (a) If you mark only one answer and it is the correct answer, you earn **2** points.
- (b) If you mark the wrong answer or neither answer, you earn **0** points.

Some helpful information:

- A reminder about prefixes: p (pico) = 10^{-12} ; n (nano) = 10^{-9} ; μ (micro) = 10^{-6} ; m (milli) = 10^{-3} ; k (kilo) = 10^{+3} ; M or Meg (mega) = 10^{+6} ; G or Gig (giga) = 10^{+9} .

The next two questions pertain to the situation described below.

Two charges Q_1 and Q_2 are placed on the x-axis, at $x = 0$ and $x = 2 \text{ cm}$, respectively, as shown in the figure. The charge $Q_2 = 5.5 \mu\text{C}$, whereas Q_1 is not known. A third charge $q = +4.5 \mu\text{C}$ is placed a distance $x = 3 \text{ cm}$ from the origin, on the x-axis.



Superposition principle
+
Coulomb's law

1) What must the value of Q_1 be such that the force on q due to charges 1 and 2 is zero?

- a. $Q_1 = -50 \mu\text{C}$
- b. $Q_1 = 17 \mu\text{C}$
- c. $Q_1 = 50 \mu\text{C}$
- d. $Q_1 = -17 \mu\text{C}$
- e. $Q_1 = -5.6 \mu\text{C}$

- (1) Superposition principle allows us to consider Q_1 and Q_2 separately.
- (2) $Q_2 > 0$ and $q > 0$, so Q_2 pushes q to the right.
- (3) Therefore, the force due to Q_1 must pull q to the left $\rightarrow Q_1 < 0$.
- (4) The rest is to determine the absolute value from $|F_2q| = |F_1q|$.
- (5) $k |Q_2| q / (1\text{cm})^2 = k |Q_1| q / (3\text{cm})^2 \rightarrow 9 Q_2 = |Q_1| = 9 \times 5.5 = 49.5$
- (6) That is, $Q_1 = -49.5 \mu\text{C}$.

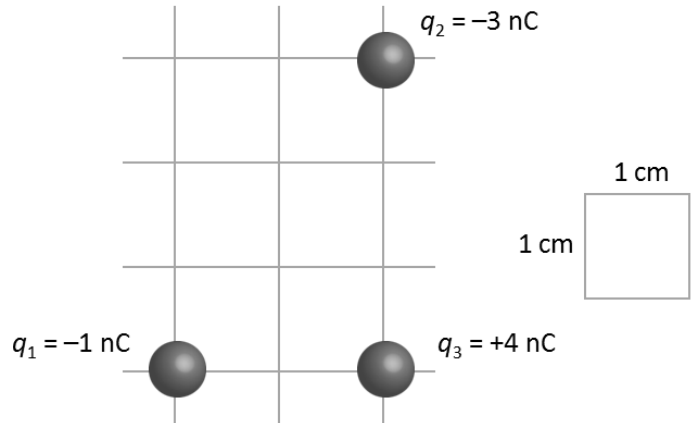
2) Does your answer change if charge q is now negative?

- a. No
- b. Yes

Notice that from the above calculation (5), q drops out from the problem. Any non-zero q gives the same result.

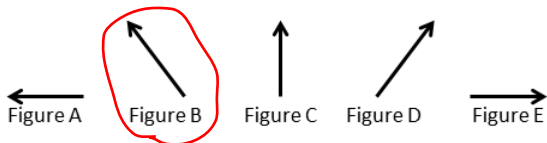
The next two questions pertain to the situation described below.

Consider the configuration of charges shown:
 $q_1 = -1 \text{ nC}$, $q_2 = -3 \text{ nC}$, and $q_3 = +4 \text{ nC}$.
 The grid is 1 cm on a side.



Superposition principle!
 Forces are vectors.

3) Which of the following vectors best represents the direction of the total force $F_{3,tot}$ on charge q_3 due to q_1 and q_2 ?



- a. Figure C
- b. Figure D
- c. Figure E
- d. Figure B
- e. Figure A

To answer this only,
 you have only to
 guess the directions
 of F_{31} and F_{32} .

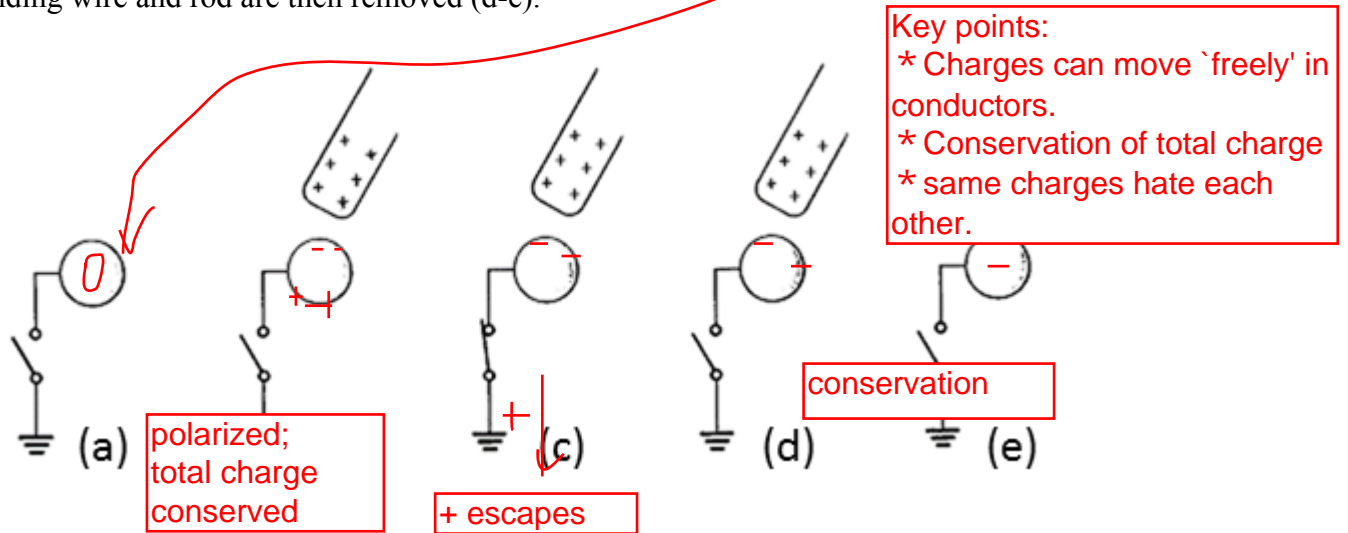
- (1) $F_{3tot} = F_{31} + F_{32}$
 - (2) Magnitudes: $|F_{31}| = kq_1q_3/(2\text{cm})^2$,
 $|F_{32}| = kq_2q_3/(3\text{cm})^2$
 - (3) Thus, $|F_{31}| : |F_{32}| = q_1/4 : q_2/9 = 1/4 : 1/3$
 $= 3 : 4$
 - (4) Directions: F_{31} attractive ← F_{32} attractive ↑
-

4) Calculate the magnitude of the total force $|F_{3,tot}|$ on charge q_3 due to q_1 and q_2 .

- a. $|F_{3,tot}| = 26 \mu\text{N}$
 - b. $|F_{3,tot}| = 150 \mu\text{N}$
 - c. $|F_{3,tot}| = 2200 \mu\text{N}$
 - d. $|F_{3,tot}| = 630 \mu\text{N}$
 - e. $|F_{3,tot}| = 93 \mu\text{N}$
- (1) $|F_{31}| = kq_1q_3/(2\text{cm})^2 = (9 \times 10^9)(1 \times 10^{-9})(4 \times 10^{-9})/(0.02)^2 = 9 \times 10^{-5} \text{ N}$
 - (2) $|F_{32}| = 12 \times 10^{-5} \text{ N}$. (we know the ratio from above)
 - (3) As a vector $F_{3,tot} = (-9, 12) \times 10^{-5} \text{ N}$.
 - (4) Therefore, its magnitude is $15 \times 10^{-5} \text{ N} = 150 \times 10^{-6} \text{ N}$

The next two questions pertain to the situation described below.

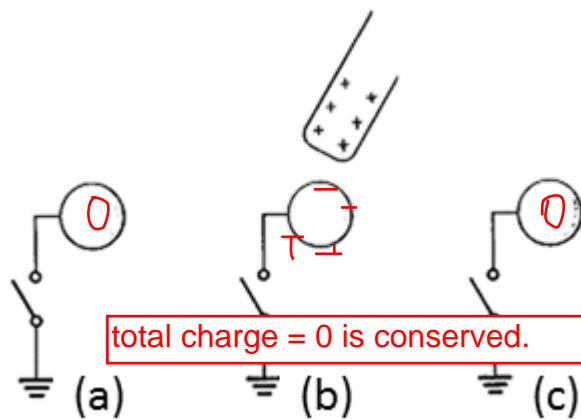
A positively charged rod is brought close but does not touch an uncharged conducting sphere (as shown in steps a-b below). As a rod approaches, the sphere is connected to ground by a conducting wire (c). The grounding wire and rod are then removed (d-e).



5) What is the charge on the conducting sphere after the sequence of steps?

- a. Zero
- b. Positive
- c. Negative

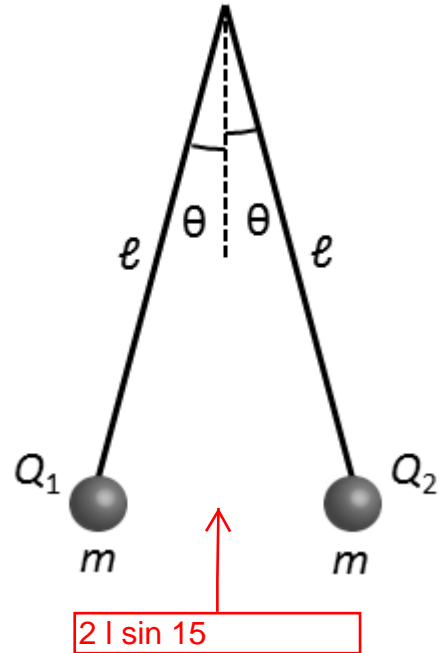
6) Now the sequence of steps is repeated, starting with the same conducting sphere (uncharged), but without grounding the sphere. What is the charge on the sphere after the sequence of steps (a-c)?



- a. Zero
- b. Negative
- c. Positive

The next three questions pertain to the situation described below.

An electroscope is built by suspending two identically sized conducting spheres of mass $m = 0.02 \text{ kg}$ from thin wires of length $\ell = 15 \text{ cm}$ as shown in the figure. After charging, both spheres make an angle of $\theta = 15^\circ$ relative to vertical and $Q_1 = Q_2$. (Note: in this problem, you may ignore any mass or charge from the thin wires.)



the total forces on spheres must be zero.
 There are gravitational force mg and Coulomb force $kQ_1Q_2/(2l \sin\theta)^2$

7) Because the system is in equilibrium:

- a. Gravity does not act on the system.
- b. The spheres will experience a net acceleration.
- c. The spheres will not experience a net acceleration.

no total force, no acceleration

8) If the charge of both Q_1 and Q_2 is increased, the angle θ will:

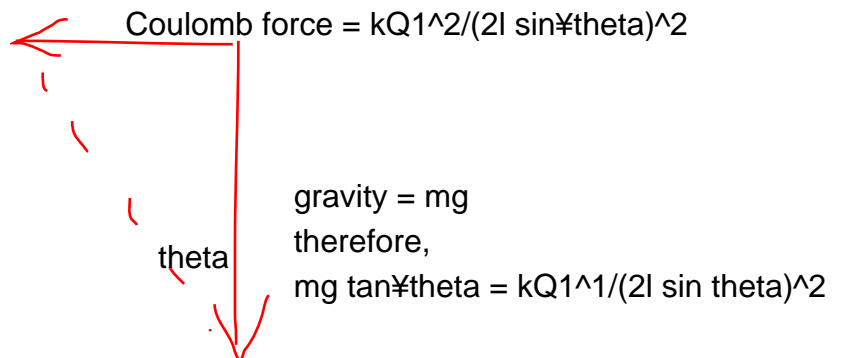
- a. decrease.
- b. increase.
- c. stay the same.

precisely speaking, we assume Q_1 and Q_2 are positive for this answer. Actually, not answerable.

9) What is the magnitude of the charge $|Q_1|$?

- a. $|Q_1| = 8.4 \times 10^{-8} \text{ C}$
- b. $|Q_1| = 1.6 \times 10^{-7} \text{ C}$
- c. $|Q_1| = 5 \times 10^{-8} \text{ C}$
- d. $|Q_1| = 3.9 \times 10^{-8} \text{ C}$
- e. $|Q_1| = 1.9 \times 10^{-7} \text{ C}$

$Q_1 = Q_2$ is written in the question.

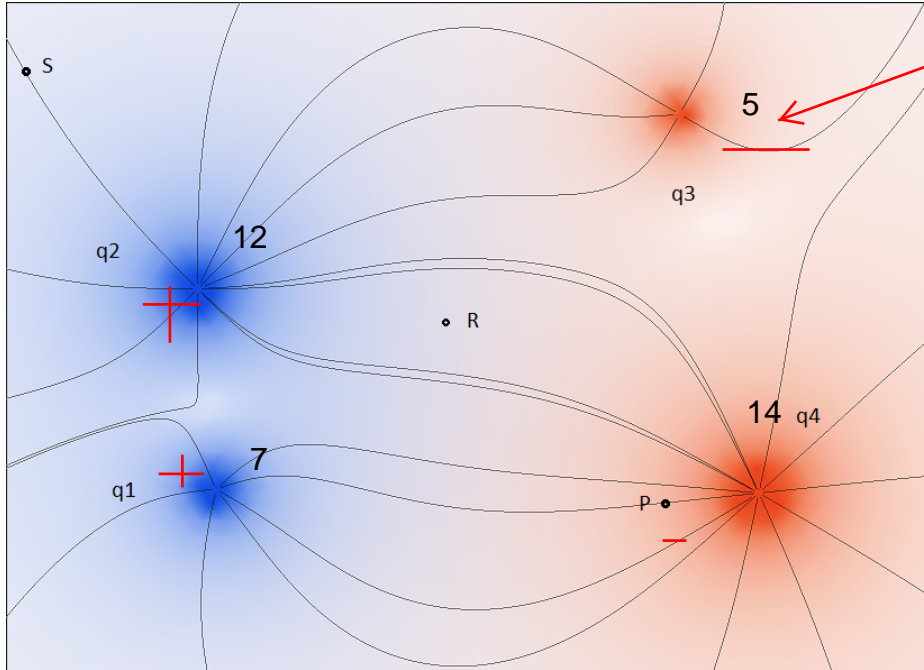


$$Q_1^2 = mg \frac{4 l^2 \tan \theta \sin^2 \theta}{k} = 0.02 \times 9.8 \times 4 \times (0.15)^2 \frac{\tan 15 \sin^2 15}{(9 \times 10^9)}$$

$$= (1.92 \times 10^{-7})^2$$

The next three questions pertain to the situation described below.

Consider the collection of 4 charges below:



If we tentatively declare q_3 to be positive, all the remaining signs are determined. (Of course, actually, q_3 can be negative.)

Charge $+Q$ produces Q/ϵ_0 force lines.
Charge $-Q$ absorbs Q/ϵ_0 force lines.

10) Using the field lines, determine the correct ordering for the magnitudes of the charges

- a. $|q_3| < |q_1| < |q_2| < |q_4|$
- b. $|q_2| < |q_1| < |q_4| < |q_3|$
- c. $|q_3| < |q_2| < |q_1| < |q_4|$
- d. $|q_1| < |q_2| < |q_3| < |q_4|$
- e. $|q_1| < |q_3| < |q_2| < |q_4|$

Count the number of lines around charges as shown in the figure.

$$5 < 7 < 12 < 14.$$

11) Based on the nature of the field lines which of the following is true:

- a. The signs of q_1 and q_2 are opposite of q_3 and q_4 .
- b. All of the charges have the same sign.
- c. The charges q_1 and q_4 have the same sign.

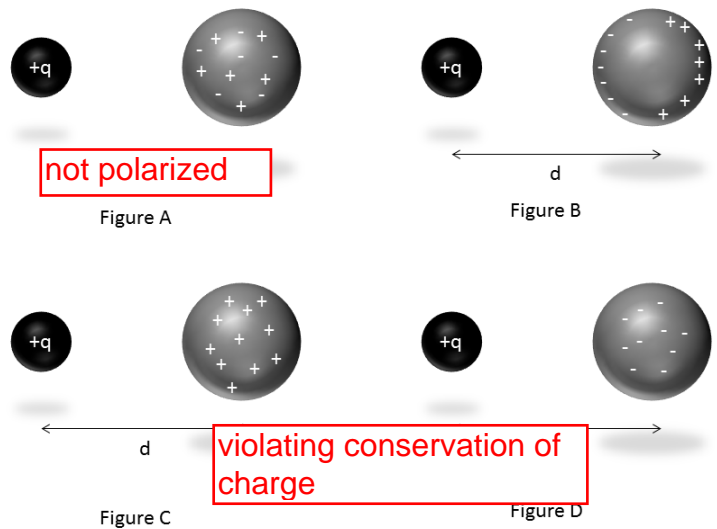
Force line directions are not given, so you cannot tell the signs; you can only tell who has the same sign as someone else or not.

12) When placed at which point will a test charge experience the largest force?

- a. P
- b. R
- c. S

line density tells us the E intensity.

13) A sphere with charge $+q$ is placed a distance d from an uncharged metal sphere. Of the four figures shown, which figure best represents the resulting charge distribution on the metal sphere?

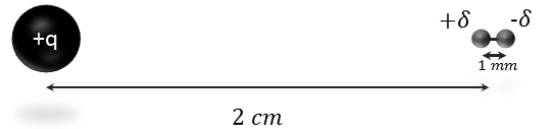


The external E polarizes a metal sphere.
The total charge must be conserved = always 0

- a. Figure C
- b. Figure A
- c. Figure D
- d. None of these
- e. Figure B**

The next two questions pertain to the situation described below.

An electric dipole has a separation distance $d = 1 \text{ mm}$. It is placed 2 cm from a fixed, positive charge $q = 9.7 \mu\text{C}$.



14) If $|\delta| = 0.21 \mu\text{C}$ what is the magnitude of the net force on the dipole due to the sphere?

- a. $F = 0 \text{ N}$
- b. $F = 87 \text{ N}$
- c. $F = 1.8 \text{ N}$
- d. $F = 0.044 \text{ N}$
- e. $F = 4.3 \text{ N}$**

Superposition principle + Coulomb's law

due to $-\delta$

due to $+\delta$

x-component of the force
 $= kq\delta(1/(0.021)^2 - 1/(0.02)^2)$
 $= - (9 \times 10^9) (9.7 \times 10^{-6}) (0.21 \times 10^{-6}) \times 232$
 $= 4252 \times 10^{9-12} = 4.25 \text{ N}.$

15) The dipole is released. In what direction will it travel?

- a. It will not move.
- b. It will move away from the charged sphere.**
- c. It will move toward the charged sphere.

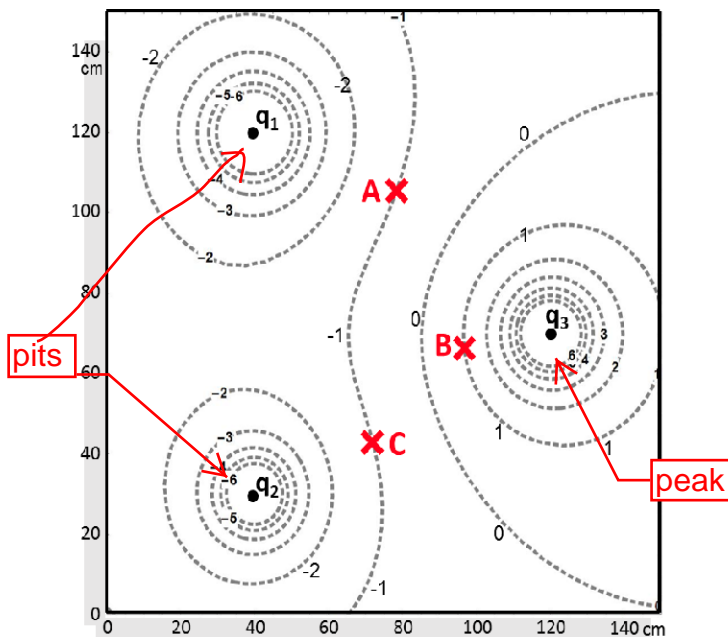
We know the x-component of the force on q is < 0 .
Therefore, action-reaction tells us the dipole to move \rightarrow

The next two questions pertain to the situation described below.

Given is a map of equal-potential lines (see figure). The potential is created by three charges in a plane (q_1 , q_2 , q_3). **Potential values are given in Volts**. Note the signs (+/-). Based on the map:

Potential is something like a landscape.
The descending directions and slopes indicate the electric fields.

+ charges create peaks
- charges create pits.



16) What is the sign (+/-) of the charge q_2 ?

- a. +
- b. -
- c. 0

17) How much total work W by you is required to move a charge of 1 C from point A to point B , and then from point B to point C ?

- a. $W = 0\text{ J}$
- b. $W = 4\text{ J}$
- c. $W = -2\text{ J}$
- d. $W = 2\text{ J}$
- e. $W = -4\text{ J}$

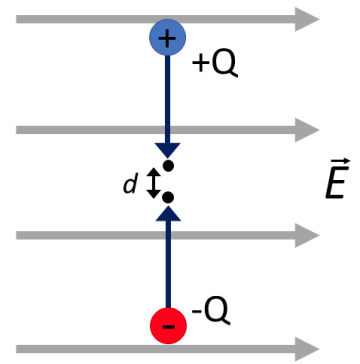
The potential energy of a charge Q at potential V is QV (relative to infinity = 0).

How high you climb up a mountain does not depend on your actual paths, but only on the height difference between the destination and the starting point.

In our case $V_C = V_A$, $\Delta V = 0$. No work is needed.

not a very precise statement, because E is not the sole fields charges feel. We should say "uniform E is imposed from outside".

18) You move two charges closer towards each other by equal distances, until they are separated by a small distance d . They have equal masses and charges of equal magnitude and opposite sign, Q and $-Q$. The charges are exposed to a uniform electric field E , as shown in the diagram. Keeping in mind interactions between the two objects, which statement best describes the work done by you on the system of charges?

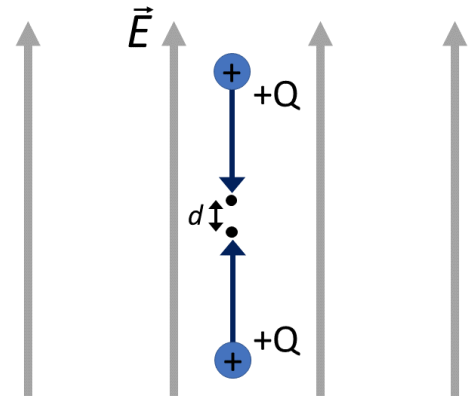


Superposition principle: you can separately consider all the fields, E and the ones due to charges.
 E is irrelevant because it is orthogonal to the displacement.

- a. I am doing negative work on the system of charges.
- b. I am doing positive work on the system of charges.
- c. I am doing no work on the system of charges.

Imagine what would happen o you: you would be dragged by the charge.

19) Choose the statement that best describes the work done by you on the system shown. The objects have equal charge Q , and the direction of electric field is vertical.



- a. I am doing positive work on the system of charges.
- b. I am doing negative work on the system of charges.
- c. I am doing no work on the system of charges.

These two charges repel each other, and you must move them against `their wishes.'

E is irrelevant.
You can imagine E to be the slope of the landscape. In this case one charge goes up and the other down by the same amounts, so no net work is done by you against the forces due to E .

20) Consider the case of two identical charges, with equal mass $M = 0.7 \text{ kg}$ and equal charge $Q = +6 \text{ C}$, in the absence of an external electric field. The charges start at an infinitely far distance apart, and move in opposite directions directly towards one another, with velocities of $+5 \text{ km/s}$ and -5 km/s , respectively. What is the closest distance d that the charges will get to one another?

- a. $d = 8700 \text{ m}$
- b. $d = 58 \text{ m}$
- c. $d = 2 \times 10^3 \text{ m}$
- d. $d = 150 \text{ km}$
- e. $d = 19 \text{ km}$**

The charges repel each other; that is, one charge climbs up the potential hill due to the other charge. That is, the potential energy stored in their mutual relation increases.

The total energy must be conserved.

Initially, all energy is $K = (1/2)mv^2 \times 2 = 0.7 \times (5 \times 10^3)^2 = 17.5 \times 10^6 \text{ J}$.
 Finally, at the closest distance $K = 0$, and the total energy becomes the Coulomb potential energy: kQ^2/d .

$$d = (9 \times 10^9)6^2 / (17.5 \times 10^6) = 18.5 \times 10^{9-6} = 18.5 \text{ km}.$$

21) What is the change in potential energy of a particle of charge $+q$ that is brought from a distance of $3R$ to a distance of R from a particle of charge $-q$?



- a. $U = -2kq^2/3R$**
- b. $U = -kq^2/4R^2$
- c. $U = -2kq^2/R$
- d. $U = kq^2/3R$
- e. $U = kq^2/3R^2$

Potential energy change
 = Final potential energy - initial potential energy

Initial Coulomb potential energy = $-kq^2/3R$.

Final Coulomb potential energy = $-kq^2/R$.

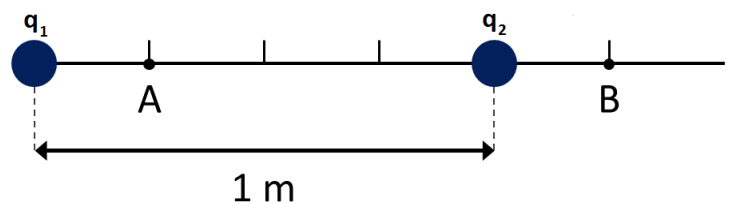
Hence,

$$U = -(kq^2/R)[1 - 1/3] = -2kq^2/3R.$$

Let us denote this as Q .

22) Two $2.9 \mu\text{C}$ charges are held fixed at the positions shown in the figure. Note that both charges are positive.

Calculate the change in potential energy $U(B) - U(A)$ of a $1.0 \mu\text{C}$ charge that is moved from A to B . Note that the ruler lines shown in the figure are equally spaced.



We can use superposition principle to divide and conquer the problem.

- a. $U = -0.014 \text{ J}$**
- b. $U = -0.042 \text{ J}$
- c. $U = 0 \text{ J}$
- d. $U = 0.042 \text{ J}$
- e. $U = 0.014 \text{ J}$

V_I = initial pot-energy of the charge = $kqQ(1/(1/4) + 1/(3/4)) = kqQ(4 + 4/3)$.

V_F = final pot-energy = $kqQ(1/(5/4) + 1/(1/4)) = kqQ(4 + 4/5)$.

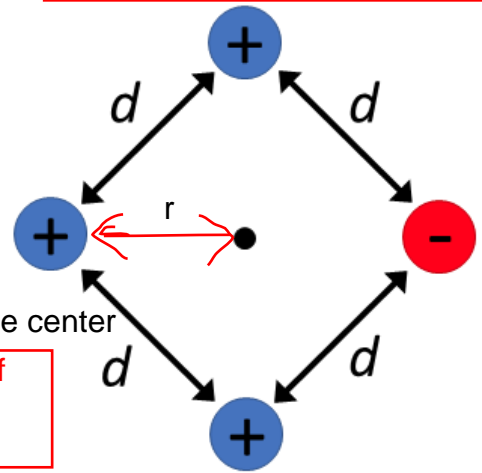
Therefore,

$$U = V_F - V_I = kqQ(4/5 - 4/3) = -(9 \times 10^9)(2.9 \times 10^{-6})(1 \times 10^{-6}) \times 0.5333 = 13.9 \times 10^{9-12} = 0.0139 \text{ J}$$

The next three questions pertain to the situation described below.

Four point charges are equally spaced by a distance $d = 4.69 \text{ mm}$ at the corners of a square, as shown in the figure. Three of the charges are positive, with $q = 2.9 \mu\text{C}$, while one is negative with charge $q = -2.9 \mu\text{C}$.

This color coding is as perverse as Discussion figures, and should be corrected: red should be +.



Always recall superposition principle.

The magnitude of the electric potential due to any charge at the center is identical: $V = kq/r$, where $r = d/\sqrt{2}$.

"at the center of the square" is better.

23) What is the electric potential at the center point between the fixed charges?

- a. $V = -1.6 \times 10^7 \text{ V}$
- b. $V = 1.6 \times 10^7 \text{ V}$**
- c. $V = 2.2 \times 10^7 \text{ V}$
- d. $V = -1.1 \times 10^7 \text{ V}$
- e. $V = 1.1 \times 10^7 \text{ V}$

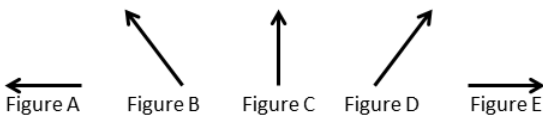
The contribution of - kills one of + charges' contribution, so we have only to consider two positive charges.

$$V_{\text{total}} = 2kq/(d/\sqrt{2}) = 2\sqrt{2} kq/d$$

$$= 2.828 \times (9 \times 10^9)(2.9 \times 10^{-6}) / (4.69 \times 10^{-3})$$

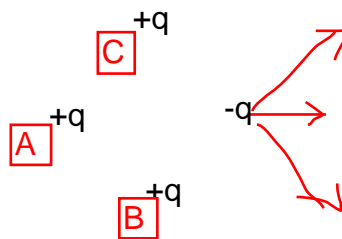
$$= 15.74 \times 10^{9-6+3} = 15.74 \times 10^6 \text{ V.}$$

24) Considering only the three positive charges, which vector arrow shown below best represents the direction of the electric field at the position of the negative charge?



The direction of the field has nothing to do with the particle that feels the field. Do not be fooled by the negative charge.

- a. Figure A
- b. Figure B
- c. Figure C
- d. Figure E**
- e. Figure D



25) Considering only the three positive charges, what is the magnitude of the electric field at the position of the negative charge?

- a. $E = 1.19 \times 10^9 \text{ N/C}$
- b. $E = 2.27 \times 10^9 \text{ N/C}$**
- c. $E = 0 \text{ N/C}$
- d. $E = 1.78 \times 10^9 \text{ N/C}$
- e. $E = 1.08 \times 10^9 \text{ N/C}$

As can be seen from the figure, the field must be in the +x direction

$$x \text{ comp due to A} = kq/(\sqrt{2}d)^2 = kq^2/2d^2.$$

$$x \text{ comp due to B} = (kq/d^2)\cos 45 = x \text{ comp due to C.}$$

Adding all of them, we get

$$E = (kq/d^2)(1/2 + 2 \cos 45)$$

$$= (9 \times 10^9)(2.9 \times 10^{-6})(1.914) / (4.69 \times 10^{-3})^2$$

$$= 2.27 \times 10^{9-6+6} = 2.27 \times 10^9 \text{ N/C (or V/m).}$$

WARNING:

Students: you must remember that **units must not be in italic** contrary to the practice in this exam. Always look at the textbook as a reliable source.

Kinematics and mechanics:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + a t$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$F = ma$$

$$a_c = \frac{v^2}{r}$$

$$E_{tot} = K.E. + P.E.$$

$$K.E. = \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

$$p = mv$$

$$W_F = Fd \cos \theta$$

Electrostatics:

$$F_{12} = \frac{kq_1 q_2}{r^2}$$

$$E \equiv \frac{F}{q_0}$$

$$U_{12} = \frac{kq_1 q_2}{r}$$

$$V \equiv \frac{U}{q_0}$$

$$W_E = -\Delta U = -W_{you}$$

Point charge:

$$E = \frac{kq}{r^2}$$

$$V = \frac{kq}{r}$$

Electric dipole:

$$p \equiv qd$$

$$\tau_{dip} = pE \sin \theta$$

$$U_{dip} = -pE \cos \theta$$

Resistance:

$$R \equiv \frac{V}{I}$$

$$I = \frac{\Delta q}{\Delta t}$$

Physical resistance: $R = \rho \frac{L}{A}$

$$P = IV = I^2 R = \frac{V^2}{R}$$

$$R_S = R_1 + R_2 + \dots$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

Capacitance:

$$C \equiv \frac{Q}{V}$$

Parallel plate capacitor: $C = \frac{\kappa \epsilon_0 A}{d}$, $V = Ed$

$$U_C = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

$$C_P = C_1 + C_2 + \dots$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

Circuits:

$$\sum \Delta V = 0$$

$$\sum I_{in} = \sum I_{out}$$

$$q(t) = q_\infty (1 - e^{-t/\tau})$$

$$q(t) = q_0 e^{-t/\tau}$$

$$I(t) = I_0 e^{-t/\tau}$$

$$\tau = RC$$

Magnetism:

$$F = qvB \sin \theta$$

$$r = \frac{mv}{qB}$$

$$F_{wire} = qvB \sin \theta$$

$$\tau_{loop} = NIAB \sin \phi$$

Magnetic dipole:

$$m \equiv NIA$$

$$\tau_{dip} = mB \sin \phi$$

$$U_{dip} = -mB \cos \phi$$

$$B_{wire} = \frac{\mu_0 I}{2\pi r}$$

$$B_{sol} = \mu_0 nI$$

Electromagnetic induction:

$$\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$$

$$|\varepsilon_{bar}| = BLv$$

$$\Phi = BA \cos \varphi$$

$$\varepsilon_{gen} = \varepsilon_{max} \sin \omega t = \omega NAB \sin \omega t$$

$$\omega = 2\pi f$$

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

$$\frac{V_p}{V_s} = \frac{I_s}{I_p} = \frac{N_p}{N_s}$$

Electromagnetic waves:

$$\lambda = \frac{c}{f}$$

$$E = cB$$

$$u_E = \frac{1}{2} \varepsilon_0 E^2$$

$$u_B = \frac{1}{2\mu_0} B^2$$

$$\bar{u} = \frac{1}{2} \varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0} B_{rms}^2 = \varepsilon_0 E_{rms}^2 = \frac{B_{rms}^2}{\mu_0}$$

$$S = I = \bar{u}c$$

$$f' = f \left(1 \pm \frac{u}{c} \right)$$

$$I = I_0 \cos^2 \theta$$

Reflection and refraction:

$$\theta_r = \theta_i$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$f = \pm \frac{R}{2}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$v = \frac{c}{n}$$

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$M = \frac{\theta'}{\theta} \approx \frac{d_{near}}{f}$$

Interference and diffraction:

Double slit interference:

$$d \sin \theta = m\lambda$$

$$d \sin \theta = (m + \frac{1}{2})\lambda$$

$$m = 0, \pm 1, \pm 2 \dots$$

Single-slit diffraction:

$$w \sin \theta = m\lambda$$

$$m = \pm 1, \pm 2 \dots$$

Circular aperture:

$$D \sin \theta \approx 1.22\lambda$$

Thin film: $\delta_1 = (0 \text{ or } \frac{1}{2})$

$$\delta_2 = (0 \text{ or } \frac{1}{2}) + 2t \frac{n_{film}}{\lambda_0}$$

$$|\delta_2 - \delta_1| = (m \text{ or } m + \frac{1}{2})$$

$$m = 0, 1, 2 \dots$$

Quantum mechanics:

$$E = hf = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{p}$$

Blackbody radiation: $\lambda_{max} T = 2.898 \times 10^{-3} m \cdot K$ Photoelectric effect: $K.E. = hf - W_0$

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

$$\hbar \equiv \frac{h}{2\pi}$$

Bohr atom: $2\pi r_n = n\lambda \quad n = 1, 2, 3 \dots$

$$L_n = m v_n r_n = n\hbar$$

$$r_n = \left(\frac{\hbar^2}{mke^2} \right) \frac{n^2}{Z} \approx (5.29 \times 10^{-11} m) \frac{n^2}{Z}$$

$$E_n = - \left(\frac{mk^2 e^4}{2\hbar^2} \right) \frac{Z^2}{n^2} \approx -(13.6 eV) \frac{Z^2}{n^2}$$

$$\frac{1}{\lambda} \approx (1.097 \times 10^7 \text{ m}^{-1}) Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Quantum atom: $L = \sqrt{\ell(\ell+1)}\hbar$

$$L_z = m_\ell \hbar$$

Nuclear physics and radioactive decay:

$$A = Z + N$$

$$r \approx (1.2 \times 10^{-15} \text{ m}) A^{1/3}$$

$$E_0 = mc^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-t/T_{1/2}}$$

$$T_{1/2} \equiv \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$$

Constants and unit conversions:

$$g = 9.8 \text{ m/s}^2$$

$$e = 1.60 \times 10^{-19} \text{ C}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2$$

$$k \equiv \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ Nm}^2 / \text{C}^2$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/s}$$

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$$

$$hc = 1240 \text{ nm} \cdot \text{eV}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

$$m_{\text{proton}} = 1.67 \times 10^{-27} \text{ kg} = 938 \text{ MeV}$$

$$m_{\text{electron}} = 9.11 \times 10^{-31} \text{ kg} = 511 \text{ keV}$$

SI Prefixes		
Power	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^0	—	—
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p