

Last Name: \_\_\_\_\_ First Name \_\_\_\_\_ Network-ID \_\_\_\_\_

Discussion Section: \_\_\_\_\_ Discussion TA Name: \_\_\_\_\_

**Turn off your cell phone and put it out of sight.**

**Keep your calculator on your own desk. Calculators cannot be shared.**

**This is a closed book exam. You have ninety (90) minutes to complete it.**

1. Use a #2 pencil. Do not use a mechanical pencil or pen. Darken each circle completely, but stay within the boundary. If you decide to change an answer, erase vigorously; the scanner sometimes registers incompletely erased marks as intended answers; this can adversely affect your grade. Light marks or marks extending outside the circle may be read improperly by the scanner. Be especially careful that your mark covers the **center** of its circle.
2. You may find the version of **This Exam Booklet at the top of page 2**. Mark the version circle in the TEST FORM box near the middle of your answer sheet. **DO THIS NOW!**
3. Print your **NETWORK ID** in the designated spaces at the *right* side of the answer sheet, starting in the left most column, then **mark the corresponding circle** below each character. If there is a letter "o" in your NetID, be sure to mark the "o" circle and not the circle for the digit zero. If and only if there is a hyphen "-" in your NetID, mark the hyphen circle at the bottom of the column. When you have finished marking the circles corresponding to your NetID, check particularly that you have not marked two circles in any one of the columns.
4. Print **YOUR LAST NAME** in the designated spaces at the *left* side of the answer sheet, then mark the corresponding circle below each letter. Do the same for your **FIRST NAME INITIAL**.
5. Print your UIN# in the STUDENT NUMBER designated spaces and mark the corresponding circles. You need not write in or mark the circles in the SECTION box.
6. Sign your name (**DO NOT PRINT**) on the **STUDENT SIGNATURE line**.
7. On the **SECTION line**, print your **DISCUSSION SECTION**. You need not fill in the COURSE or INSTRUCTOR lines.

*Before starting work, check to make sure that your test booklet is complete. You should have 14 **numbered pages** plus three (3) Formula Sheets.*

**Academic Integrity—Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including dismissal from the University.**

**This Exam Booklet is Version A.** Mark the **A** circle in the **TEST FORM** box near the middle of your answer sheet. **DO THIS NOW!**

*Exam Grading Policy—*

The exam is worth a total of **121** points, composed of three types of questions.

**MC5:** *multiple-choice-five-answer questions, each worth 6 points.*

**Partial credit will be granted as follows.**

- (a) If you mark only one answer and it is the correct answer, you earn **6** points.
- (b) If you mark *two* answers, one of which is the correct answer, you earn **3** points.
- (c) If you mark *three* answers, one of which is the correct answer, you earn **2** points.
- (d) If you mark no answers, or more than *three*, you earn **0** points.

**MC3:** *multiple-choice-three-answer questions, each worth 3 points.*

**No partial credit.**

- (a) If you mark only one answer and it is the correct answer, you earn **3** points.
- (b) If you mark a wrong answer or no answers, you earn **0** points.

**MC2:** *multiple-choice-two-answer questions, each worth 2 points.*

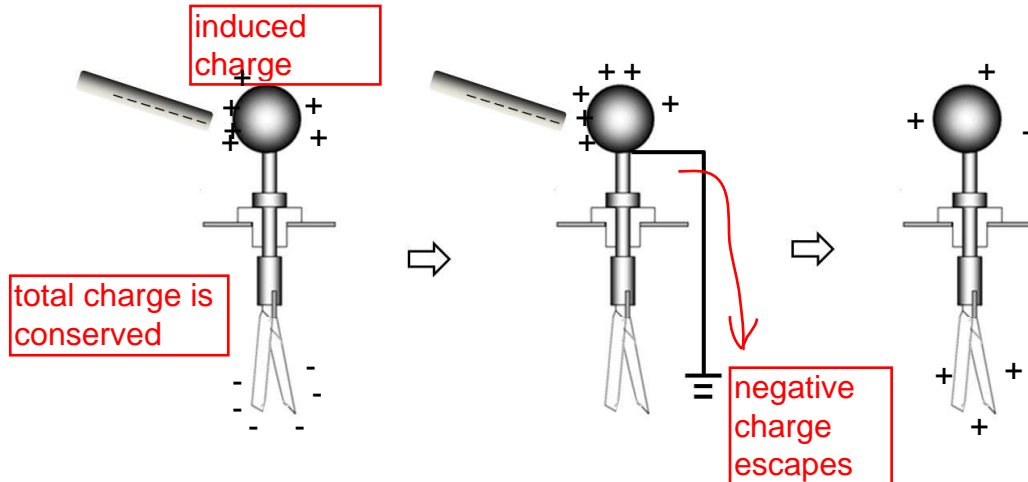
**No partial credit.**

- (a) If you mark only one answer and it is the correct answer, you earn **2** points.
- (b) If you mark the wrong answer or neither answer, you earn **0** points.

Some helpful information:

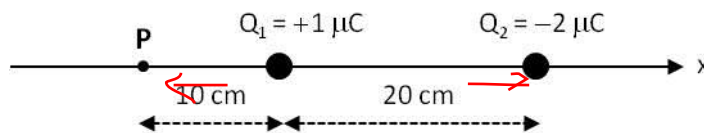
- A reminder about prefixes: p (pico) =  $10^{-12}$ ; n (nano) =  $10^{-9}$ ;  $\mu$  (micro) =  $10^{-6}$ ; m (milli) =  $10^{-3}$ ; k (kilo) =  $10^{+3}$ ; M or Meg (mega) =  $10^{+6}$ ; G or Gig (giga) =  $10^{+9}$ .

1. A negatively charged rod is brought near (but does not touch) an electroscope as shown. Then, the scope is briefly grounded. Regarding the whole sequence of three steps, which statement is NOT true?



- Negative charges will flow from the scope to the ground.
- Negative charges will be induced on the scope when the rod is moved away.
- The leaves will repel each other when the rod is moved away.

2. Two point charges are placed on the  $x$ -axis as shown. If a positive charge is brought to the point **P**, what is the direction of the net electric force felt by this charge?



- Along the negative  $x$ -axis.
- Along the positive  $x$ -axis.
- It depends on the magnitude of the positive charge at point **P**.

Let us compute the  $x$ -component

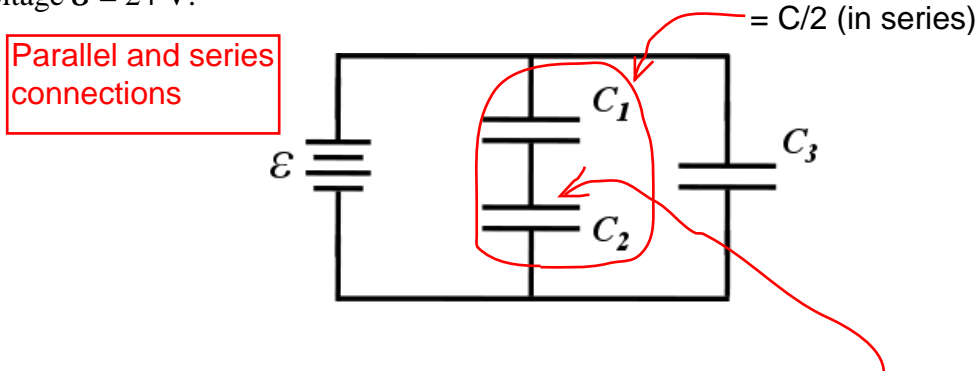
$$Q_1: -kQ_1/(0.1^2) = -(9 \times 10^9) (1 \times 10^{-6})/(0.1^2) = -9 \times 10^5 \text{ N/C}$$

$$Q_2: +kQ_2/(0.3^2) = (9 \times 10^9) (2 \times 10^{-6})/(0.3^2) = 2 \times 10^5 \text{ N/C}$$

The sum is  $-7 \times 10^5 \text{ N/C}$ .

The next three questions pertain to the following situation:

A system of capacitors, all of equal capacitance  $C$ , is connected to an ideal battery of voltage  $\mathcal{E} = 24 \text{ V}$ .



3. Calculate  $C$  given that the charge on capacitor  $C_2$  is measured to be  $Q_2 = 98 \text{ nC}$ .

- a.  $C = 43.9 \text{ nF}$
- b.  $C = 2.7 \text{ nF}$
- c.  $C = 312 \text{ nF}$
- d.  $C = 8.2 \text{ nF}$
- e.  $C = 126 \text{ nF}$

Since  $C_1 = C_2 = C$ , the voltage across  $C_2$  is  $E/2 = 12 \text{ V}$ .  
 $Q_2 = VC_2 \rightarrow C = Q_2/V = 98 \times 10^{-9}/12 = 8.17 \times 10^{-9}$

4. Now let all of the capacitors have value  $C = 25 \mu\text{F}$  and unknown charge. What is  $Q_3$ , the amount of charge collected on the capacitor  $C_3$ ?

- a.  $Q_3 = 150 \mu\text{C}$
- b.  $Q_3 = 300 \mu\text{C}$
- c.  $Q_3 = 600 \mu\text{C}$

The voltage across  $C_3$  is  $24 \text{ V}$ , so  $Q_3 = 24 \times (25 \times 10^{-6}) = 600 \times 10^{-6} \text{ C}$

5. Again the capacitors have value  $C = 25 \mu\text{F}$  and unknown charge. How much energy  $U_{total}$  is stored in the capacitor network?

- a.  $U_{total} = 0.9 \text{ mJ}$
- b.  $U_{total} = 3.2 \text{ mJ}$
- c.  $U_{total} = 10.8 \text{ mJ}$
- d.  $U_{total} = 58.2 \text{ mJ}$
- e.  $U_{total} = 36.8 \text{ mJ}$

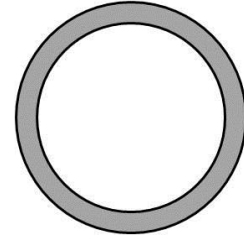
The total capacitance is

$$C_{tot} = C + C/2 = 3C/2 \text{ (in parallel)} = 37.5 \mu\text{F}$$

Therefore,

$$U_{tot} = (1/2) C_{tot} V^2 = (1/2)(37.5 \times 10^{-6}) \times 24^2 = 10800 \times 10^{-6} = 10.8 \times 10^{-3} \text{ J}$$

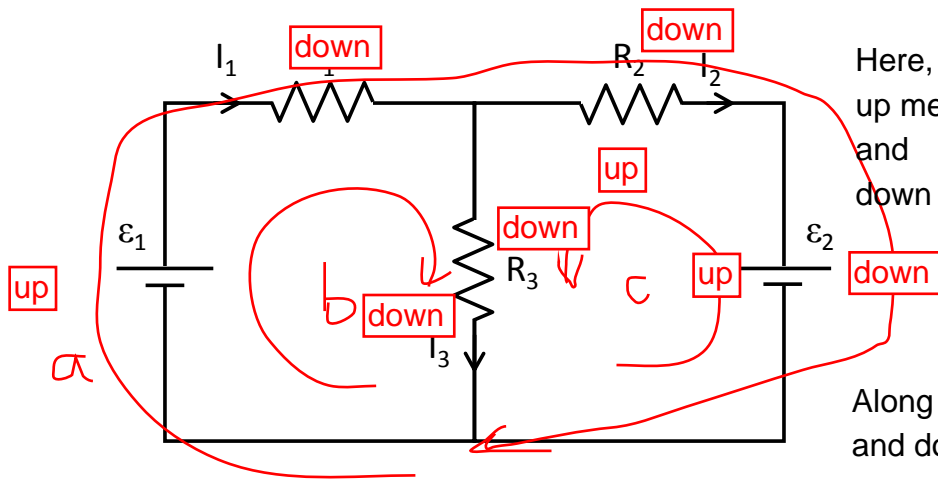
6. Consider an uncharged spherical conducting shell as shown. If charges are transferred to it, which statement is TRUE regarding their behavior?



- a. They will be distributed uniformly throughout the conductor.
- b. They will spread on the inner surface.
- c. They will spread on the outer surface.

The same charges wish to be as far away as possible from each other.

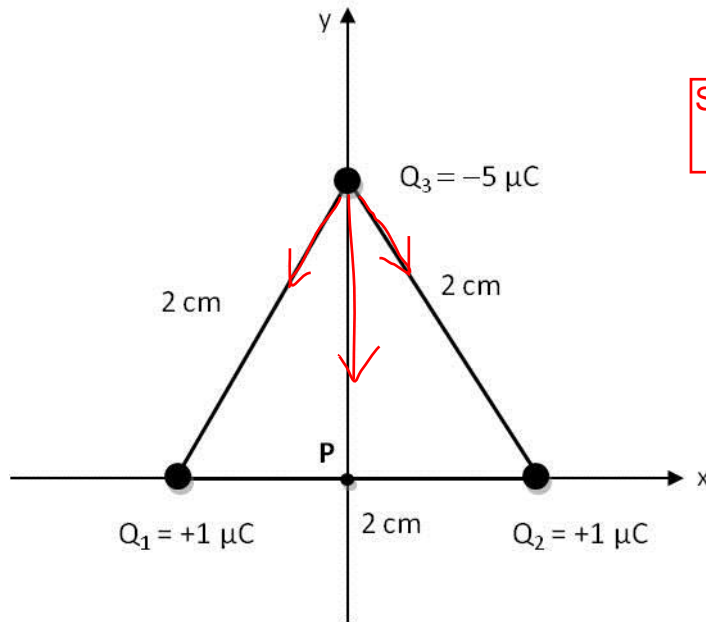
7. Consider the circuit below. Which of the following equations is *incorrect*?



- |  |                                     |
|--|-------------------------------------|
| a. $\mathcal{E}_1 - \mathcal{E}_2 - I_1R_1 - I_2R_2 = 0$ | loop a: $E1 - I1R1 - I2R2 - E2 = 0$ |
| b. $\mathcal{E}_1 - I_1R_1 - I_3R_3 = 0$                 | loop b: $E1 - I1R1 - I3R3 = 0$      |
| c. $\mathcal{E}_2 - I_2R_2 - I_3R_3 = 0$                 | loop c: $E2 + I2R2 - I3R3 = 0$      |

The next five questions pertain to the following situation.

Three point charges are positioned on the vertices of an equilateral triangle as shown.



Sketch the forces

8. What is the magnitude of the net electric force  $F$  on the charge  $Q_3$ ?

- a.  $F = 3.89 \text{ N}$
- b.  $F = 112 \text{ N}$
- c.  $F = 195 \text{ N}$

The magnitude of the force due to  $Q_1$  is

$$|F| = kQ_1Q_2/(0.02^2) = (9 \times 10^9) (1 \times 10^{-6}) (5 \times 10^{-6}) / (0.02^2) = 11.25 \times 10^{9-6-6+4} = 11.25 \times 10^1 = 112.5 \text{ N}.$$

Therefore., the  $y$  component of the total force (= total force) is

$$112.5 \cos 30^\circ \times 2 = 112.5 \times \sqrt{3} = 194.9 \text{ N}$$

9. What is the direction of the electric field at the origin,  $P$ ?

- a. Along the positive  $y$ -axis.
  - b. Along the negative  $y$ -axis.
- At the origin  $Q_1$  and  $Q_2$  kill each other, so the force is only due to  $Q_3$ . Upward.

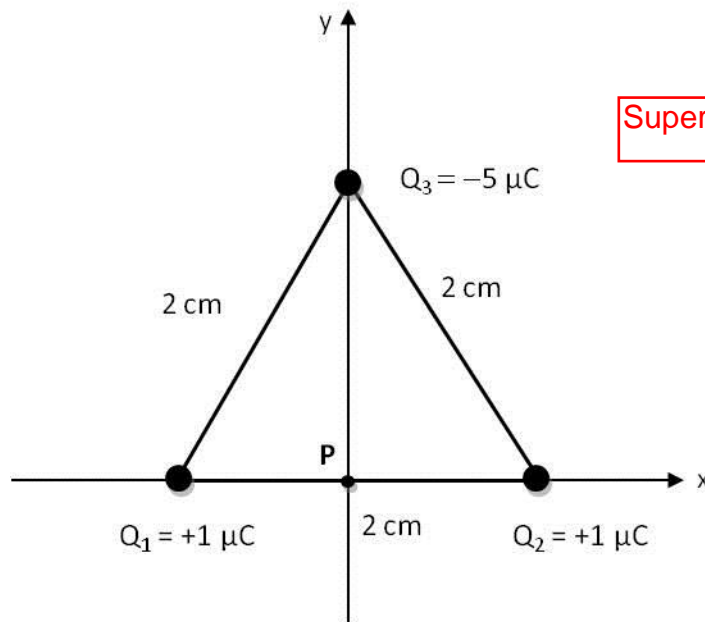
10. What is the magnitude of the electric field  $E$  at the origin,  $P$ ?

- a.  $E = 1.35 \times 10^7 \text{ N/C}$
- b.  $E = 5.39 \times 10^8 \text{ N/C}$
- c.  $E = 1.50 \times 10^8 \text{ N/C}$
- d.  $E = 1.12 \times 10^8 \text{ N/C}$
- e.  $E = 7.01 \times 10^8 \text{ N/C}$

$$|E| = kQ_3 / (0.02 \cos 30)^2$$

$$= (9 \times 10^9) (5 \times 10^{-6}) / (\sqrt{3} \times 10^{-2})^2 = 15 \times 10^{9-6+4} = 15 \times 10^7 \text{ N/C}$$

The next two questions continue from the previous page.



Superposition principle

11. How much work  $W$  is required *by you* to assemble the three charges to this configuration?

- a.  $W = -0.0405 \text{ J}$
- b.  $W = -4.05 \text{ J}$**
- c.  $W = 4.05 \text{ J}$
- d.  $W = -202.3 \text{ J}$
- e.  $W = 202.3 \text{ J}$

If the total potential energy is  $U$ , the work needed to make this assembly is  $U$ .

$$\begin{aligned} U &= (k/0.02)[Q_1Q_3 + Q_3Q_2 + Q_2Q_1] \\ &= (9 \times 10^9 / 0.02)[-5 - 5 + 1] \times 10^{-12} \\ &= -40.5 \times 10^{9-12+2} = -4.05 \text{ J.} \end{aligned}$$

12. What is the electric potential  $V$  due to the three charges at origin,  $P$ ?

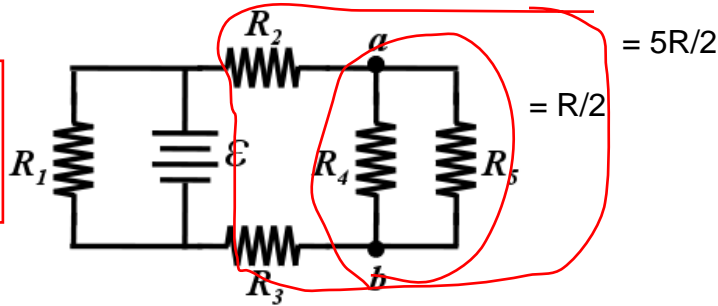
- a.  $V = -202.3 \text{ V}$
- b.  $V = -2.34 \text{ V}$
- c.  $V = 1.35 \text{ V}$
- d.  $V = -1.35 \times 10^5 \text{ V}$
- e.  $V = -7.98 \times 10^5 \text{ V}$**

$$\begin{aligned} V &= k [Q_1/1\text{cm} + Q_2/1\text{cm} - Q_3/\sqrt{3}\text{cm}] \\ &= (9 \times 10^9) [1 + 1 - 5/\sqrt{3}] (1 \times 10^{-6}) / (0.01) \\ &= -7.98 \times 10^{9-6+2} = -7.98 \times 10^5 \text{ V.} \end{aligned}$$

The next three questions pertain to the following situation:

An ideal battery of voltage  $\mathcal{E} = 12 \text{ V}$  is connected to a circuit of resistors.

Formula for parallel connection  
 $1/R_{tot} = 1/R_1 + 1/R_2$   
 or  
 $R_{tot} = R_1 R_2 / (R_1 + R_2)$



Why C instead of R? Unbelievable.

13. Assume all of the resistors have resistance  $R$ . What is the equivalent resistance,  $C_{eq}$ , for the circuit?

- a.  $C_{eq} = 3R/2$
  - b.  $C_{eq} = 5R$
  - c.  $C_{eq} = 5R/7$
  - d.  $C_{eq} = 4R/3$
  - e.  $C_{eq} = 13R/9$
- $5R/2$  and  $R$  are in parallel, so  
 $R_{tot} = (5R/2)R / (7R/2) = 5R/7.$

14. If the resistance of each resistor  $R = 75 \Omega$ , what is  $P_1$ , the power dissipated by resistor  $R_1$ ?

- a.  $P_1 = 1.9 \text{ W}$
  - b.  $P_1 = 9.0 \text{ W}$
  - c.  $P_1 = 5.7 \text{ W}$
- $12 \text{ V}$  is applied to  $R_1 = R = 75 \text{ ohm}$ . Therefore,  
 $P = V^2/R = 12^2/75 = 1.92 \text{ W}.$

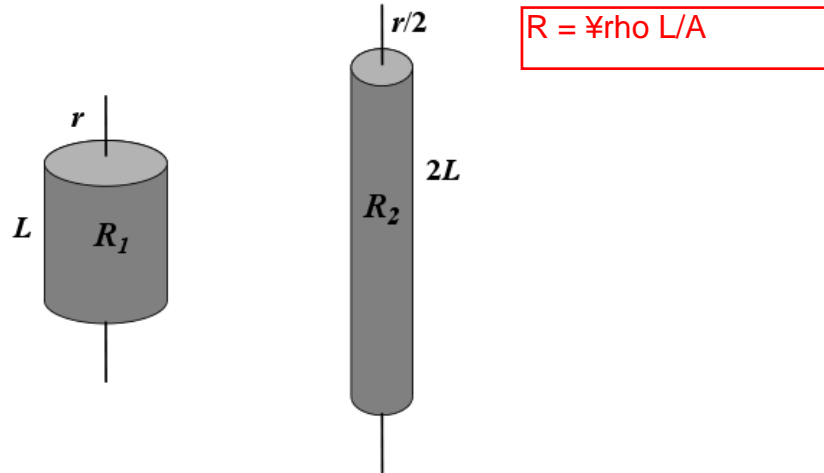
$P = V^2/R$

15. What is the voltage  $V_{ab}$  difference between points  $a$  and  $b$ , as labeled on the circuit?

- a.  $V_{ab} = 2.4 \text{ V}$
  - b.  $V_{ab} = 18.2 \text{ V}$
  - c.  $V_{ab} = 6.0 \text{ V}$
  - d.  $V_{ab} = 12.0 \text{ V}$
  - e.  $V_{ab} = 4.8 \text{ V}$
- The current  $I$  through  $R_2$  satisfies  $I(5R/2) = E$  or  
 $I = 2E/5R = 24/(5 \times 75) = 0.064 \text{ A}$   
 The voltage drop due to  $R_2$  and  $R_3$  must be  
 $(R_2 + R_3) \times 0.064 = 150 \times 0.064 = 9.6 \text{ V}.$   
 Therefore,  $12 - 9.6 = 2.4 \text{ V}$  is the voltage across  $R_4$ .



16. Two resistors are created using copper, whose resistivity is  $\rho = 1.72 \times 10^{-8} \Omega \cdot \text{m}$ . The first resistor has radius  $r$  and length  $L$ . The second resistor has radius  $r/2$  and length  $2L$ . What is the ratio of the second resistor's resistance  $R_2$  to that of the first resistor's resistance  $R_1$ ?



- a.  $R_2/R_1 = 1/4$
- b.  $R_2/R_1 = 1/2$
- c.  $R_2/R_1 = 2$
- d.  $R_2/R_1 = 8$
- e.  $R_2/R_1 = 16$

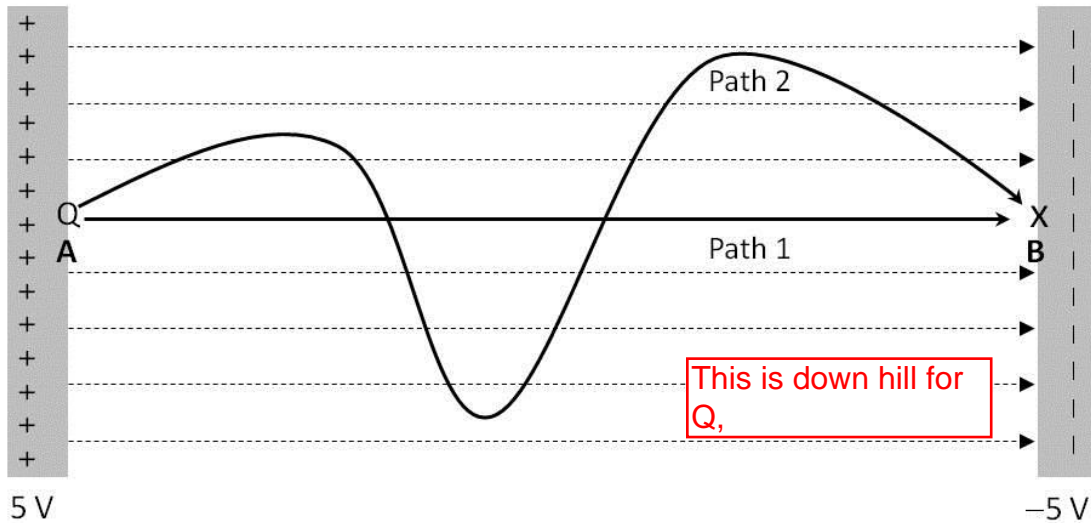
$$R_1 = \rho L / (\pi r^2)$$
$$R_2 = \rho 2L / (\pi (r/2)^2).$$

Therefore,

$$R_2/R_1 = 2 \times 2^2 = 8$$

The next two questions pertain to the following situation.

A uniform electric field is generated by two parallel plate electrodes, positive and negative, respectively, as shown. The dashed lines indicate the electric field. The electric potential at the positive and the negative electrode is 5 V and -5 V, respectively. Consider a charge  $Q = +3 \text{ mC}$  with mass of 1 mg.



17. Imagine that you move the charge  $Q$  from point **A** to point **B** along the two paths shown. Let  $W_1$  and  $W_2$  be the work done *by the electric field* following Path 1 and Path 2, respectively. What is the relationship between  $W_1$  and  $W_2$ ?

- a.  $W_1 > W_2$
- b.  $W_1 < W_2$
- c.  $W_1 = W_2$

The work needed is the potential difference, so it does not depend on the actual paths.

18. If the charge  $Q$  is released freely at **A**, what is its speed,  $v$ , when arriving at **B**?

- a. Not enough information is given.
- b.  $v = 7.75 \text{ m/s}$
- c.  $v = 5.48 \text{ m/s}$
- d.  $v = 173 \text{ m/s}$
- e.  $v = 245 \text{ m/s}$

The positive charge tumbles down the potential slope from A to B.

The key point is the conservation of energy.

Initial  $U = 10Q = 30 \times 10^{-3} \text{ J}$ , initial  $K = 0$

Final  $U = 0$ , final  $K = (1/2) mv^2$ .

Conservation energy implies

$$10Q = (1/2) mv^2$$

Therefore,

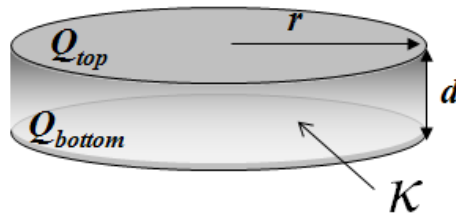
$$v^2 = 20Q/m = 20 \times 3 \times 10^{-3} / (1 \times 10^{-3})$$

$$= 60 = (7.746)^2.$$

The answer says e ????

The next two questions pertain to the following situation:

A capacitor is created by placing two circular metal plates of radius 2 mm a distance 5  $\mu\text{m}$  apart. A material of dielectric constant  $\kappa = 2.5$  is placed between the plates. The capacitor is then charged by placing a charge  $Q_{top} = +3 \text{ nC}$  on the top plate and  $Q_{bottom} = -3 \text{ nC}$  on the bottom plate. After charging, the entire element is disconnected from all other element, such as wires or a battery.



$$Q/(\pi r^2) \kappa \epsilon_0 \text{ is } E.$$

This times d must be E.

Or you can use the capacitance formula:

$$C = \kappa \epsilon_0 A/d$$

and  $Q = CV$ .

19. What is the voltage difference  $V$  measured between the two plates of this capacitor?

- a.  $V = 15 \text{ V}$        $V = [Q/(\pi r^2) \kappa \epsilon_0] d$   
**b.**  $V = 54 \text{ V}$        $= [(3 \times 10^{-9})/(\pi (2 \times 10^{-3})^2 \times 2.5 \times 8.85 \times 10^{-12})] \times (5 \times 10^{-6})$   
 c.  $V = 95 \text{ V}$        $= 0.05395 \times 10^{-9 + 6 + 12 - 6} = 0.054 \times 10^3 = 54 \text{ V}$   
 d.  $V = 65 \text{ V}$   
 e.  $V = 225 \text{ V}$

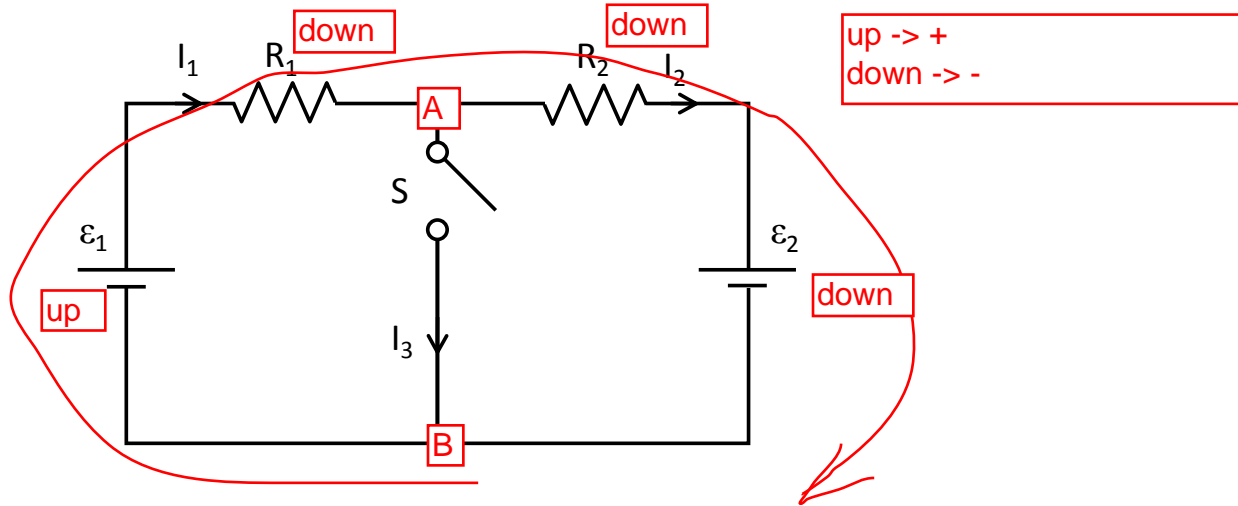
20. The distance between the plates is increased from 5  $\mu\text{m}$  to 20  $\mu\text{m}$ . How does the new charge on the top plate  $Q_{top,new}$  compare to the original charge on the top plate  $Q_{top}$ ?

- a.  $Q_{top,new} < Q_{top}$       Charge is conserved!  
 b.  $Q_{top,new} > Q_{top}$   
**c.**  $Q_{top,new} = Q_{top}$

The next three questions pertain to the following situation:

Consider the circuit below.  $\mathcal{E}_1 = 15 \text{ V}$ ,  $\mathcal{E}_2 = 5 \text{ V}$ ,  $R_1 = 1 \Omega$ ,  $R_2 = 2 \Omega$ .

Initially the switch  $S$  is open.



21. What is the current  $I_1$  in resistor  $R_1$ ?

- a.  $I_1 = 0 \text{ A}$
- b.  $I_1 = 6.25 \text{ A}$
- c.  $I_1 = 3.33 \text{ A}$
- d.  $I_1 = 1.50 \text{ A}$
- e.  $I_1 = 17.5 \text{ A}$

Kirchhoff's voltage law implies (clockwise around the outer loop)

$$\mathcal{E}_1 - I_1 R_1 - I_2 R_2 - \mathcal{E}_2 = 0$$

$$I_1 = I_2$$

so

$$I_1 = (\mathcal{E}_1 - \mathcal{E}_2) / (R_1 + R_2) = 10/3 = 3.33 \text{ A}$$

but this should also be obvious intuitively.

See how to choose the sign above.

22. Now the switch  $S$  is closed. What is the current  $I_3$ ?

- a.  $I_3 = 0 \text{ A}$
- b.  $I_3 = 6.25 \text{ A}$
- c.  $I_3 = 3.33 \text{ A}$
- d.  $I_3 = 1.50 \text{ A}$
- e.  $I_3 = 17.5 \text{ A}$

Notice that the voltages at A and B are identical (you may assume it is zero). Therefore, Kirchhoff's current law applied at A implies

$$I_1 = I_2 + I_3$$

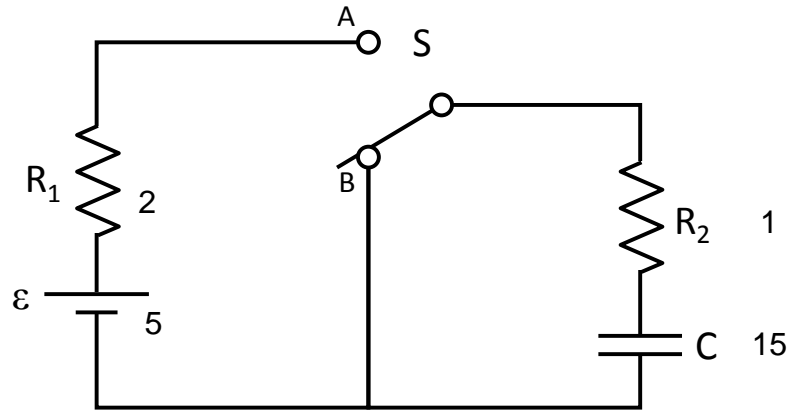
Since  $V_A = 0$ ,  $I_1 = \mathcal{E}_1 / R_1 = 15/1 = 15 \text{ A}$

$$I_2 = -\mathcal{E}_2 / R_2 = -5/2 = -2.5 \text{ A}.$$

Therefore,  $I_3 = I_1 - I_2 = 17.5 \text{ A}$ .

The next five questions pertain to the following situation:

Consider the circuit below.  $\mathcal{E} = 5 \text{ V}$ ,  $R_1 = 2 \text{ } \Omega$ ,  $R_2 = 1 \text{ } \Omega$ , and  $C = 15 \text{ } \mu\text{F}$ . Initially the switch  $S$  is at position B and the capacitor  $C$  is fully discharged.



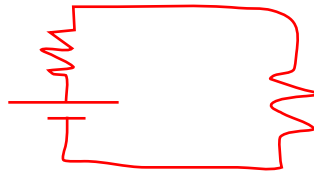
This initial configuration implies that  $C$  is not charged at all.

At  $t = 0$ , the switch  $S$  is flipped to position A.

23. What is the current  $I_2$  in resistor  $R_2$  *immediately* after setting the switch to A?

- a.  $I_2 = 0 \text{ A}$
- b.  $I_2 = 1.67 \text{ A}$**
- c.  $I_2 = 12.5 \text{ A}$
- d.  $I_2 = 6.33 \text{ A}$
- e.  $I_2 = 5.00 \text{ A}$

Immediately after switching, the voltage across  $C$  is 0, so the effective circuit is

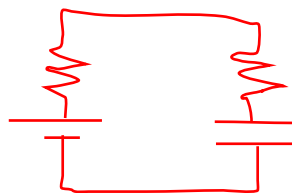


$$I_2 = \mathcal{E} / (R_1 + R_2) = 5 / 3 = 1.67 \text{ A}$$

24. At some time  $t > 0$  later, the current through  $R_2$  is found to be  $I_2 = 1.0 \text{ A}$ . What is the charge  $Q$  on the capacitor  $C$  at that precise time?

- a.  $Q = 30 \text{ } \mu\text{C}$**
- b.  $Q = 250 \text{ } \mu\text{C}$
- c.  $Q = 75 \text{ } \mu\text{C}$

This implies that the voltage across  $R_2$  is  $I_2 R_2 = 1 \text{ V}$ .



Voltage due to  $R_1$  and  $R_2$  is 3 V

Therefore the voltage across  $C$  is  $5 - 3 = 2 \text{ V}$ .  
 $Q = 2C = 30 \times 10^{-6} \text{ C}$ .

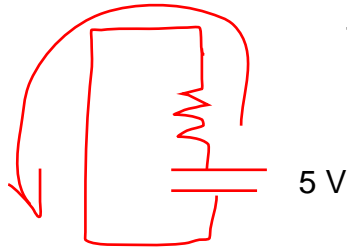
*The next three questions continue from the previous page:*

After a long time, the switch  $S$  is reset to position B. The next three questions pertain to this situation.

25. What is the magnitude of the current  $I_2$  in resistor  $R_2$  *immediately* after resetting the switch to B?

- a.  $I_2 = 0$  A
- b.  $I_2 = 6.67$  A
- c.  $I_2 = 5.00$  A
- d.  $I_2 = 12.5$  A
- e.  $I_2 = 1.33$  A

After a long time, C is charged up (the voltage across it is 5V, and the current is zero.

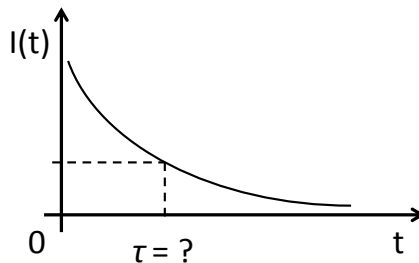


Thus, immediately after resetting to B, the effective circuit reads as in the left. Hence,  $I = 5/R_2 = 5$  A.

26. In what direction around the circuit does the current  $I$  flow immediately after resetting the switch?

- a. Clockwise
- b. Counterclockwise

27. Eventually, the current decays gradually to zero as shown in the figure below. Which formula best represents the time constant  $\tau$  for this decay?



- a.  $\tau = R_1 C$
- b.  $\tau = R_2 C$
- c.  $\tau = (R_1 + R_2) C$

**Check to make sure you bubbled in all your answers.  
Did you bubble in your name, exam version and network-ID?**