

Last Name: _____ First Name _____ Network-ID _____

Discussion Section: _____ Discussion TA Name: _____

Turn off your cell phone and put it out of sight.

Keep your calculator on your own desk. Calculators cannot be shared.

This is a closed book exam. You have ninety (90) minutes to complete it.

1. Use a #2 pencil. Do not use a mechanical pencil or pen. Darken each circle completely, but stay within the boundary. If you decide to change an answer, erase vigorously; the scanner sometimes registers incompletely erased marks as intended answers; this can adversely affect your grade. Light marks or marks extending outside the circle may be read improperly by the scanner. Be especially careful that your mark covers the **center** of its circle.
2. You may find the version of **this Exam Booklet at the top of page 2**. Mark the version circle in the TEST FORM box near the middle of your answer sheet. **DO THIS NOW!**
3. Print your **NETWORK ID** in the designated spaces at the *right* side of the answer sheet, starting in the left most column, then **mark the corresponding circle** below each character. If there is a letter "o" in your NetID, be sure to mark the "o" circle and not the circle for the digit zero. If and only if there is a hyphen "-" in your NetID, mark the hyphen circle at the bottom of the column. When you have finished marking the circles corresponding to your NetID, check particularly that you have not marked two circles in any one of the columns.
4. Print **YOUR LAST NAME** in the designated spaces at the *left* side of the answer sheet, then mark the corresponding circle below each letter. Do the same for your **FIRST NAME INITIAL**.
5. Print your UIN# in the STUDENT NUMBER designated spaces and mark the corresponding circles. You need not write in or mark the circles in the SECTION box.
6. Sign your name (**DO NOT PRINT**) on the **STUDENT SIGNATURE line**.
7. On the **SECTION line**, print your **DISCUSSION SECTION**. You need not fill in the COURSE or INSTRUCTOR lines.

*Before starting work, check to make sure that your test booklet is complete. You should have 17 **numbered pages** plus three (3) Formula Sheets.*

Academic Integrity—Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including dismissal from the University.

This Exam Booklet is Version A. Mark the **A** circle in the **TEST FORM** box near the middle of your answer sheet. **DO THIS NOW!**

Exam Grading Policy—

The exam is worth a total of **122** points, composed of three types of questions.

MC5: *multiple-choice-five-answer questions, each worth 6 points.*

Partial credit will be granted as follows.

- (a) If you mark only one answer and it is the correct answer, you earn **6** points.
- (b) If you mark *two* answers, one of which is the correct answer, you earn **3** points.
- (c) If you mark *three* answers, one of which is the correct answer, you earn **2** points.
- (d) If you mark no answers, or more than *three*, you earn **0** points.

MC3: *multiple-choice-three-answer questions, each worth 3 points.*

No partial credit.

- (a) If you mark only one answer and it is the correct answer, you earn **3** points.
- (b) If you mark a wrong answer or no answers, you earn **0** points.

MC2: *multiple-choice-two-answer questions, each worth 2 points.*

No partial credit.

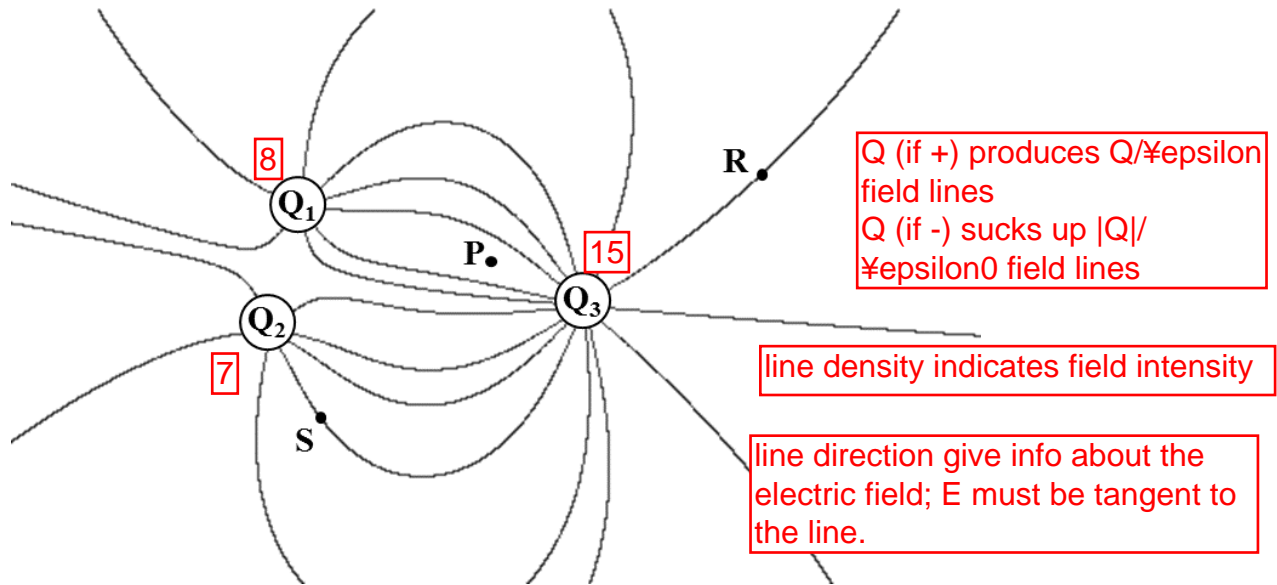
- (a) If you mark only one answer and it is the correct answer, you earn **2** points.
- (b) If you mark the wrong answer or neither answer, you earn **0** points.

Some helpful information:

- A reminder about prefixes: p (pico) = 10^{-12} ; n (nano) = 10^{-9} ; μ (micro) = 10^{-6} ; m (milli) = 10^{-3} ; k (kilo) = 10^{+3} ; M or Meg (mega) = 10^{+6} ; G or Gig (giga) = 10^{+9} .

The next two questions pertain to the following situation:

Consider the electric field lines connecting the charged objects shown above.



1. Which of the following statements is correct?

- a. $|Q_1| < |Q_2|$ $8 < 7$ X
- b. $|Q_1| < |Q_3|$ $8 < 15$ yes
- c. $|Q_3| < |Q_2|$ $15 < 8$ X

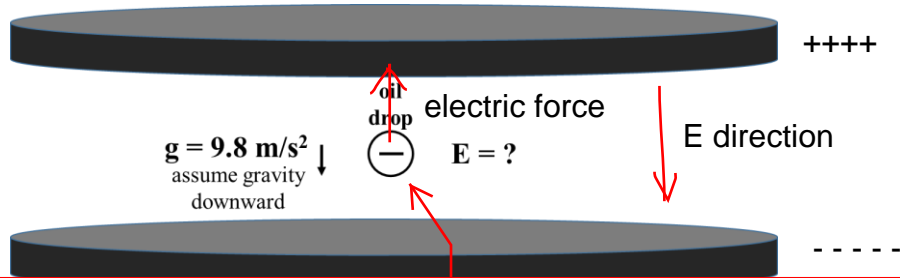
2. At which of the points P, R, or S labeled in the diagram would the smallest magnitude of electric field be measured?

- a. P
- b. R
- c. S

lowest line density

The next two questions pertain to the following situation:

A large, negatively charged drop of oil is suspended in the electric field created by two charged capacitor plates near the earth's surface. Assume the object is suspended in the constant electric field between the plates, such that the pull of gravity is balanced by the electric force on the object.



Warning: Since the droplet is large, its induced polarization cannot be ignored, so this question is, strictly speaking, not answerable. It must be clearly stated that we may ignore the induced dipole.

3. Which plate must be the positively charged in order to cause the electric field to be oriented in the correct direction for the suspension to occur?

- a. the top plate
- b. the bottom plate
- c. both plates

The direction of the force is opposite to that of the electric field.
E field is from + to -.

4. Given that the mass m of the charged object 4×10^{-13} kg and its charge Q is measured to be 8×10^{-18} C, what is the magnitude of the electric field E needed to cause such a suspension?

- a. $E = 5.8 \times 10^{-4}$ N/C
- b. $E = 3.4 \times 10^3$ N/C
- c. $E = 4.9 \times 10^5$ N/C
- d. $E = 6.2 \times 10^6$ N/C
- e. $E = 9.7 \times 10^9$ N/C

If we ignore the induction effect, the force balance implies
gravity + electric force = 0

or

$$mg = qE \rightarrow E = mg/q$$

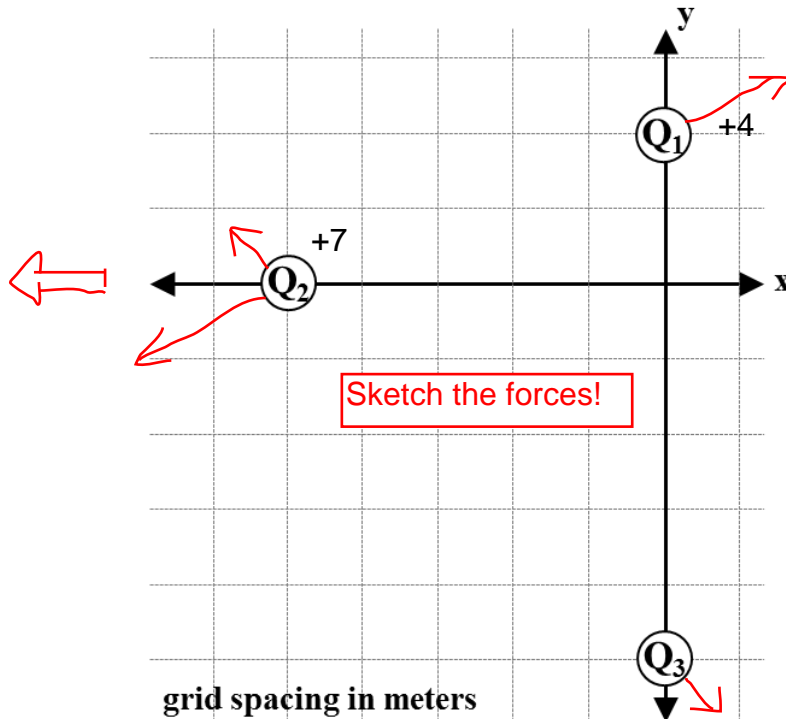
That is,

$$E = 4 \times 10^{-13} \times 9.8 / 8 \times 10^{-18} \\ = 4.9 \times 10^{-13 + 18} = 4.9 \times 10^5 \text{ N/C}$$

at most

The next two questions pertain to the following situation:

Three charges are placed in a coordinate system as shown and held in place. The charge $Q_1 = +4 \text{ nC}$ is located at $(0 \text{ m}, 2 \text{ m})$, the charge $Q_2 = +7 \text{ nC}$ is located at $(-5 \text{ m}, 0 \text{ m})$, and the charge Q_3 is located at $(0 \text{ m}, -5 \text{ m})$ with unknown charge.



5. What must be the charge on Q_3 for the net force on Q_2 due to Q_1 and Q_3 to point directly to the left, in the $-x$ direction?

- a. $Q_3 = +6.5 \text{ nC}$
- b. $Q_3 = -9.1 \text{ nC}$
- c. $Q_3 = -0.78 \text{ nC}$
- d. $Q_3 = +3.6 \text{ nC}$**
- e. $Q_3 = +17 \text{ nC}$

From the figure above Q_3 must be positive.
y-component must kill each other.

$$E_y \text{ due to } Q_1 = -[kQ_1/(25+4)] \times (2/\sqrt{29})$$

$$E_y \text{ due to } Q_3 = [kQ_3/(25+25)] \times (1/\sqrt{2})$$

$$\text{That is } Q_3 = Q_1(50/29) (2\sqrt{2}/\sqrt{29}) = 3.62 \text{ nC.}$$

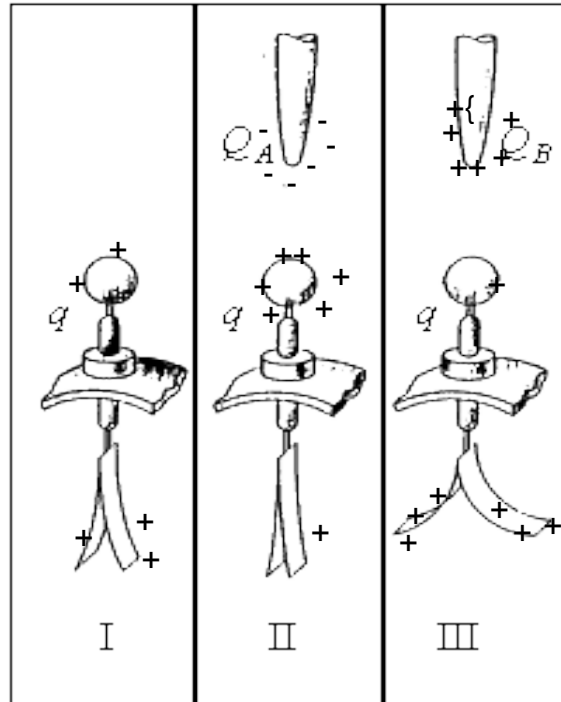
6. Let Q_2 be replaced with another charge $Q_{2,\text{new}}$ of the same sign but larger in magnitude. Q_3 is replaced with a new charge $Q_{3,\text{new}}$ such that $Q_{2,\text{new}}$ feels a net force only to the left, as above. How does $|Q_{3,\text{new}}|$ compare to $|Q_3|$ in the previous problem?

- a. $|Q_{3,\text{new}}| < |Q_3|$
- b. $|Q_{3,\text{new}}| = |Q_3|$**
- c. $|Q_{3,\text{new}}| > |Q_3|$

The problem says the field at Q_2 does not change. Q_2 has nothing to do with the field, so to maintain the field we must not change Q_1 nor Q_3 .

7. The conducting sphere of an electroscope is loaded with charge q so that the conducting leaves stand apart as shown in Figure I. As a rod with charge Q_A is brought near (but does not touch) the conducting sphere in figure II, the leaves move towards one another. This rod is removed and a different rod holding charge Q_B is brought near the conducting sphere in Figure III. The leaves are seen to move farther apart than in Figure I.

Since we cannot determine the signs of charges, let us tentatively assume q to be +.

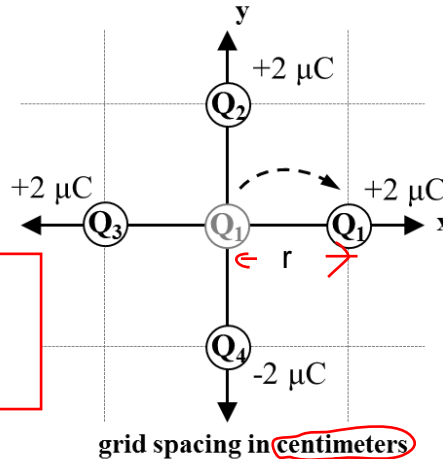


Which one of the following statements must be true?

- a. The magnitude of Q_A must be less than the magnitude of q . need not be.
- b. Q_A and q must have the same sign. NO
- c. Q_B and q must have the same sign. YES
- d. Q_A must be positive and Q_B must be negative. As mentioned above we cannot determine the signs. We can only say $qQ_A < 0$, $qQ_B > 0$.
- e. Q_A must be negative and Q_B must be positive.

The next three questions pertain to the following situation:

Four charges of equal magnitude are arranged in the coordinate system as shown below. The charge Q_2 is located at (0 cm, 1 cm), the charge Q_3 at (-1 cm, 0 cm), and the charge Q_4 at (0 cm, -1 cm). Initially the charge Q_1 is located at the origin (0 cm, 0 cm) and you apply an external force to move the charge one centimeter in the +x direction to (1 cm, 0 cm). The values of the charges are indicated in the diagram.



Superposition principle
We can separately compute the potentials contributed by individual charges

8. Calculate the potential at the origin for the charges in the final configuration.

- a. $V(0,0) = 0 \text{ V}$
 - b. $V(0,0) = -7.2 \times 10^6 \text{ V}$
 - c. $V(0,0) = +1.4 \times 10^6 \text{ V}$
 - d. $V(0,0) = -9.5 \times 10^5 \text{ V}$
 - e. $V(0,0) = +3.6 \times 10^6 \text{ V}$
- V due to Q_1 kQ_1/r , etc. Therefore, the total V is
 $V = (k/r)(\sum \text{of } Q) = (9 \times 10^9) (4 \times 10^{-6}) / (1 \times 10^{-2})$
 $= 36 \times 10^{9-6+2} = 36 \times 10^5 \text{ V}$

9. How much work is done by your external force in moving Q_1 from the origin to its final position at (1 cm, 0 cm)?

- The initial potential energy of Q_1 (Q_2 and Q_4 kill each other)
- a. $W_{ext} = 0 \text{ J}$
 - b. $W_{ext} = +1.8 \text{ J}$
 - c. $W_{ext} = +3.6 \text{ J}$
 - d. $W_{ext} = -1.8 \text{ J}$
 - e. $W_{ext} = -9.2 \text{ J}$
- The final potential energy of Q_1 (Q_2 and Q_4 kill each other)
- $(k/r)Q_1(Q_2/\sqrt{2} + Q_3/2 + Q_4/\sqrt{2})$
 $= (9 \times 10^9)(2 \times 10^{-6})(2 \times 10^{-6}) / (2 \times 10^{-2})$.

The potential energy difference is
 $\Delta V = -(9 \times 10^9)(2 \times 10^{-6})(2 \times 10^{-6}) / (2 \times 10^{-2}) = -18 \times 10^{-1} \text{ J}$.

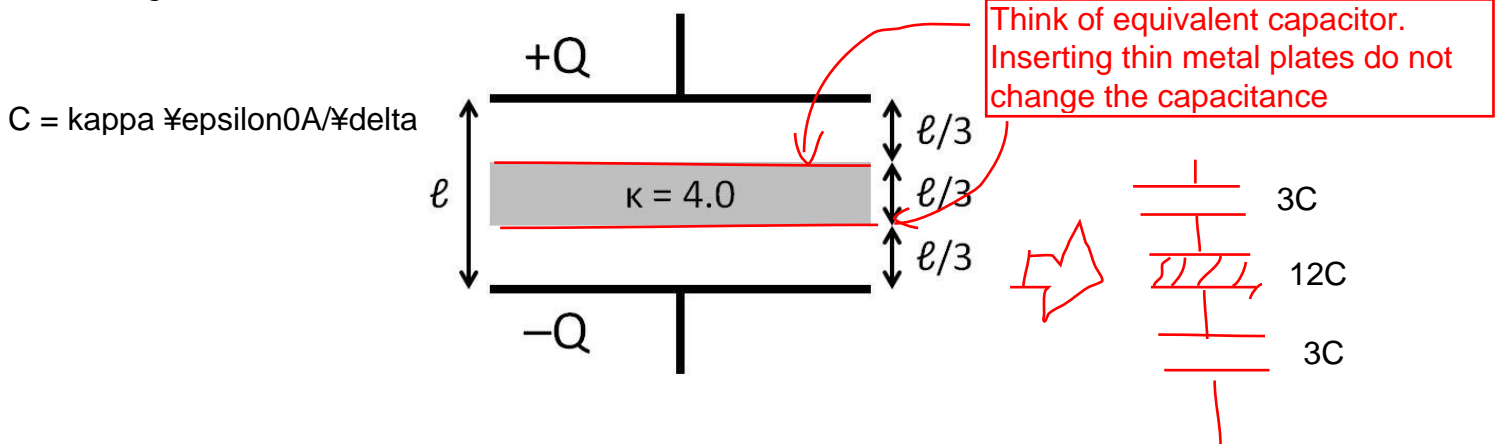
The potential decrease is due to the work done by you.

10. If the charge Q_1 were instead opposite in charge, the work done to move it as in the previous problem would have the same value but the opposite sign.

- a. True
 - b. False
- As can be seen from the above formula.

The next two questions pertain to the following situation:

An isolated capacitor consists of two parallel plates carrying charge $\pm Q$, separated by a distance ℓ . Its capacitance is C_0 . Suppose a slab of dielectric material with dielectric constant $\kappa = 4.0$ and width $\ell/3$ is inserted midway between the two plates, as shown in the figure below.



11. In terms of C_0 , what is the new capacitance C_{new} with the dielectric slab?

- a. $C_{new} = 4C_0$
- b. $C_{new} = (4/3)C_0$
- c. $C_{new} = C_0$
- d. $C_{new} = (2/3)C_0$
- e. $C_{new} = (1/4)C_0$

$$\begin{aligned} 1/C_{tot} &= 1/3C_0 + 1/12C_0 + 1/3C_0 \\ &= (1/C_0) (9/12) \\ \text{That is } C_{tot} &= 4C_0/3 \end{aligned}$$

Warning
 Strictly speaking, there is no correct solution among a-e, because according to the proper writing of the formulas $4/3C$ implies $4/(3C)$, not $(4/3)C$.
 You must learn how to write math formulas correctly.

12. With the dielectric slab inserted, the charge Q on the plates

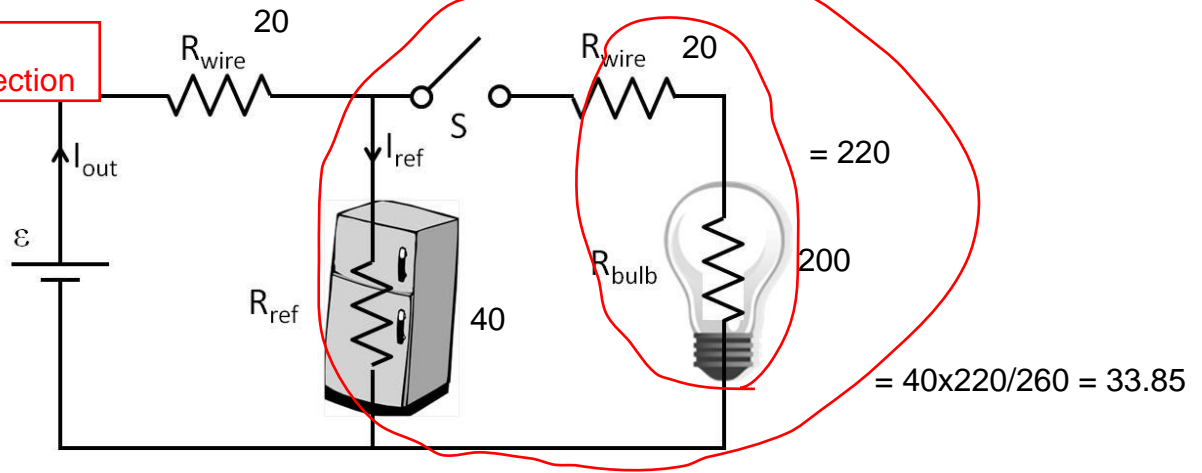
- a. increases
- b. decreases
- c. remains the same

Isolated -> the total charge on each plate must be conserved.

The next two questions pertain to the following situation:

The following represents a circuit in your kitchen. A wall outlet generates a voltage $\varepsilon = 120 \text{ V}$ and powers a refrigerator with resistance $R_{ref} = 40 \text{ }\Omega$. A light bulb in the room has resistance $R_{bulb} = 200 \text{ }\Omega$ and can be turned on or off with the light switch S . The wires in the circuit have resistance $R_{wire} = 20 \text{ }\Omega$.

key
series vs parallel connection



13. Let $I_{out,open}$ be the current out of the outlet when the light switch S is open (i.e when the light is off). Now the light switch S is closed, turning on the light. The current $I_{out,closed}$ out of the wall outlet

- a. $I_{out,closed} > I_{out,open}$
- b. $I_{out,closed} < I_{out,open}$
- c. $I_{out,closed} = I_{out,open}$

More paths to flow, more current we have.

14. What is the power P dissipated in the circuit when the switch is closed?

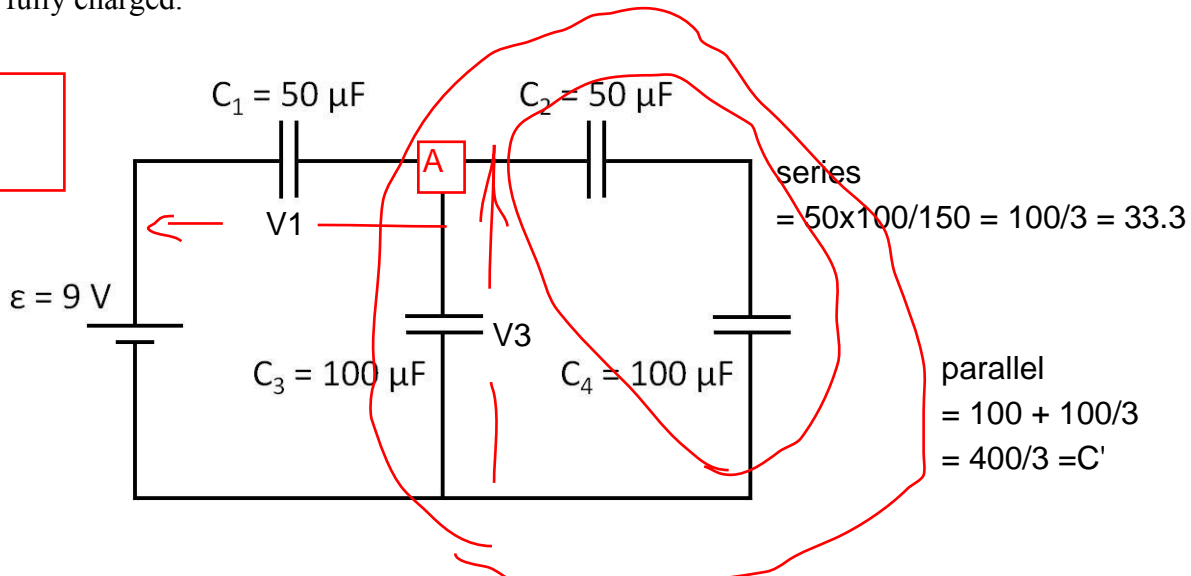
- a. $P = 175 \text{ W}$
- b. $P = 267 \text{ W}$
- c. $P = 486 \text{ W}$
- d. $P = 2210 \text{ W}$
- e. $P = 7.93 \times 10^5 \text{ W}$

We need the total resistance $R_{tot} = 20 + 33.85 = 53.85$.
 $P = V^2/R_{tot} = 120^2/53.85 = 267 \text{ W}$

The next two questions pertain to the following situation:

Consider a network of capacitors connected to a battery, as shown below. All the capacitors are fully charged.

key
series vs parallel
connections



15. What is the total energy U_{tot} stored in the capacitor network?

- a. $U_{tot} = 0.850$ mJ
- b. $U_{tot} = 3.115$ mJ
- c. $U_{tot} = 1.472$ mJ
- d. $U_{tot} = 7.257$ mJ
- e. $U_{tot} = 10.58$ mJ

We need the total capacitance.

$$C_{tot} = 50 \times (400/3) / (550/3) = 400/11 = 36.36 \text{ } \mu\text{F}$$

$$\text{Therefore, } U = (1/2)CV^2 = (1/2) (36.36 \times 10^{-6}) 9^2 = 1472.7 \times 10^{-6} \text{ J} = 1.47 \times 10^{-3} \text{ J}$$

16. What is the charge Q_3 stored on the capacitor C_3 ?

- a. $Q_3 = 245$ μC
- b. $Q_3 = 350$ μC
- c. $Q_3 = 492$ μC

see the above figure

$$V_1 + V_3 = 9$$

$$V_1 \times C_1 = V_3 \times C' \rightarrow V_3 = V_1(50 \times 3 / 400) = 3V_1/8.$$

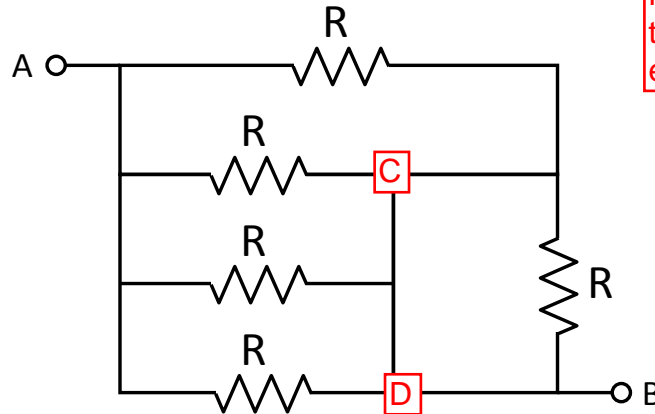
$$V_1 + 3V_1/8 = 11V_1/8 = 9 \rightarrow V_1 = 72/11 = 6.545 \text{ V}$$

This implies that C_1 has $327 \text{ } \mu\text{C}$.

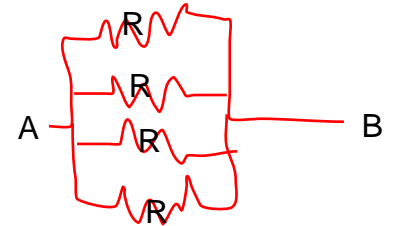
The ratio of the charges on C_3 and C_2 must be $100 : 33 = 3 : 1$, so $327 \times (3/4) = 245.25 \text{ } \mu\text{C}$ must be in C_3 .

charge conservation: total charge at A must be zero, so the charge on C_1 and the sum of charges on C_3 and C_2 must be identical.

The key point is to recognize clearly what points are with the same electrical potential.



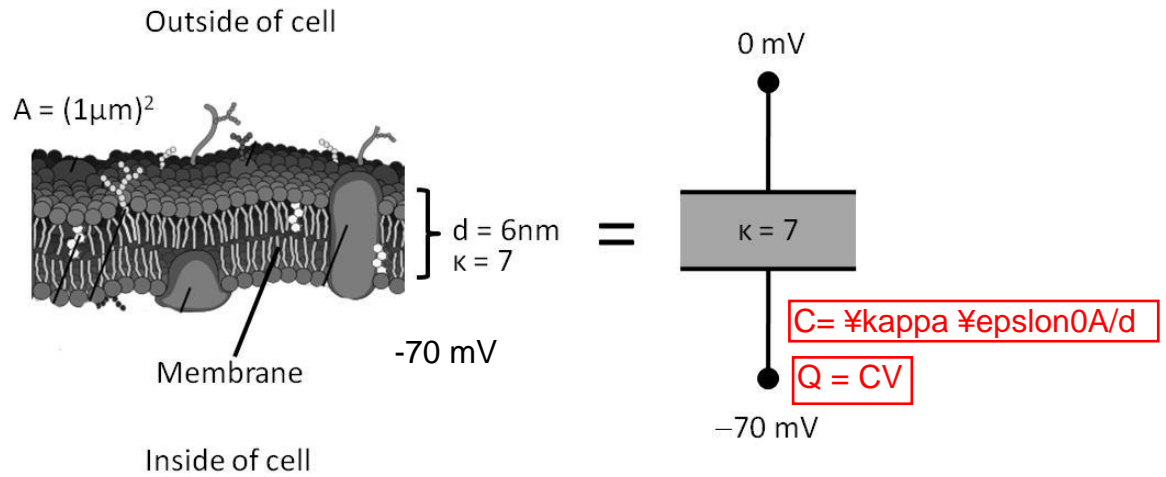
Notice that C and D must have the same voltage, so the equivalent circuit is



17. In the network of resistors above, the 5 resistors have the *same* resistance R . Calculate the equivalent resistance between points A and B in terms of R .

- a. $R_{tot} = R/4$
- b. $R_{tot} = 3R/5$
- c. $R_{tot} = R$
- d. $R_{tot} = 4R/3$
- e. $R_{tot} = 5R/4$

18. A neuron can be modeled as a parallel plate capacitor, with the membrane acting as a dielectric and ions as charges on the plates. The membrane has a dielectric constant $\kappa = 7.0$ and a thickness $d = 6.0$ nm.



In its resting state, the inside of the cell is at a potential of -70mV relative to the outside because potassium (K^+) ions outside the cell exceed those inside.

Calculate the excess number of potassium ions ($1 \text{ K}^+ = 1.6 \times 10^{-19} \text{ C}$) on the outer side of a $1\text{-}\mu\text{m}^2$ patch of membrane.

- a. $N = 4.5 \times 10^3$
- b. $N = 9.3 \times 10^3$
- c. $N = 2.6 \times 10^4$
- d. $N = 7.4 \times 10^5$
- e. $N = 1.2 \times 10^6$

we can compute the capacitance per $1 \mu\text{m}^2$:

$$C = 7 \times (8.85 \times 10^{-12}) \times (1 \times 10^{-12}) / (6 \times 10^{-9})$$

$$= 10.325 \times 10^{-12 - 12 + 9} = 1.0325 \times 10^{-14} \text{ F}$$

Therefore, the charge per $1 \mu\text{m}^2$ patch is

$$Q = CV = (1.0325 \times 10^{-14}) \times (70 \times 10^{-3})$$

$$= 72.3 \times 10^{-17} \text{ C.}$$

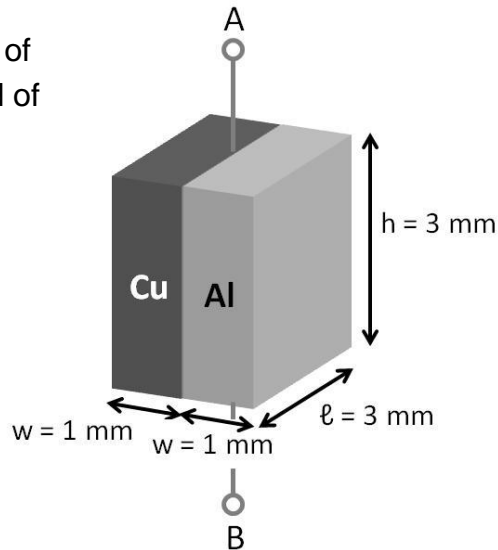
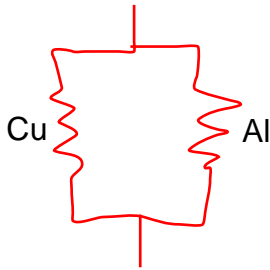
Each K ion is $1.6 \times 10^{-19} \text{ C}$. so

$$N = 72.3 \times 10^{-17} / (1.6 \times 10^{-19}) = 45.1 \times 10^2$$

A resistor is made from a slab of aluminum and a slab of copper pasted together as shown in the figure below.

The two slabs have identical dimensions: $w = 1 \text{ mm}$, $\ell = 3 \text{ mm}$, and $h = 3 \text{ mm}$. The resistivities of aluminum and copper are $\rho_{Al} = 2.65 \times 10^{-8} \Omega \cdot \text{m}$ and $\rho_{Cu} = 1.68 \times 10^{-8} \Omega \cdot \text{m}$, respectively.

This is a parallel connection of two resistors made of Al and of Cu



For each piece
 $R = \rho h / (wl)$.

Parallel connection
 $1/R_{tot} = 1/R_1 + 1/R_2$
 or
 $R_{tot} = R_1 R_2 / (R_1 + R_2)$

19. Suppose wires are attached to the top and bottom surfaces of the resistor, at positions A and B. What is the resistance measured across A and B?

- a. $R_{A-B} = 0.48 \times 10^{-5} \Omega$
 b. $R_{A-B} = 1.03 \times 10^{-5} \Omega$
 c. $R_{A-B} = 6.74 \times 10^{-5} \Omega$

$$R_{Al} = (2.65 \times 10^{-8}) (3 \times 10^{-3}) / (1 \times 10^{-3} \times 3 \times 10^{-3}) \\ = 2.65 \times 10^{-8} \cdot 3 \cdot 10^3 / 3 = 2.65 \times 10^{-5} \text{ ohm.}$$

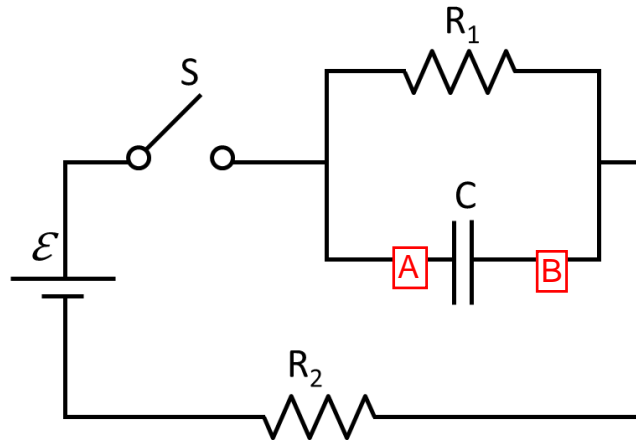
$$R_{Cu} = (1.68 \times 10^{-8}) \text{ the same} = 1.68 \times 10^{-5} \text{ ohm}$$

Therefore,

$$R_{tot} = [(1.68 \times 2.65) / (1.68 + 2.65)] \times 10^{-5} = 1.028 \times 10^{-5} \text{ ohm}$$

The next four questions pertain to the following situation:

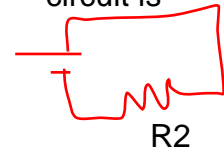
Consider the following circuit. Initially the switch S is open and the capacitor C is fully discharged.



$R_1 = 17 \Omega$
 $R_2 = 23 \Omega$
 $C = 5 \mu\text{F}$
 $\mathcal{E} = 13 \text{ V}$

C is not charged immediately, so initially the voltage across it is 0. This implies initially A and B are connected directly, so no current can go through R1.

The initial equivalent circuit is



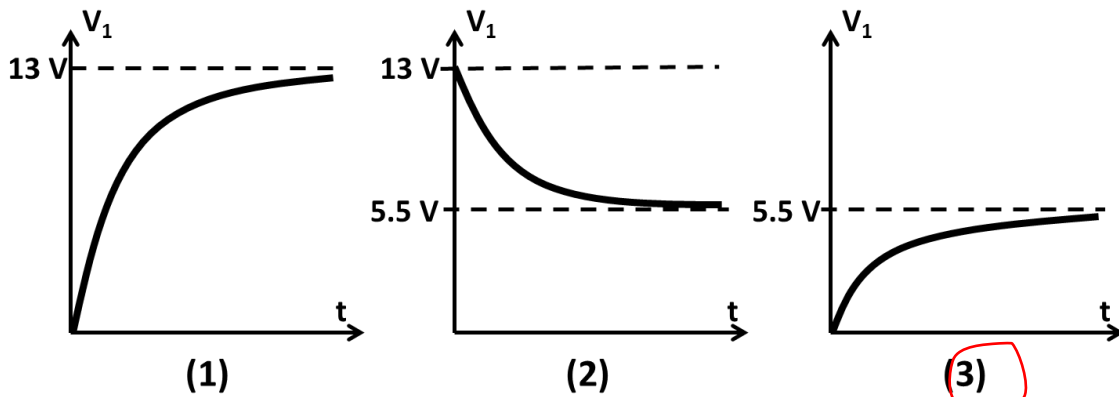
At $t = 0$, the switch is closed.

20. What is the current I_C through the capacitor C immediately after the switch is closed?

- a. $I_C = 0 \text{ A}$
- b. $I_C = 0.355 \text{ A}$
- c. $I_C = 0.565 \text{ A}$**
- d. $I_C = 0.705 \text{ A}$
- e. $I_C = 1.430 \text{ A}$

All the current goes through the capacitor, so
 $I_C = \mathcal{E}/R_2 = 13/23 = 0.565 \text{ A}$

21. Which of the following plots best represents the voltage drop V_1 across the resistor R_1 as a function of time?



- a. (3)**
- b. (2)
- c. (1)

As noted above, initially no current.

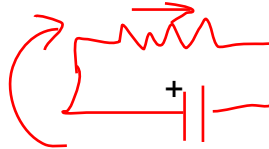
(problem continues from previous page)

After a very long time, the switch S is now opened.

22. Immediately after the switch is opened, in which direction will the current flow through the capacitor?

- a. To the left
- b. To the right
- c. There is no current

now, the capacitor is fully charged, so it behaves as a battery, and discharges through R1



This is a semantic question. If you understand the physics illustrated in the left, IGNORE this bad question.

23. How long will it take for the current across the capacitor to drop to 1/10 of the value just after the switch was opened?

- a. 80.5 μs
- b. 131 μs
- c. 196 μs
- d. 316 μs
- e. 11.4 μs

How long for the current through R1 is a better question.

$$\tau = RC$$

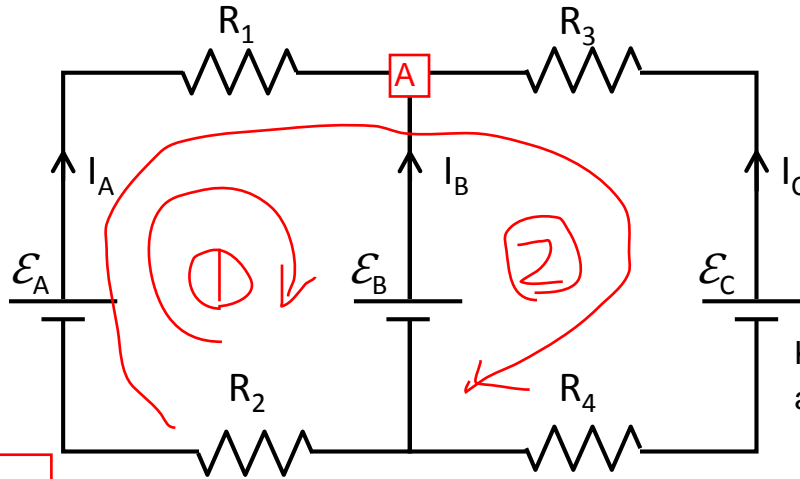
$$e^{-t/\tau} = 1/10$$

$$\ln 10 = t/\tau \rightarrow t = \ln 10 RC = \ln 10 \times 17 \times (5 \times 10^{-6}) \\ = 195.7 \times 10^{-6} \text{ s}$$

The next two questions pertain to the following situation:

Consider the following circuit. All resistors have a resistance of 1Ω .

If you wish to solve the problem completely, you need 3 equations.



Kirchhoff's current law applied at A implies $I_A + I_B + I_C = 0$

Kirchhoff's voltage law: around a loop no net change of the voltage.

$\mathcal{E}_A = 18 \text{ V}, \mathcal{E}_B = 10 \text{ V}, \mathcal{E}_C = 4 \text{ V}$

There are many ways to solve this problem. Here, we use the two loops in the figure.

24. What is the current I_A out of the battery with emf \mathcal{E}_A ?

- a. $I_A = 9 \text{ A}$
- b. $I_A = 4 \text{ A}$**
- c. $I_A = 14 \text{ A}$
- d. $I_A = 3.5 \text{ A}$
- e. $I_A = 5 \text{ A}$

Loop 1

$EA - IA - EB - IA = 0 \rightarrow IA = 4A$

25. If \mathcal{E}_C is increased, the current across R_1 will:

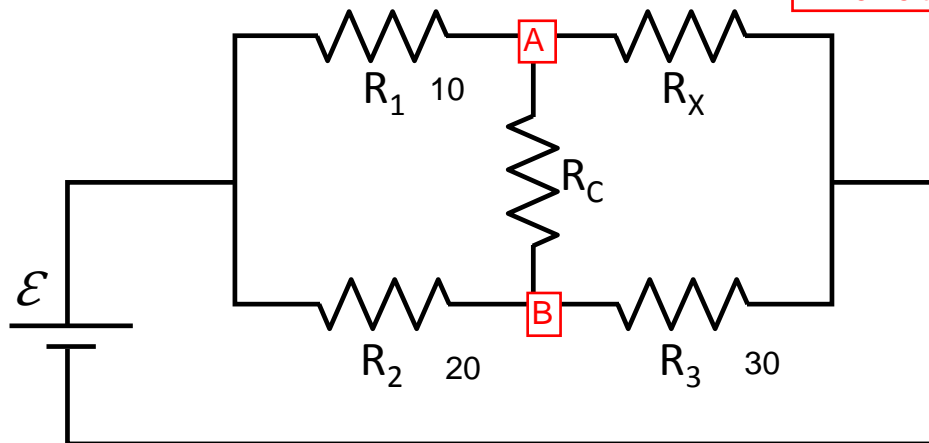
- a. increase
- b. stay the same**
- c. decrease

Note that I_A is completely determined already; \mathcal{E}_C has nothing to do with it.

The next two questions pertain to the following situation:

In the following circuit, the current through the resistor R_C is zero.

No current across R
-> no voltage drop across R



$$R_1 = 10 \Omega ; R_2 = 20 \Omega ; R_3 = 30 \Omega ; \mathcal{E} = 12 \text{ V}$$

26. What is the current through resistor R_3 ?

- a. 1.0 A
- b. 0.48 A
- c. 0.24 A
- d. 2.2 A
- e. 3.7 A

The voltage at A and that at B must be identical.

The voltage at B is $(30/50) E = 7.2 \text{ V}$.

Therefore $7.2/30 = 0.24 \text{ A}$ must be the current.

27. What is the value of R_X ?

- a. 30 Ω
- b. 15 Ω
- c. 10 Ω

$R_X = 15 \text{ ohm}$ is required for the voltage A to be 7.2 V.

or $R_1/R_2 = R_X/R_3$ is needed. $R_X = R_1 R_3 / R_2 = 300 / 20 = 15$.

**Check to make sure you bubbled in all your answers.
Did you bubble in your name, exam version and network-ID?**

Mechanics:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + a t$$

$$F = m a$$

$$a_c = \frac{v^2}{r}$$

$$E_{tot} = K.E. + P.E.$$

$$K.E. = \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

$$p = m v$$

$$W_F = F d \cos \theta$$

Electrostatics:

$$F_{12} = \frac{k q_1 q_2}{r^2}$$

$$E \equiv \frac{F}{q_0}$$

$$V \equiv \frac{U}{q_0}$$

$$\text{Point charge: } E = \frac{k q}{r^2}, \quad V = \frac{k q}{r}$$

$$U_{12} = \frac{k q_1 q_2}{r}$$

$$W_E = -\Delta U = -W_{you}$$

Capacitance:

$$C \equiv \frac{Q}{V}$$

$$\text{Parallel plate capacitor: } C = \frac{\kappa \epsilon_0 A}{d}, \quad V = E d$$

$$U_C = \frac{1}{2} Q V = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C}$$

$$C_P = C_1 + C_2 + \dots$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

Resistance:

$$R \equiv \frac{V}{I}$$

$$I = \frac{\Delta q}{\Delta t}$$

$$\text{Physical resistance: } R = \rho \frac{L}{A}$$

$$P = IV = I^2 R = \frac{V^2}{R}$$

$$R_S = R_1 + R_2 + \dots$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

Circuits:

$$\sum \Delta V = 0$$

$$\sum I_{in} = \sum I_{out}$$

$$q(t) = q_\infty (1 - e^{-t/\tau})$$

$$q(t) = q_0 e^{-t/\tau}$$

$$I(t) = I_0 e^{-t/\tau}$$

$$\tau = RC$$

Magnetism:

$$F = q v B \sin \theta$$

$$r = \frac{m v}{q B}$$

$$F = I L B \sin \theta$$

$$\tau = N I A B \sin \phi$$

$$B_{wire} = \frac{\mu_0 I}{2 \pi r}$$

$$B_{sol} = \mu_0 n I$$

Induction and inductance:

$$\varepsilon = -N \frac{\Delta \Phi}{\Delta t}$$

$$\Phi = B A \cos \phi$$

$$\varepsilon_{bar} = B L v$$

$$\varepsilon_{gen} = \varepsilon_{max} \sin \omega t = \omega N A B \sin \omega t$$

$$\omega = 2 \pi f$$

$$L \equiv \frac{N \Phi}{I}$$

$$\varepsilon = -L \frac{\Delta I}{\Delta t}$$

$$\text{Solenoid inductor: } L = \mu_0 n^2 A \ell$$

$$U_L = \frac{1}{2} L I^2$$

AC circuits and transformers:

$$V_{rms} = \frac{V_{max}}{\sqrt{2}} \quad I_{rms} = \frac{I_{max}}{\sqrt{2}} \quad \frac{V_p}{V_s} = \frac{I_s}{I_p} = \frac{N_p}{N_s}$$

$$V_R(t) = V_{R,max} \sin(\omega t) = I_{max} R \sin(\omega t) \quad \omega = 2\pi f$$

$$V_C(t) = V_{C,max} \sin(\omega t - \pi/2) = I_{max} X_C \sin(\omega t - \pi/2) \quad X_C \equiv \frac{1}{\omega C}$$

$$V_L(t) = V_{L,max} \sin(\omega t + \pi/2) = I_{max} X_L \sin(\omega t + \pi/2) \quad X_L \equiv \omega L$$

$$V_{gen}(t) = V_{gen,max} \sin(\omega t + \phi) = I_{max} Z \sin(\omega t + \phi) \quad Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \quad \tan \phi = \frac{X_L - X_C}{R}$$

$$\bar{P} = I_{rms} V_{R,rms} = I_{rms} V_{gen,rms} \cos \phi \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Electromagnetic waves:

$$\lambda = \frac{c}{f} \quad E = cB$$

$$u_E = \frac{1}{2} \epsilon_0 E^2 \quad u_B = \frac{1}{2\mu_0} B^2 \quad \bar{u} = \frac{1}{2} \epsilon_0 E_{rms}^2 + \frac{1}{2\mu_0} B_{rms}^2 = \epsilon_0 E_{rms}^2 = \frac{B_{rms}^2}{\mu_0} \quad S = I = \bar{u}c$$

$$f' = f \left(1 \pm \frac{u}{c} \right) \quad I = I_0 \cos^2 \theta$$

Reflection and refraction:

$$\theta_r = \theta_i \quad \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad v = \frac{c}{n} \quad \sin \theta_c = \frac{n_2}{n_1} \quad M = \frac{\theta'}{\theta} \approx \frac{d_{near}}{f}$$

Interference and diffraction:

Double slit interference: $d \sin \theta = m\lambda$ $d \sin \theta = (m + \frac{1}{2})\lambda$ $m = 0, \pm 1, \pm 2, \dots$

Single-slit diffraction: $w \sin \theta = m\lambda$ $m = \pm 1, \pm 2, \dots$

Circular aperture: $D \sin \theta \approx 1.22\lambda$

Thin film: $\delta_1 = (0 \text{ or } \frac{1}{2})\lambda$ $\delta_2 = (0 \text{ or } \frac{1}{2})\lambda + 2t \frac{n_{film}}{\lambda_0}$ $|\delta_2 - \delta_1| = (m \text{ or } m + \frac{1}{2})\lambda$ $m = 0, 1, 2, \dots$

Quantum mechanics:

$$E = hf = \frac{hc}{\lambda} \quad \lambda = \frac{h}{p}$$

Blackbody radiation: $\lambda_{max} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$ Photoelectric effect: $K.E. = hf - W_0$

$$\Delta p_x \Delta x \geq \frac{\hbar}{2} \quad \hbar \equiv \frac{h}{2\pi}$$

Bohr atom: $2\pi r_n = n\lambda \quad n = 1, 2, 3, \dots$

$$r_n = \left(\frac{\hbar^2}{mke^2} \right) \frac{n^2}{Z} \approx (5.29 \times 10^{-11} \text{ m}) \frac{n^2}{Z}$$

$$\frac{1}{\lambda} \approx (1.097 \times 10^7 \text{ m}^{-1}) Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Quantum atom: $L = \sqrt{\ell(\ell+1)}\hbar$

$$L_n = mv_n r_n = n\hbar$$

$$E_n = - \left(\frac{mk^2 e^4}{2\hbar^2} \right) \frac{Z^2}{n^2} \approx -(13.6 \text{ eV}) \frac{Z^2}{n^2}$$

$$L_z = m_\ell \hbar$$

Nuclear physics and radioactive decay:

$$A = Z + N$$

$$r \approx (1.2 \times 10^{-15} \text{ m}) A^{1/3}$$

$$E_0 = mc^2$$

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

$$N(t) = N_0 e^{-\lambda t} = N_0 2^{-t/T_{1/2}}$$

$$T_{1/2} \equiv \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$$

Special relativity:

$$\Delta t = \gamma \Delta t_0$$

$$L = \frac{L_0}{\gamma}$$

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Constants and unit conversions:

$$g = 9.8 \text{ m/s}^2$$

$$e = 1.60 \times 10^{-19} \text{ C}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2$$

$$k \equiv \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ Nm}^2 / \text{C}^2$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/s}$$

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$$

$$hc = 1240 \text{ nm} \cdot \text{eV}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

$$m_{\text{proton}} = 1.67 \times 10^{-27} \text{ kg} = 938 \text{ MeV}$$

$$m_{\text{electron}} = 9.11 \times 10^{-31} \text{ kg} = 511 \text{ keV}$$

SI Prefixes		
Power	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^0	—	—
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p