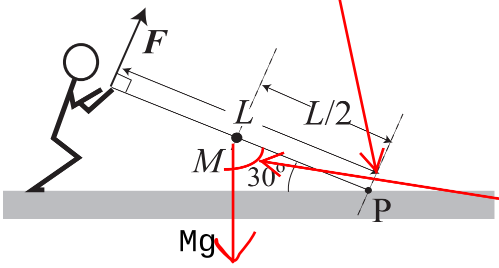
Balance around this point.
For this type of questions:
(1) Itemize all the forces acting on the bar except at P.
(2) For each force compute the torque around P.

Physics 101 (S11) **Q7A**

Name: ____________________ **Section:** __________ **Score:** __________/20

1. At the center of a light bar of length L is attached a point mass (= a block whose size you can ignore) of mass M. The bar makes an angle 30° with the horizontal. The bar can rotate freely around the point P in the figure. You can ignore the mass of the bar.



60 degrees

(a) You are supporting this bar at one end with the force $\boldsymbol{F}$ which is perpendicular to the bar as noted in the figure. Write the magnitude of the force F in terms of M and g (or find the ratio F/Mg), where g is the acceleration of gravity. [5]

clockwise   arm length   \sqrt{3}/2

```
The torque due to F around P is -FL.
The torque due to gravity on M around P is Mg(L/2)sin(60deg)
                                            = (\sqrt{3}/4)MgL.
The total torque = 0, so F = \sqrt{3}Mg/4.
```

Recall its definition.

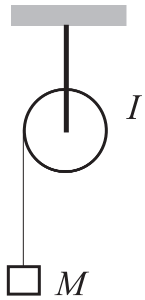
(b) You stop supporting it, and just leave the bar, which starts to rotate around P. Write the initial angular acceleration α of the bar in terms of g and L. [5]

```
We use I\alpha = \tau.
I of the ball around P is M(L/2)^2 = ML^2/4.
\tau = Mg (L/2)sin(60deg) = \sqrt{3}LMg/4 (as already
                                                computed in (a)).

Therefore, \alpha = \sqrt{3}g/L.
```

(**2** on the next page)

2. There is a drum whose moment of inertia around its axle is I. Around it is wound a flexible string from which hangs a block of mass M as illustrated below. The radius of the drum is R.



(a) Initially, the block and the drum are stationary. The block is gently released. After descending a certain distance L, the kinetic energy of the block is E. Write its momentum in terms of M and E. [5]

```
Don't be fooled by the stage setting.  E = p^2/2M, so
p = \sqrt{2ME}.
```

(b) Find the kinetic energy of the drum at the same moment in terms of I, R, M and E. [5]

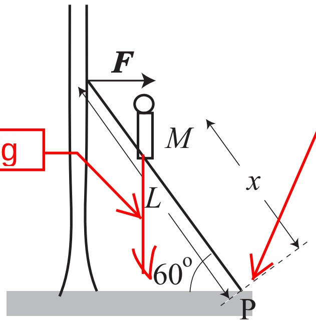
```
The kinetic energy of the disk is K = (1/2)I\omega^2, but
R\omega = V, so K = IV^2/2R^2,  MV^2= 2E, so
     K = I(2E/M)/2R^2 = IE/MR^2.
```

Read the general comment in Q7A

Physics 101 (S11) **Q7B**

Name: ____________________ **Section:** __________ **Score:** __________/20

1. A light ladder of length L leans against a smooth wall, making an angle of 60° with the horizontal. A person of mass M climbs up the ladder. You may ignore the weight of the ladder and the friction between the wall and the ladder. You may assume that there is a sufficient static friction between the ground and the ladder at point P.



Mg

You need not worry about the forces acting here.

clockwise

(a) If the magnitude of the force from the wall F exceed F_c, the wall breaks. Express the maximum distance x from the bottom of the ladder along the ladder the person can climb up in terms of F_c, M, L and the acceleration of gravity g. [5]

```
The torque due to F around P is -FL sin 60 = -\sqrt{3}FL/2.
The torque due to the gravity acting on the person is
      +Mgx sin 30 = Mgx/2.
The total torque around P is zero:
     Mgx/2 - \sqrt{3}FL/2 = 0.
Therefore, x = \sqrt{3}FL/Mg.  This implies that the `cirical' length
is \sqrt{3}F_cL/Mg.
```

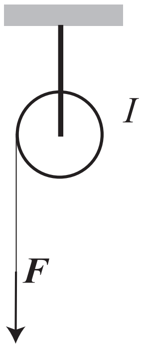
(b) When the person climbs up to L/2 along the ladder, the wall collapses and the force $\boldsymbol{F}$ vanishes. What is the angular acceleration α of the ladder around its bottom point P? Write α in terms of L and g. [5]

```
We use I\alpha = \tau.
     I  = M(L/2)^2 = ML^2/4.
     \tau = Mg(L/2)\sin 30 = MgL/4.

Therefore, \alpha = g/L.
```

(**2** on the next page)

2. There is a drum whose moment of inertia around its axle is I. A flexible string is wound around the drum. The radius of the drum is R. The drum can rotate without friction around the axle.



(a) Initially, the drum is at rest. A constant force whose magnitude is F is then applied downward on the string. What is the angular speed ω of the drum when the string has been pulled by a distance L. Write the angular speed ω of the drum in terms of L, I and F. [5]

```
This is a problem about the work-energy theorem: \Delta E = W.

The work done to the drum by the force is W = FL.
The initial kinetic energy is zero.
The final kinetic energy is (1/2)I\omega^2.
Therefore,
     FL = (1/2)I \omega^2,
or \omega = \sqrt{2FL/I}.
```

(b) Suppose the mass of the drum is doubled, but I is maintained by appropriately changing its size. What happens to the angular speed if you repeat the same experiment. You must justify your answer. [5]

```
No change, because I does not change.

Sorry for too trivial a problem.
```

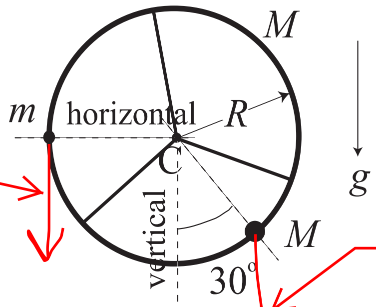
The uniformity of the hoop must be assumed.

Physics 101 (S11) **Q7C**

Name: ________________ **Section:** ________ **Score:** ________/**20**

1. Two point masses (= blocks whose sizes you can ignore) with mass m and M, respectively, are fixed on the hoop of radius R and mass M (the same M) as illustrated below. The hoop can rotate freely around its horizontal axle through the center C. The line connecting the center C and mass M makes an angle of 30° with the vertical as illustrated below. Ignore the masses of the spokes.

mg



Mg

Read the general comment given for Q7A.

(a) The hoop with the fixed point masses is stationary. What is the ratio m/M? [5]

[Notice that the hoop has nothing to do with the question except for keeping the mass=center distance constant.]

The torque around C due to mg = mgR clockwise
The torque around C due to Mg = -MgR sin(30 deg) = -MgR/2.
The sum of these torques must vanish, so mgR = MgR/2 or m/M = 1/2.

Recall the defining formula: sum mr^2

(b) Now, the point mass of mass m is removed, and the hoop starts to rotate. What is the initial angular acceleration α of the hoop? [5]

We use I\alpha = \tau.
    I = MR^2 + MR^2 = 2MR^2.
    \tau = MgR/2 (already computed in (a)).

Therefore,
    2MR^2\alpha = MgR/2,
so
    \alpha = g/4R.

(**2** on the next page)

2. A uniform disk of radius R and mass M is initially rotating with a kinetic energy E.

(a) A 'break shoe' (a small box in the figure) applies a kinetic friction force F along the rim of the disk. How many rotations ρ_0 does the disk make before its kinetic energy is halved (see the illustration (a) above)? [5]

This is a work-energy theorem question. The work W done by the disk to the break shoe is \Delta \theta times torque.

torque = FR,
\Delta \theta = 2 \pi \rho_0.
\Delta E = -E/2 (but let us ignore its sign).

E/2 = 2\pi FR \rho_0, or \rho_0 = E/(4\pi FR).

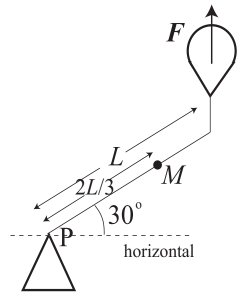
(b) Now, the break shoe is moved inside the disk and the same friction force is applied at the position which is distance $R/2$ away from the center (see the illustration (b) above). The disk makes ρ rotations before coming to a halt. Which is larger, ρ this time or ρ_0 in (a)? You must justify your answer. [5]

Now, the torque is FR/2 (halved!), so \Delta \theta must be doubled to do the same amount of work E/2. Thus, \rho/\rho_0 = 2.

Physics 101 (S11) **Q7D**

Name: ________________ **Section:** __________ **Score:** __________/20

1. On a light rod (ignore its mass) of length L is a point mass (= a block whose size you may ignore) of mass M fixed at a point that is $2L/3$ from the pivot P, around which the rod can rotate freely. At the other end of the rod is a balloon exerting a vertically upward force of magnitude F as illustrated below.



Read the general comment given in Q7A.

clockwise

(a) The rod makes an angle 30° with the horizontal and the whole system is stationary. What is F in terms of M and the acceleration of gravity g? [5]

Don't compute

```
Torque due to F around P = LF sin 60 deg.
Torque due to Mg around P = -Mg(2L/3) sin(60 deg).

The total torque around P must be zero: LF = 2MgL/3 or
     F = 2Mg/3.
```

(b) Now, the string holding the balloon is cut. What is the angular acceleration of the rod immediately after the cut? [5]

Recall the definition of I = sum mr^2

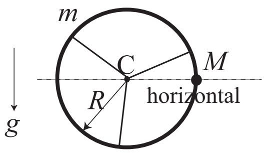
```
We use I\alpha = \tau.

I = M(2L/3)^2,
\tau = Mg(2L/3) sin(60 deg) (as already computed in (a)).

Therefore, M(2L/3)^2\alpha = Mg(2L/3) \sqrt{3}/2, or
     (2L/3)\alpha = \sqrt{3}g/2.
Hence,
     \alpha = 3\sqrt{3}g/4L.
```

(**2** on the next page)

2. A uniform hoop of radius R and mass m is in the vertical plane and can rotate freely around the horizontal axle through the center C of the hoop. A point mass of mass M is fixed on the hoop. Its initial position is as illustrated below. Ignore the masses of the spokes.



(a) Suppose $m = M$. The system is initially at rest. The hoop is gently released to rotate. When the point mass reaches the lowest point, what is its speed V_0? [5]

This is a conservation of energy problem. hoop
point mass
Mass M loses its potential energy, so \Delta U = -MgR.
The initial kinetic energy is of course 0.
The final kinetic energy is K = (1/2)MR^2\omega^2 + (1/2)MV_0^2.
R\omega = V_0, so K = MV_0^2. This uses up the potential energy, so
MV_0^2 = MgR.

That is, V_0 = \sqrt{gR}.

(b) Suppose M is much larger than m (say, $M = 1000m$). Is the speed V of the point mass at the bottom faster than or slower than the answer V_0 in (a)? You must justify your answer. [5]

This must be an easy question, because now you can almost ignore the hoop, so all the potential energy is converted to the kinetic energy of the point mass itself (V is about \sqrt{2Rg}). Faster!