Name: ____________________ **Section:** __________ **Score:** __________/**20**

1. Audio CD players read their discs at a constant rate (linear speed) and thus must vary the disc's rotational speed from around 500 rpm, when reading at the innermost edge, to 200 rpm at the outer edge.

(a) The radius of the innermost track (read first) is 2.3 cm. What is the radius of the outermost track? [5]

```
The linear speed is angular speed times radius.  The angular speed is
proportional to the rotational speed in rpm (X rpm = 2\pi X/60 rad/s).
Therefore, 500 x 2.3 = 200 x r.  That is, r = 5.75 cm.
```

(b) When a music is over, the disc rotational speed is 200 rpm. To stop this rotation within 5 complete rotations, at least what average angular acceleration must be applied to the disk? Answer in rad/s^2. Pay attention to the sign. [5]

```
This is a rotational kinematics problem.  The initial angular speed is
omega_0 = 200 x 2\pi/60 rad/s = 20.94 rad/s.  The final angular speed
is zero.  The angular displacement is Delta \theta = 5 rotations = 10\pi.
Use omega^2 = omega_0^2 + 2 \alpha Delta\theta.
     \alpha = -(20.94)^2/20\pi = -6.98 rad/s^2.
This must be negative, because the angular speed winds down to zero.
```

The acceleration discussed here is the acceleration of the rotation, so it is related to the linear acceleration in the tangential direction (tangential acceleration).

The acceleration calculated as V^2/r is the acceleration needed to maintain circular motion, and is in the radial direction (centripetal acceleration).

Do not confuse these two.

(**2** on the next page)

2. You are in a car moving at a speed V along a circular curve of radius R on a horizontal ground. Thanks to the frictional force (magnitude f) between the seat and your body, you need not lean on the wall.

(a) Write down the magnitude f of the frictional force. Your answer may be in terms of V, R, your body mass M, the acceleration of gravity g, and the coefficient μ_s of static friction between the seat and your body (you may not need all of them). [5]

```
f is the needed centripetal force that keeps you in the circular
trajectory: f = MV^2/R.  If this happens to be the maximum static
friction, then f = \mu_s Mg, but this is not generally true; it is
correct only just before you slip.
```

(b) You jump up perpendicularly to the floor of the car (assume you are sufficiently away from the walls). Suppose the car is turning constantly to the left, where will you land, to the right or to the left of the original position? You must justify your answer briefly. [5]

```
If you jump up, the friction force does not work any more, so you
do not have any acceleration in the horizontal direction.  Therefore,
your horizontal velocity is the tangential velocity of the car just
when you jump.  However, the car keeps turning left.  Inevitably, you
move to the right relative to the car.

In such a problem, imagine you are in the car and just do what is
described in the problem. You must know the correct answer. Respect
your intuition.
```

Name: ____________ **Section:** ________ **Score:** ________/20

1. DVD players read discs at a constant rate (linear speed) and thus must vary the disc's rotational speed as it reads from the inner edge of the disc toward the outer edge of the disc. At the innermost edge of the disc, the rotational speed is 1500 rpm.

(a) The radius of the innermost track (read first) is 2.3 cm. The radius of the outermost track is 5.8 cm. What is the rotational speed (in rpm) of the disk when the movie is ending? [5]

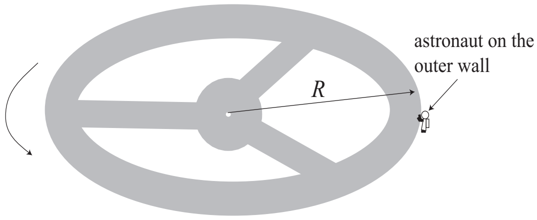
```
The linear speed = angular speed times radius.  This must be maintained.
The rotational speed is proportional to the angular speed (1 rpm =
2\pi/60 rad/s).  The linear speed must be maintained, so
     1500 x 2.3 = (rotational speed at 5.8 cm) x 5.8.
That is, the required rotational speed is 595 rpm.
```

(b) You start to play a wrong movie; the disc rotational speed is 1500 rpm. To stop this rotation within 18 complete rotations, at least what angular acceleration α (in rad/s^2) must be applied to the disk? [5]

```
This is a rotational kinematics problem.  The initial angular
speed = 1500 x 2\pi/60 = 50\pi (rad/s). The final angular speed
= 0.
     Use \omega^2 = \omega_0^2 + 2 \alpha Delta \theta.
Delta \theta = 18 x 2\pi = 36 \pi (rad).
Therefore,
     \alpha = - (50\pi)^2/72\pi = -109.1 (rad/s^2).
```

(**2** on the next page)

2. We wish to make a space station with 'simulated' gravity by rotation.



(a) The inside of the outer wall (radius R) of the space station is used as the ground. We want the effective weight (i.e. normal force) of a person standing on this wall to be equivalent to his actual weight on the surface of the earth. What must be the rotational speed V of the outer wall? Your answer may include earth's gravitational acceleration g. [5]

The effective weight on the body of mass M should be Mg. This must be the normal force from the wall to be used as floor, i.e., the centripetal force to keep the body going around the circle of radius R.

Mg = MV^2/R.

Thus, V^2 = Rg. That is, V = \sqrt{Rg}.

(b) An astronaut of mass M holds on the outside of the wall of the station (assume the wall is not thick, so its radius is R). What force is needed to keep him on the wall? Give its direction and magnitude. [5]

The astronaut must go around the circle of radius R at a speed of V, so she needs a centripetal force of MV^2/R = Mg. This force is a centripetal force, so it is along the radial direction of the space station, pointing to the center.

So, for example, she must be tied to the wall that pulls her toward the wall with the force whose magnitude is Mg.

Name: ____________ **Section:** ________ **Score:** ________/20

1. DVD disk is read starting from the innermost track at a radius of 2.3 cm. The required rotational frequency of the disk at which the innermost track is read is 25.5 Hz.

(a) Initially, the disk is at rest. From this state we wish to reach the required rotational frequency to play the innermost track in 5 s with a constant angular acceleration α. What is the required α (in rad/s^2)? [5]

omega = 2 \pi f

```
The required final omega = 2\pi x 25.5  = 51 \pi rad/s.
\alpha = \omega/time = 51\pi/5 = 32.0 rad/s^2.
```

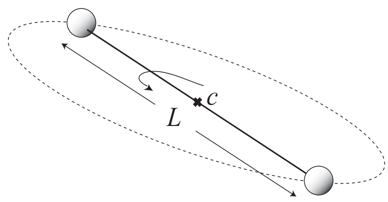
Here, alpha is the acceleration of the rotation itself (i.e., this can change the tangential speed; tangential acceleration) The formula V^2/R is the acceleration in the radial direction (toward the center: centripetal acceleration). These two accelerations are perpendicular. Don't confuse them.

(b) When a movie is coming to the end, the outermost track is being read at a radius of 5.8 cm. To read the information at a constant rate the DVD player keeps the constant linear speed (tangential speed) to scan the track. What must be the rotational frequency f (in Hz) of the disk at the end of the movie?

```
The linear speed is omega times radius.  This is maintained constant,
rotational frequency f times radius is also maintained (because
omega = 2\pi times f). Therefore,
     25.5 x 2.3 = f x 5.8.
This implies f = 10.1 Hz (= 63.5 rad/s).
```

(**2** on the next page)

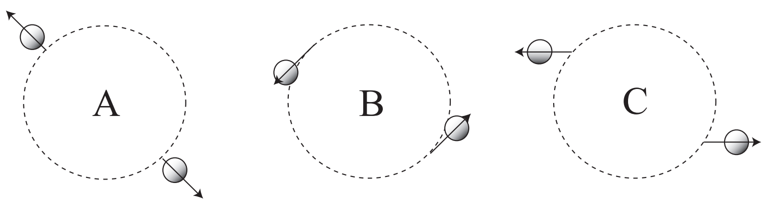
2. Two identical blocks of mass M is connected with a massless and flexible rope of length L. The system is in space and rotates in a plane around the stationary midpoint c (which does not move relative to distant stars) of the rope as illustrated.



(a) The rope can withstand the tension up to T. What is the largest angular speed ω_M of the rotation beyond which the rope snaps? Write ω_M down in terms of T, L and M. [5]

```
This is in space, so no external force acts upon the system.
Although two masses are connected, the motion of each block is just the
same as a block tied to c with a tether of length L/2.  Therefore,
     centripetal force = T = MV^2/(L/2) = M \omega_M^2 (L/2).
This implies
     \omega_M = \sqrt{2T/ML}.
```

(b) Immediately after the rope snaps, what is the trajectories of the masses from a stationary observer (relative to point c). Choose the correct illustration from below and give a brief justification of your choice. [5]




```
When the tether snaps, the tension T disappears, so there is no
acceleration. Thus, the law of inertia is everything!  The tangential
velocity just before snapping must be maintained.  That is,
the blocks fly away in the tangential directions as B.
```

Name: ____________ **Section:** ________ **Score:** ________/20

1. Audio CD players read their discs at a constant rate (linear speed) and thus must vary the disc's rotational frequency from 8 Hz, when reading the innermost track, to 3.5 Hz at the outer edge.

(a) The radius of the innermost track (read first) is 2.3 cm. What is the angular speed ω (in rad/s) of the disk, when the track at radius 3 cm is being played? [5]

```
The linear speed must be maintained, so angular speed times radius
must be constant.  The angular speed \omega is proportional to the
rotational frequency f: \omega = 2\pi f.  Therefore, f times radius
must be constant:
     8 x 2.3 = \omega x 3,
so x = 6.13 Hz.  Therefore, the angular speed is 6.13 x 2\p = 38.5
rad/s.
```

(b) Since the music is boring, you wish to jump from a track of radius 3 cm to the track close to the end. The angular speed of the disc to play the track properly is 23 rad/s. To make this transition within 5 complete rotations with a constant angular acceleration (or deceleration), what angular acceleration α is required (in rad/s^2)? [5]

```
This is a rotational kinematics problem. Use
     \omega^2 = \omega_0^2 + 2 \alpha Delta \theta.
\omega = 23, \omega_0 = 38.5, Delta \theta = 5 x 2\pi = 10\pi.
Therefore,
\alpha = - 15.2 rad/s^2
```

(**2** on the next page)

2. There is a planet of mass M whose rotational angular speed around its axis is ω (that is, its one day is $2\pi/\omega$).

(a) We wish to place a stationary satellite to this planet. How far (distance R) is it from the center of the planet? [Hint. A stationary satellite is a satellite which is always above a fixed location on the planet's equator, so such a satellite must rotate with the angular speed ω. You may use Newton's formula for gravitational force GmM/r^2 to supply the needed centripetal force. Express R in terms of G, M and ω.] [5]

```
A stationary satellite must go around the planet at an angular speed
of \omega. If its circular orbit has a radius R, then the required
centripetal force is m\omega^2 R, where m is the mass of the satellite.
This centripetal force must be supplied by the gravitational
interaction, so
     m\omega^2 R = GmM/R^2
where M is the mass of the planet.  Thus,
     R^3 = GM/\omega^2
gives R.
```

(b) Unfortunately, although your formula was right, an engineer made a numerical mistake and the satellite rotates ahead of the planet. Now, you must fix it. Will you increase or decrease R? You must justify your answer. [5]

```
The satellite rotates faster than the planet, so it must be slowed
down. Intuitively speaking, if the satellite is closer to the planet,
it is pulled by the planet more strongly, so it must `run faster' not
to fall.  Thus, we must increase R.

The formula you obtained in (a) just tells you the same.
```