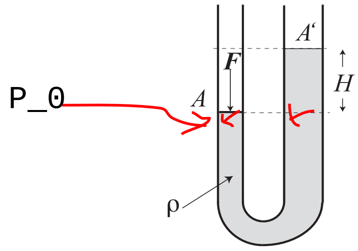
Name: ________________ **Section:** __________ **Score:** __________ **/20**

1. A U-tube contains a liquid of density ρ. With a piston of cross section A in the left tube a vertical force whose magnitude is F $(= |\boldsymbol{F}|)$ is applied. Consequently, the surface of the liquid in the right column of cross section A' rises, and the height difference between the left and the right liquid surfaces is H as illustrated below.



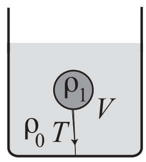
Pascal says: the pressures in the same liquid at the same height are the same (P_0).

Write down the magnitude F $(= |\boldsymbol{F}|)$ of the force in terms of A, A', H, ρ and g (the acceleration of gravity); you need not use all of them. [5]

Let P_A be the atmospheric pressure.
The right column: P_0 = P_A + H\rho g.
The left column: P_0 = P_A + F/A. Therefore, F/A = H\rho g, or
F = \rho H A g.

2. A ball of volume V with density ρ_1 is immersed in the liquid of density ρ_0 $(> \rho_1)$. The ball is tethered to the bottom of the container as illustrated below. Write down the tension T in the tether in terms of ρ_0, ρ_1, V, and g. [5]

plus



Archimedes says:

The volume of the displaced liquid = V, so the buoyancy force is
V\rho_0 g. The mass of the ball is M = V\rho_1. Therefore, the total
force acting on the ball is
-T + V\rho_0 g -V\rho_1 g = 0. That is
T = (\rho_0 - \rho_1)Vg.

(**2** on the next page)

3. A block of mass M on a frictionless horizontal floor is attached to the wall by a spring with spring constant k. Initially, the block is pulled to the right with a horizontal force $\boldsymbol{F}$ (as illustrated below) and is at rest. At $t = 0$, the force is removed without giving any initial speed to the block.

Write down the maximum speed V of the block in terms of F $(= |\boldsymbol{F}|)$, k and M. [5]

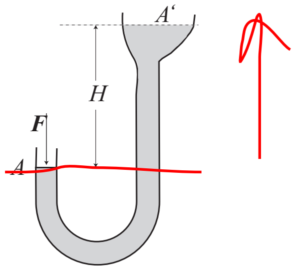
```
The amplitude A = F/k. \omega = \sqrt{k/M}, so max V = A\omega
= F/\sqrt{kM}.

This is ok, but I recommend the use of energy conservation:
The initial potential energy is U = (1/2) kA^2 = F^2/2k.  K = 0,
so the total mechanical energy E is E = F^2/2k. When V is max, E
= K = (1/2) MV^2.  Hence, MV^2 = F^2/k, or V^2 = F^2/Mk.
```

4. A pendulum made of a string of length L and a small block of mass M hangs from the ceiling. Initially, the pendulum is stationary and the string makes a small angle θ with the vertical. Then, the block is gently released. After what length of time τ does the block reach the maximum speed for the first time? Using a small angle approximation, write τ down in terms of M, L, θ and g (the acceleration of gravity). You need not use all the quantities. [5]

```
The max speed is realized when the block is at the lowest position.
Thus, we need the time to reach the lowest point from the highest
point (= the initial position in this problem). The shortest time
must be 1/4 of the period T: T = 2\pi \sqrt{L/g}.  That is,
(\pi/4)\sqrt{L/g}.
```

Name: ____________________ **Section:** __________ **Score:** __________/20

1. A U-tube contains a liquid of density ρ. A force $\boldsymbol{F}$ is applied vertically downward on the liquid in the left tube (by a weightless, free-moving piston). In equilibrium, the surface of the liquid in the right tube is higher than the surface of the liquid in the left tube by a height H. The liquid surface area at the top of the right tube is A'.

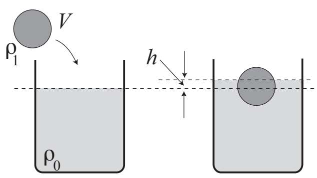



```
This extra head must be
supported by F.
The pressure at the red
line must be equal.
```

Write down the magnitude F of the force $\boldsymbol{F}$ in terms of A, A', H, ρ and g (the acceleration of gravity); you need not use all of the quantities. [5]

```
F/A = H\rho g, so F = HA\rho g

(A more detailed explanation is in Quiz A)
```

2. A ball of volume V with density ρ_1 is put in a beaker of cross section A containing a liquid of density ρ_0 ($> \rho_1$). When the ball floats on the liquid surface as illustrated below right, the liquid surface rises by h. Write down h in terms of A, V, ρ_1, ρ_0, and g (the acceleration of gravity). You need not use all the quantities. [5]




```
hA is the volume of the ball below the liquid surface, so the volume
of the liquid displaced by the ball is hA. Therefore, hA\rho_0g is
the buoyancy force.  This must support the weight of the ball, which
is V\rho_1 g.  Thus,
     hA\rho_0 = V\rho_1, or h = V\rho_1/A\rho_0.
```

(**2** on the next page)

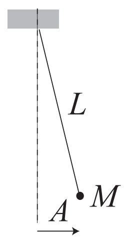
3. A block of mass M on a frictionless horizontal floor is attached to the wall by a spring with spring constant k. Initially, the block is pulled to the right with a force $\boldsymbol{F}$ as illustrated just below. The block is initially at rest. At $t = 0$, the force is removed.

\w denotes omega

Write down the magnitude of the maximum acceleration a of the block in terms of F $(= |\boldsymbol{F}|)$, k and M (you need not use all of them). [5]

A = F/k, \w = \sqrt{k/M}, so max acceleration = A\w^2 =F/M, but this must be obvious. If F is remove, the spring pulls the mass by the force whose magnitude is F, which is the largest initially, so F/m must be the largest acceleration.

4. A pendulum made of a string of length L and mass M hangs from the ceiling. The magnitude of the maximum acceleration of the mass in the horizontal direction is a. What is the amplitude A (see the figure below) of the oscillation? Assume the oscillation is small. Write down A in terms of a, L, g and M (you need not use all the quantities). [5]

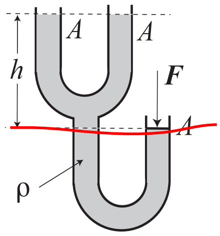


A\omega^2 is the max acceleration a; \omega = \sqrt{g/L}, so
a = Ag/L or A = aL/g.

Another approach is a = g \theta = gA/L, where \theta is the max angle displacement in radians.

Name: ____________ **Section:** ________ **Score:** ________/20

1. A 'double' U-tube illustrated below contains a liquid of density ρ. The cross sections of the tubes are all A. A force F is applied in the right tube by a weightless, free-moving piston of cross section A. Consequently, the surfaces of the liquid in the left columns rise, and the height difference between the left and the right liquid surfaces is h (as illustrated).



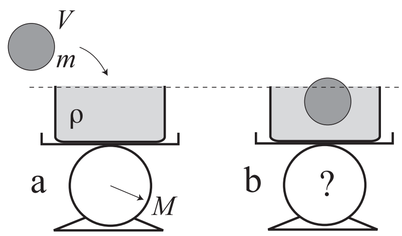
In both tubes, the pressure at the red line must be identical.
The pressure depends only on the height difference.

Write the force F down in terms of A, h, ρ and g (the acceleration of gravity); you need not use all of them. [5]

F/A = \rho g h, so F = \rho g hA.

(A more detailed explanation may be found in Quiz A).

2. A ball of volume V with mass m is put in a beaker filled to its rim with a liquid of density ρ. Initially, without the ball, the reading of the scale is M (situation 'a' illustrated below). When the ball is put in the beaker, some liquid spills and the ball floats on the liquid surface (assume that the spill is neatly cleaned). What is the reading of the scale (situation 'b' illustrated below)? [5]

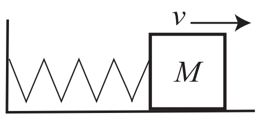


The volume spilled over is the same as the volume of the ball below the liquid surface. The lost mass \times g is the buoyancy force, which is identical to the gravitational force Mg. Thus, the total mass above the scale cannot change.

M is the answer.

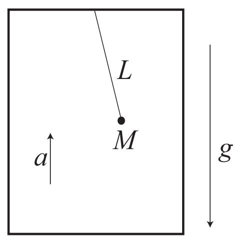
(**2** on the next page)

3. A block of mass M on a frictionless horizontal floor is attached to the wall by a spring with spring constant k. Initially, the block is situated at the spring's equilibrium position with an instantaneous velocity v to the right. No horizontal force is being applied. Write down the maximum magnitude of the acceleration a experienced by the block in terms of v, k and M (you need not use all of them). [5]




```
a = A\omega^2, \omega = \sqrt{k/M}, so a = Ak/M.
v = A\omega, because this v is the max velocity, so
a = v\omega = v\sqrt{k/M}.

Another way: a = F/M, where F is the max force (= kA).  Also
F^2/2k = Mv^2/2 (conservation of energy), so v = F/\sqrt{kM}.
Therefore, a = v\sqrt{kM}/M = v\sqrt{k/M}.
```

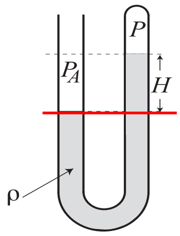
4. A pendulum made of a string of length L and mass M hangs from the ceiling of an elevator. Its period is T. Assume small oscillations. What is the acceleration a of the elevator? Choose the upward direction to be the positive direction. Write a down in terms of M, L, T and g. You need not use all of them. [Hint: the acceleration experienced by the mass is effectively $g + a$ downward. Assume $a > -g$.] [5]




```
T = 2\pi\sqrt{L/(a+g)}, so T^2 = 4\pi^2L/(a + g).
a + g = 4\pi^2L/T^2, or a = 4\pi^2L/T^2 - g.
```

Name: ______________ **Section:** ________ **Score:** ________/20

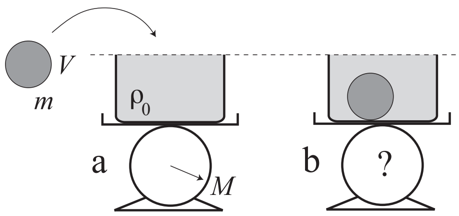
1. In a U-tube whose one end is closed as illustrated below contains a liquid of density ρ. The cross section of the tube is A. Let P_A be the atmospheric pressure. Write down the pressure P in the sealed chamber in terms of ρ, P_A, H, A and g (the acceleration of gravity). You need not use all the quantities. [5]




```
The pressure at the red line
in both the tubes must be the
same.
```

```
Left: P_A
Right: P + \rho Hg
These pressures must be the same, so P = P_A - \rho gH.
```

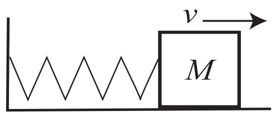
2. A ball of volume V with mass m is put in a beaker which is initially filled to its rim with a liquid of density ρ_0. For the beaker + the liquid the reading of the scale is M (situation a illustrated below). When the ball is put in the beaker, some liquid spills over and the ball sinks to the bottom (assume that the resultant mess is totally removed). What is the reading of the scale (situation b illustrated below)? [Hint: How much liquid is lost?] [5]




```
V\rho_0 of the liquid mass is lost.
The mass added as the ball is m.

M - V\rho_0 + m  must be the answer.
```

(**2** on the next page)

3. A block of mass M on a frictionless horizontal floor is attached to the wall by a spring with spring constant k. Initially, the block is situated at the spring's equilibrium position with an instantaneous velocity v to the right (as illustrated below). What is the amplitude A of the oscillation? Write A down in terms of M, k and v. You need not use all of them. [5]




```
v is the max speed, so v = A\omega; \omega = \sqrt{k/M}, so
A = v\sqrt{M/k}.

You can use conservation of energy: E = (1/2)Mv^2 = (1/2)kA^2.
Therefore, A^2 = (M/k)v^2.
```

4. A pendulum made of a string of length L and a block of mass M hangs from the ceiling. At $t = 0$ the mass is at its lowest position, with tangential speed v. At what time τ does the block return to the lowest position for the first time? Assume small oscillations. Write τ down using v, M, L and g. You need not use all of them. [5]

```
The time must be 1/2 of the period T.
\tau = \pi \sqrt{L/g}
```