Physics 101 (S11) **Q4A**

Name: ____________________ **Section:** __________ **Score:** __________/**20**

1. A cannon and the target are on a horizontal terrain. A cannon ball is shot from the ground to aim at the target which is distance L away from the cannon. The initial velocity of the cannon ball makes 45 degrees from the horizontal, and its initial speed is V. You must write your answers in terms of symbols V, L, and g, the acceleration of gravity (you need not use all of them).

(a) What is the speed (in terms of the initial speed V) of the cannon ball when it reaches the highest point? [5]

```
If we take the x axis to be parallel to the ground, and y orthogonal
and upward, the initial velocity reads (V/\sqrt{2}, V/\sqrt{2}).
Notice that the x component of the velocity stays constant, since
there is no acceleration in the x-direction.
     At the highest point, the y-component of the velocity must
vanish, so at the highest point, the velocity must be
(V/\sqrt{2}, 0).  Thus, the speed must be V/\sqrt{2}.
```

(b) Find the initial speed V required to hit the target. [5]

```
Since the x-component of the velocity is V/\sqrt{2}, \sqrt{2}L/V is the
time needed to reach the target.
```

time

We use
x = x_0 +v_0t +(1/2)at^2

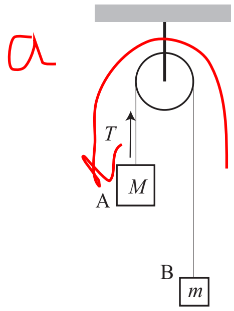
```
After \sqrt{2}L/V the y-coordinate must be zero:

0 = 0 +(V/\sqrt{2})[\sqrt{2}L/V] -(1/2)g [\sqrt{2}L/V]^2

That is,
     0 = 0 + L -gL^2/V^2,
so
     V^2 = gL   or V = \sqrt{gL}.
```

(**2** on the next page)

2. From an ideal pulley (massless and frictionless) hang two blocks A (with mass M) and B (with mass m) as shown in the figure.



(a) The masses are undergoing an accelerated motion. What is the magnitude of the tension T in the figure? [5]

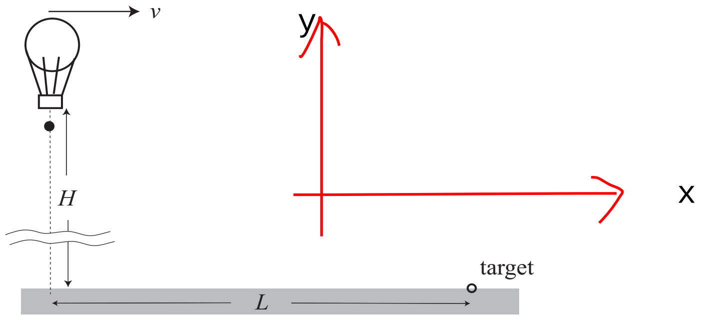
```
We must write down the equation of motion for Block A and B.  Let
the (positive direction of the) acceleration a be chosen as drawn
in the figure.
     For A: Ma = Mg - T,
     For B: ma = T - mg,
so
     mMa = mMg - mT
     mMa = MT - mMg
Subtracting the second equation from the first, we obtain
     0 = 2mMg -(m + M)T.
Therefore,
     T = 2mMg/(m + M).
```

(b) Suppose M/m is far larger than 1. What is the magnitude of the acceleration of Block B approximately? [5]

```
It is almost determined by M, so the problem is almost the free
fall of Block A: its acceleration must be almost g, so must be
the acceleration of Block B.
```

Name: ____________________ **Section:** __________ **Score:** __________ /**20**

1. A balloon is flying horizontally to the east at a speed v at a height H. We wish to throw a ball of mass m from the balloon to the target that is distance L away to the east on the level ground from the point exactly below the balloon as illustrated.



(a) It is calculated that if the ball is gently released (that is, with zero velocity *relative* to the balloon) from the balloon now, it will hit the target. Write the speed v of the balloon in terms of L, H, m, and g (acceleration of gravity); you need not use all of them. [5]

```
x and y components may be handled separately.
The time needed to land is (1/2)gt^2 = H, so t = \sqrt{2H/g}.
During this time the ball must traverse L in the x direction.  Since
the x-component of the velocity must be v (inertia),
     L = v times \sqrt{2H/g}.
That is,
     v = L\sqrt{g/2H}.
```

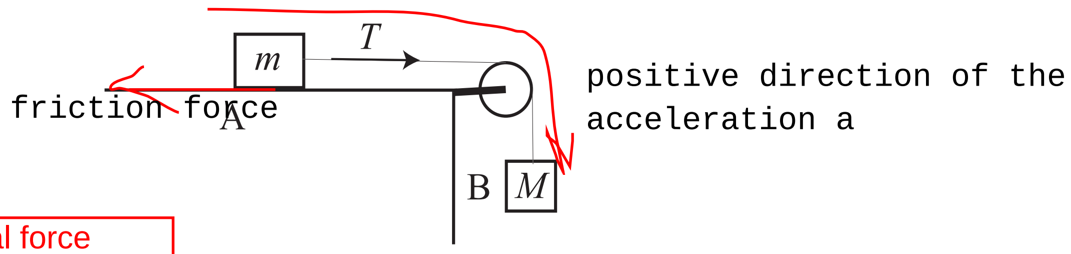
(b) Write the speed of the ball when it hits the target in terms of v, H and g (that is, you need not express v as required in (a)]. [5]

```
 The x-component is v.
 The y-component is - g \sqrt{2H/g}=-\sqrt{2gH}.

 Therefore,
      \sqrt{v^2 + 2gH}.
```

(**2** on the next page)

2. On a horizontal table is Block A of mass m, which is connected to Block B of mass M with a massless flexible string through a frictionless and massless pulley as illustrated below. The coefficient of kinetic friction between the table and Block A is $1/3$.



frictional force

(a) Suppose $M = 2m$. When Block B is gently released, Block A starts to slide on the table. What is the magnitude of the tension T in the string in terms of symbols m and g, the acceleration of gravity? [5]

```
Equation of motion for A
     ma = T - (1/3)mg
Equation of motion for B
     Ma = -T + Mg,
that is,
     2ma = -T + 2mg.
Therefore,
     2ma = 2T - (2/3)mg,
     2ma = -T + 2mg.
Their difference reads
     0 = 3T - (8/3)mg  That is, T = (8/9)mg.
```

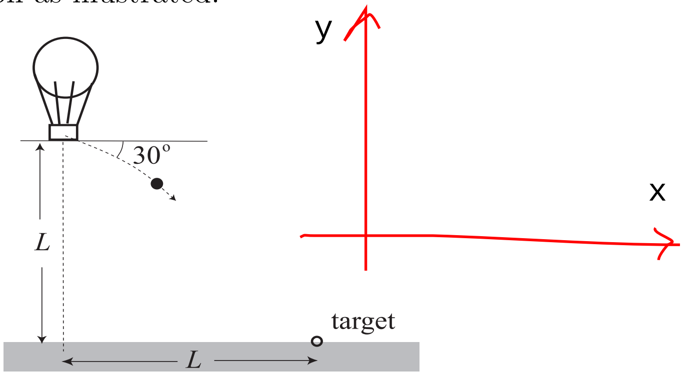
2 times the equation for A

(b) Suppose the mass M of Block B is far larger than that of Block A (i.e, M/m is much larger than 1). What is the estimate of the (magnitude of the) acceleration of Block A? You must justify your answer. [5]

This is almost a free fall problem for Block B, so its acceleration must be g. This must also be Block A's acceleration.

Name: ________________ **Section:** ________ **Score:** ________/20

1. A balloon is stationary relative to the ground at height L. We wish to throw a ball of mass m from the balloon to the target that is distance L away on the level ground from the point exactly below the balloon as illustrated.



(a) We wish to throw the ball 30 degrees downward from the horizontal (as illustrated). What initial speed V_0 do we need to hit the target? Write this V_0 in terms of L, g (the acceleration of gravity) and m (you need not use all of them). [5]

y component of the initial velocity

```
x and y directions may be handled separately!
The x-velocity is constant and V_0 cos 30 = \sqrt{3}V_0/2.  Thus, it
takes the ball L/(\sqrt{3}V_0/2) to reach the target.
After this time the ball must reach the ground, so
0 = L - (V_0/2)[L/(\sqrt{3}V_0/2)] - (1/2)g [L/(\sqrt{3}V_0/2)]^2
or
0 = L - L/\sqrt{3} - 2gL^2/3V_0^2
That is,
3(1 - 1/\sqrt{3})V_0^2 = (3 - \sqrt{3})V_0^2  = 2gL.
     V_0 = \sqrt{2/(3-\sqrt{3}}\sqrt{gL} = 1.26\sqrt{gL}.
```

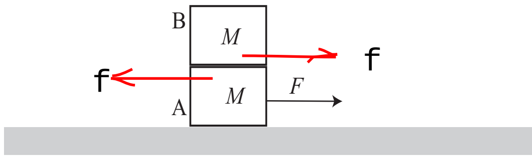
This is simply
x = x_0 + v_0t + (1/2)at^2

(b) Write the speed V of the ball when it hits the target in terms of V_0, L and g (that is, you need not write V_0 in terms of L, g, etc., as in (a)). [5]

```
The x-component of the velocity is the initial value \sqrt{3}V_0/2.
The y-component may be computed as
     V_y^2 = V_{0y}^2 +2gL ,
Therefore,
     V^2 + V_x^2 + V_y^2 = V_0^2 + 2gL.
That is,
     V = \sqrt{V_0^2 + 2gL}
```

(**2** on the next page)

2. On a frictionless horizontal floor is Block A of mass M, on which sits Block B of mass M (the same as A). The coefficient of static friction between the two blocks is $2/3$.



(a) Write down the equation of motion (Newton's second law) for Block A and Block B in the horizontal direction separately, assuming they move together. You may assume that the friction between the two blocks is f. Use a for the acceleration of Block A. [Hint: F and f work on Block A in the opposite directions.] [5]

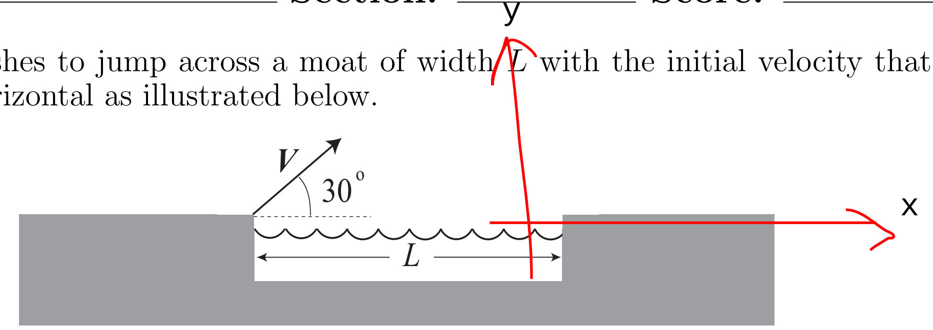
```
Block A   Ma = F - f,
Block B   Ma = f.
```

(b) If f exceeds the maximum static friction, Block B will slide off of Block A. Write the maximum magnitude F_M of the force F that allows Block B to stay on Block A in terms of M and g (the acceleration of gravity) (compute the number F_M/Mg). [5]

```
We get 2Ma = F, so f = F/2.  The max f is 2Mg/3,
so F_M = 4Mg/3.
```

Name: ____________ **Section:** ________ **Score:** ________/20

1. A person wishes to jump across a moat of width L with the initial velocity that makes $30°$ with the horizontal as illustrated below.



(a) What is the speed of the person at his highest point while crossing the moat in terms of the initial speed V?[5]

```
If we choose the coordinate system as in the figure, the initial
velocity reads (\sqrt{3}V/2, V/2).
The x-component does not change.  The y-component vanishes at the
highest point, so there, the velocity is (\sqrt{3}V/2, 0).
Thus, the speed is \sqrt{3}V/2.
```

(b) Write down the minimum initial speed V (the absolute value of the initial velocity) he needs to go beyond the moat in terms of L and g, the acceleration of gravity. [5]

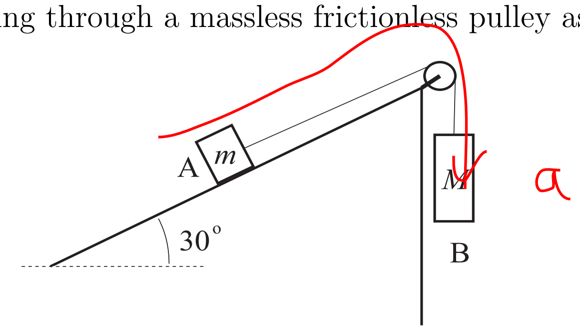
```
The needed time to cross the moat is L/(\sqrt{3}V/2).
After this time, the person's y-coordinate must be non-negative:
 0 < 0 + (V/2)[L/(\sqrt{3}V/2)] - (1/2)g [L/(\sqrt{3}V/2)]^2.
That is,
     0 < 1/\sqrt{3} - 2gL/3V^2.

The minimum velocity V is given by
     V^2 = 2gL/\sqrt{3}, that is V = 1.07\sqrt{gL}
```

This is simply due to
x = x_0 + v_0 t +(1/2) a t^2

(**2** on the next page)

2. On a frictionless incline is Block A of mass m, to which Block B of mass M is attached with a massless flexible string through a massless frictionless pulley as illustrated below.



(a) Suppose $m = M$. Write down the magnitude of the acceleration of Block B in terms of g, the acceleration of gravity. [5]

```
Let us take the positive direction of the acceleration as indicated
by the arrow.
     Block A: Ma = -Mg/2 + T
     Block B: Ma = Mg -T
Therefore, 2Ma = Mg/2.  That is, a = g/4.
```

(b) Suppose m of Block A is much bigger than that of Block B (that is, m/M is much larger than 1). What is the approximate acceleration of Block B? [5]

```
It is almost the free sliding down of Block A, so a  = g/2, almost.
Block B must move with A, so its acceleration must also be almost g/2.
```