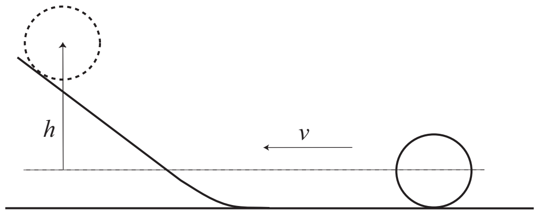
Name: uniformity assumed **Section:** ________ **Score:** ________/20

1. A disk of mass M and radius R rolls without slip on the flat horizontal surface as illustrated below. Its translational speed is v.



(a) Suppose the inclined surface is with friction, and the disk rolls up the surface without slip. What is the height h of the highest possible point that the disk can reach relative to the initial height (the height is that of the center of mass) (in terms of v and g, the acceleration of gravity)? [5]

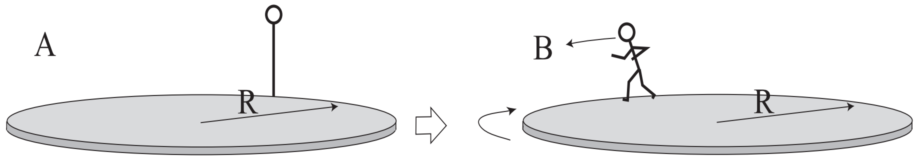
```
This is a conservation-of-energy question.  The initial kinetic energy
is completely converted to the potential energy U.  Let us measure U
from the lowest point. Then, initially U = 0 and finally it is Mgh. The
final kinetic energy is obviously 0. The initial kinetic energy consists
of the translational part (1/2)Mv^2 and the rotational part (1/2)I omega^2.
I = (1/2)MR^2, and v = R omega, so this is equal to (1/4)Mv^2.
Thus, conservation of energy tells us that
     Mgh = (1/2)Mv^2 + (1/4)Mv^2 = (3/4)Mv^2.
or
     h = 3v^2/4g.
```

(b) Suppose the inclined surface is frictionless, but the horizontal plane is as in (a). Can the disk with the same horizontal speed v climb up to the point higher than in the case (a)? You must justify your answer. [5]

```
No.
Since the slope is slippery, the rotational kinetic energy
cannot be used to climb up the slope, so inevitably h must be
smaller. In formulas the new height h' satisfies:
     Mgh' = (1/2)Mv^2  so h' = v^2/2g < 3v^2/4g.
```

(**2** on the next page)

2. A horizontal circular stage of moment of inertia I and radius R can rotate freely around its center. An actor of mass M is on the stage. He is initially standing still on the edge of the stage which is stationary as shown in A in the figure below.



(a) The actor starts to jog on the stage along its edge (of radius R), and reaches the tangential speed V observed by a stationary observer outside the stage (on the ground). The situation is illustrated in B above. What is the total angular momentum of the actor + the stage [5]?

This is a conservation-of-angular-momentum problem. It is stationary initially, so the total angular momentum of the system must be zero.

L_A = 0.

Situation A and situation B have exactly the same angular momentum.

L_A = L_B = 0.

(b) What is the angular speed ω of the stage? [5]

The angular momentum of the actor is: (his moment of inertia MR^2 around the center of the stage) times (his angular speed V/R). That is, MRV is his angular momentum. The angular momentum of the stage is I times omega, so

L = MVR + I omega = 0.

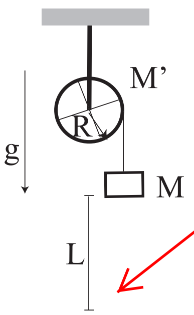
That is, MVR/I is the angular speed of the stage.
(Here, the - sign appears because the actor and the stage rotate in the opposite directions.)

Physics 101 (S11) **Q8B**

Name: ____________________ **Section:** __________ **Score:** __________/**20**

1. A hoop of mass M' and radius R is used as a pulley which can rotate freely around its horizontal axle (ignore the mass of the spokes). A massless string is wound around the hoop and a block of mass M hangs on it. Initially, the system is stationary.

K_init = 0.



Let us measure the potential energy from the lowest point: U_init = MgL, U_final = 0.

(a) Suppose $M = M'$. The block is released gently. When it descends by distance L, its speed is V. Write L in terms of V and g. [5]

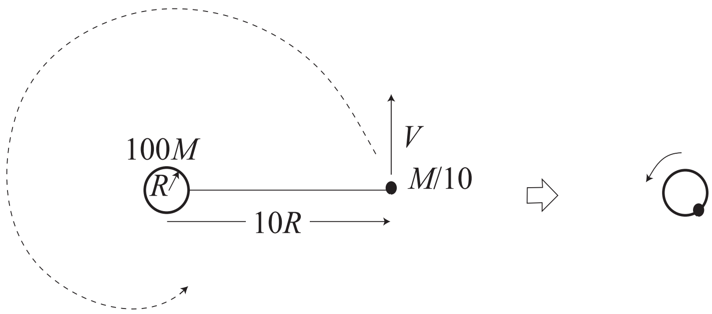
```
This is a conservation-of-energy problem. E_init = U_init = MgL.
The final kinetic energy is
     the energy of the block = (1/2)MV^2
     the rotational energy of the pulley = (1/2)I omega^2
               = (1/2)MR^2(V/R)^2 = (1/2)MV^2.
Therefore, E_final = MV^2.  The conservation of mechanical energy
implies
     MgL = MV^2.
That is, L = V^2/g.
```

(b) Suppose M' is much larger than M (say, $M' = 1000M$). Is the speed V' of the block when it descends by distance L as in (a) about the same as, much faster than, or much slower than V in (a)? You must justify your answer. [5]

```
Obviously, much slower, since it is hard to rotate the pulley.
Quantitatively, the final mechanical energy is
     (1/2)MV'^2 + (1/2)M'V'^2 = MgL,
so
     V'^2 = 2gL/(1+M'/M),
which is much smaller than gL.
```

(**2** on the next page)

2. On a horizontal frictionless table is a disk of radius R and mass $100M$ that can rotate freely around its vertical axle which is fixed on the table. On its edge is tethered a small ball of mass $M/10$ with a massless string (of length $9R$). Initially, the disk is stationary, but the ball is given a speed V perpendicular to the tight tether as illustrated below (top view). Notice that the ball is initially $10R$ away from the center of the disk. You may ignore the size of the ball.



(a) Eventually, the tether winds around the disk and the ball sticks to the edge of the disk as illustrated above right. They must be rotating together around the disk axle. What is the final total angular momentum of the system?

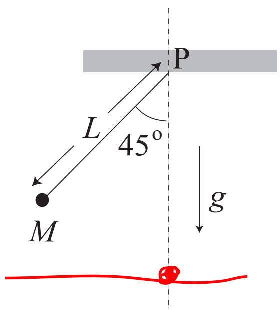
```
This is a conservation-of-angular-momentum problem. The initial
angular momentum of the ball around the center of the disk is
(M/10)V(10R) = MRV (or I = (M/10)(10R)^2, omega = V/(10R), so
I omega = MRV).  The disk is stationary, so the total initial
angular momentum is MRV.
     The final angular momentum must be the same.
```

(b) Does the total kinetic energy of the system increase? You must justify your answer. [Hint: $K = L^2/2I$] [5]

```
The initial I (the ball only) is 10MR^2. The final one
is almost determined by the heavy disk I = (1/2)100M R^2 = 50MR^2
(actually, it is slightly larger than this due to the ball).
Thus, Since K = L^2/2I, the energy decreases.
[How is the energy lost? It is lost when the ball sticks to the
cylinder.]
```

Name: ____________________ **Section:** __________ **Score:** __________/**20**

1. One end of a light bar of length L is attached to the ceiling at point P around which the bar can rotate freely. To the other end is fixed a small ball of mass M. Initially, the bar makes a 45° angle with the vertical as illustrated below. You may ignore the size of the ball.



cos 45 deg = 1/\sqrt{2}
Remember such representative values of cos and sin.

```
L - L/\sqrt{2}
```

(a) Let us assume that we may ignore the mass of the light bar. The bar is gently released and is allowed to rotate freely around P. Write down the square V^2 of the speed V of the ball when it reaches the lowest point in terms of L and g, the acceleration of gravity. [5]

```
This is a conservation of energy problem. Let us measure the
potential energy U from the lowest point.  The initial potential
energy = U = MgL(1 - 1/\sqrt{2}). The kinetic energy of the system
is (1/2)I\omega^2 = (1/2)MR^2(V/R)^2 = (1/2)MV^2.  Therefore,
conservation of mechanical energy implies
     (1/2)MV^2 = MgL(1-1/\sqrt{2}),
or
     V^2 = gL(2-\sqrt{2}).
```

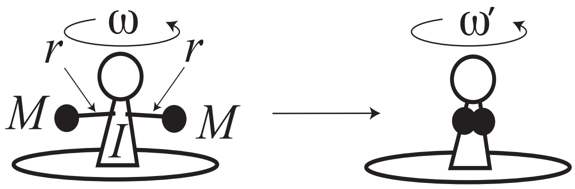
Recall I = \sum mr^2; although here there is only one mass: I = MR^2

(b) Suppose a small mass of m is attached to the ball. Is the speed of the ball at the lowest position faster than in the case (a)? You must justify your answer. [Hint: don't think too much.] [5]

```
This is equivalent to changing the mass, but we know the final
answer does not depend of the mass.  No change.
```

(**2** on the next page)

2. On a light horizontal turntable (with zero moment of inertia) is a child whose moment of inertia around its body center is I. She is at the center of the turntable, and holds two small balls of mass M. Initially, she holds them with her arms extended as illustrated below left. The length of extended arm is r, but you may ignore its mass. Initially, she is rotating at an angular speed of ω.



(a) She retracts her arms so that the balls may be assumed to be at the center of the turntable. What is the angular speed ω' of the child in the situation illustrated above right in terms of ω, I, M and r?

```
This is about conservation of angular momentum: L = I omega.
Thus, the problem boils down to the calculation of I, the moment
of inertia.

Initially, I_0 = I + 2Mr^2 (recall the definition I = \sum mr^2).
Finally, I_1  = I.
Thus, the conservation of angular momentum tells us that
      (I + 2Mr^2) omega = I omega'.
That is,
     omega' = omega(1+2Mr^2/I).
```

(b) To retract the balls does she have to do some mechanical work? You must justify your answer. [5]

```
Yes.  According to K = L^2/2I with L being conserved, decreasing I
by retraction increases K.  This increase must have come from her
arm motion.
```

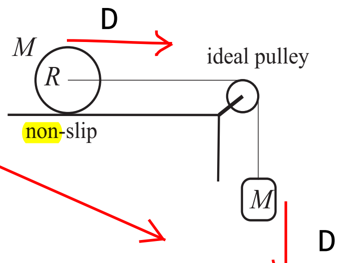
Physics 101 (S11) **Q8D**

Name: ________________ **Section:** ________ **Score:** ________/**20**

1. On a horizontal table is a cylidrical roller of mass M and radius R. The axle of the roller is pulled by a block of mass M with a massless string through an ideal pulley (no mass/no friction as usual). We assume that the roller rolls without slip on the table. Initially, the system is stationary.

K_init = 0

U should be measured from the lowest point. U_final = 0



(a) The block is now allowed to descend without constraint. When the roller moves by distance D, its speed is V. Write V down in terms of D and g, the acceleration of gravity. [5]

The potential energy of the roller does not change, so we do not consider it.

```
This is a conservation-of-energy question.
U_init = MgD, K_init = 0
U_fin = 0.
K_fin consists of the kinetic energy of the block (1/2)MV^2 and
the kinetic energy of the roller.  The latter consists of the
translational part K_T = (1/2)MV^2 and the rotational part
K_R = (1/2)I omega^2. I = (1/2)MR^2, and V = R omega, so
K_R = (1/2)(1/2)MR^2 omega^2 = (1/4)MV^2.  Thus,
K_fin = (1/2)MV^2 + (1/2)MV^2 + (1/4) MV^2 = (5/4)MV^2.
E_init = E_fin implies MgD = (5/4)MV^2.  Therefore,
     V = \sqrt{4gD/5}
```

(b) What is the angular momentum of the cylinder around its axle at this moment in terms of M, R and V. [5]

Don't forget that there are two 1/2's

```
L = I omega = (1/2)MR^2 omega = (1/2)MRV.
```

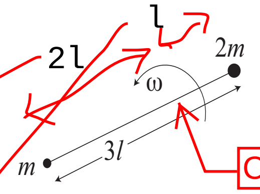
The kinetic energy of the block

Translational kinetic energy of the roller

(**2** on the next page)

2. On a frictionless horizontal table are two masses with mass m and $2m$, respectively, which are tied to each other by a massless wire of length 3ℓ initially. The system rotates with angular speed ω around the center of mass of the system. The top view is given below.

I = \sum mr^2



CM is here.

(a) Now, the wire is shortened to length ℓ. What is the angular speed Ω of the rotation of this system around its center of mass?

This is an angular-momentum-conservation question. The initial angular momentum around the CM is I omega, where I is the moment of inertia around the CM. By definition
I = m(2l)^2 +2ml^2 = 6ml^2. If the length scale becomes 1/3, then the formula tells us that the final moment of inertia I' = I/9.
The angular momentum is conserved: I omega = I'Omega = I Omega/9.
That is, Omega = 9 omega.

(b) Does the kinetic energy increase by this process? You must justify your answer. [5]

Yes, because K = L^2/2I with L being conserved but I becoming smaller.