Name: ____________ **Section:** ________ **Score:** ________/20

1. In a frictionless straight trough is a block. A constant force F pushes the block for t seconds to the right and the work done by the force is W. Initially, the block is stationary.

(a) Write down the momentum p of the block after t in terms of the symbols given above (F, W, and t; you need not use all of them).[5]

```
We know
     Delta p/Delta t = F
(eq of motion or Newton's second law).

Since F is constant, p = Ft.
```

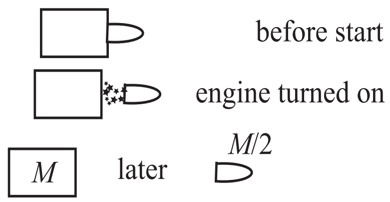
(b) The mass of the block is M. Write t down in terms of M, F and W. [5]

```
Due to the work-energy relation the final kinetic energy must be
equal to W done by the force F to the block.  Therefore,
     p^2/2M = W.
That is, t^2 F^2/2M = W. Therefore,
     t = \sqrt{2MW}/F.

The above solution is recommended, but you could use
     W = F times the displacement D.
D is (1/2)at^2, where a is the acceleration: a = F/M.
Therefore, F times (1/2)(F/M)t^2 = W, the same equation as obtained
above.
```

(**2** on the next page)

2. A spaceship consists of a main ship of mass M and a small explorer of mass $M/2$. Initially, they are connected and the center of mass is stationary (relative to distant stars). Then, the rocket engine of the explorer is turned on. After the rocket fuel is spent to produce mechanical energy E, the engine is turned off. The situation is illustrated in the figure.



(a) Suppose the magnitude of the momentum of the explorer is p. What is the magnitude of the momentum of the main ship (relative to their center of mass)? [5]

Since there is no external force acting on the main ship + explorer system, the total momentum of the system is conserved.

Therefore, the main ship also must have the momentum with the same magnitude p. (as a vector, it is \`-p'.)

(b) Express E (= the final total mechanical energy) in terms of p and M. [5]

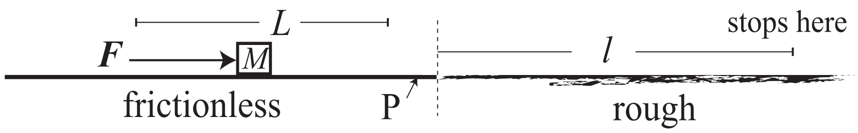
There is no potential energy.
The kinetic energy of the main ship = p^2/(2M).
The kinetic energy of the explorer = p^2/M.

Adding these two, we obtain E = 3p^2/2M.

Physics 101 (S11) **Q6B**

Name: ____________________ **Section:** __________ **Score:** __________/**20**

1. In a straight trough is a block of mass M. The left half of the trough is frictionless, and the block is initially stationary. On the frictionless portion of the trough a constant force F (horizontal and parallel to the trough) pushes the block while it undergoes a displacement of length L. Then, the force is turned off, and the block keeps moving and eventually into the rough portion of the trough. The block stops after running a distance ℓ due to a constant kinetic friction force f. See the figure below.



(a) Write down f in terms of the symbols given above (F, M, L, and ℓ; you need not use all of them.). [5]

The mechanical energy earned on the smooth patch is totally consumed on the rough patch.

The gained mechanical energy on the left half is the work done on the block by the force F: W = FL.

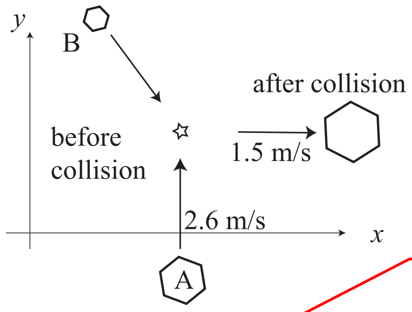
The work the block does to the ground on the rough patch is, analogously, W = fl. These works must be identical, so
f = FL/l.

(b) What is the magnitude of the momentum of the block just before going into the rough portion (say, at P in the figure)? [5]

The kinetic energy is just W, so FL = p^2/2M. That is.
p = \sqrt{2MFL}.

(**2** on the next page)

2. Two blocks A and B collide around the star mark in the following figure (top view) and stick to each other. Block A is 5 kg and is initially moving in the positive y-direction at speed 2.6 m/s. Block B is 1 kg. After sticking the single piece moves in the positive x-direction with speed 1.5 m/s (see the figure below). Assume that the experiment is performed on a frictionless horizontal plane (the sheet of the quiz paper is the plane).



The vector components are always ordered as:
(x-component, y-component).

(a) Find the initial velocity of Block B. [Hint: Set the answer velocity to be (u, v).] [5]

The momentum is conserved, since no external force acts in the horizontal plane. Therefore,

1(u, v) + 5(0, 2.6) = 6(1.5, 0) [unit = kg m/s]

Therefore, u = 9, v = -13 (m/s).

Kinetic energy is additive.

Mass is additive: 1 + 5 kg

(b) How much mechanical energy is lost by the collision?[5]

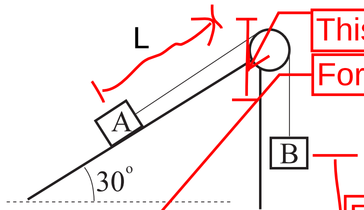
The initial kinetic energy = (1/2)(u^2+v^2) +(5/2)2.6^2 = 141.9 J
The final kinetic energy = (6/2)1.5^2 = 6.75 J

Hence, 6.75 - 141.9 = - 135.15 J That is 135.15 J lost.

Change is always calculated as:
Final - Initial

Name: ______________ **Section:** __________ **Score:** __________/20

1. On a frictionless inclined surface that makes 30° with the horizontal is Block A of mass M, which is connected with Block B with the same mass M by a massless and flexible string through a massless and frictionless pulley. Block B hangs vertically from the pulley. Initially, these blocks are stationary.



This is L/2.

For A

This convention is up to you.

0

Here, do not forget `minus,' because the final position is lower than our reference point 0.

For B

(a) The blocks are gently released. What is the speed of Block A when Block B descends by length L? [5]

-L

A and B both must have the same speed V.

```
This is a conservation-of-energy question.
Initial K = 0, since the blocks are stationary.
Initial U (This should be measured from convenient reference points;
     Let us measure the U of A and B from their current positions)
     U = 0 (of course, according to our convention).

Final K = (1/2)MV^2 + (1/2)MV^2.
Final U = Mg(L/2) + Mg(-L) = -MgL/2
   Energy conservation 0 = MV^2 - MgL/2, so V = \sqrt{Lg/2}
```

Initial E = Final E

(b) At the moment when B descends by length L, the string is cut. Block A can still climb up the slope. What is the height H of the highest point of Block A measured from its position at the moment the string is cut? [Answer H/L]. [5]

due to inertia

```
The initial mechanical energy is totally kinetic (again, we choose
a convenient reference point for U, that is, the starting point).
     K = (1/2)MV^2 = MLg/4.
Finally, the block stops, so K = 0, but the potential energy is
U = MgH.  Therefore, conservation of energy tells us that
     MLg/4 = MgH
Hence, H/L = 1/4.
```

(**2** on the next page)

There is only kinetic energy.

Conservation implies: Initial = Final

2. In a frictionless straight trough are two blocks A and B. Block A is with mass m and is running to the right with a speed V and collides with Block B of mass M, which is initially stationary. The collision is perfectly elastic and the total mechanical energy is conserved.

(a) Assume $M = 2m$. Let the final velocity of Block A be a and that of Block B b. Show $2a + b = 0$ and find the final speed of Block A. [5]

```
Momentum conservation: mV = ma + 2mb.
Energy conservation: (1/2)mV^2 = (1/2)ma^2 + (1/2)2m b^2.
They read: V = a + 2b, which implies V^2 = a^2 + 4ab + 4b^2.
           V^2 = a^2 +2b^2.
Equating the two formulas for V^2, we get
      a^2 + 4ab + 4b^2 = a^2 + 2b^2, so 4ab + 2b^2 = 0.
Hence, 2a + b = 0, because b is not zero.  Since b = -2a,
     V = a + 2(-2a) = -3a.
That is, a = -V/3.  Consequently, we get
     b = 2V/3.
```

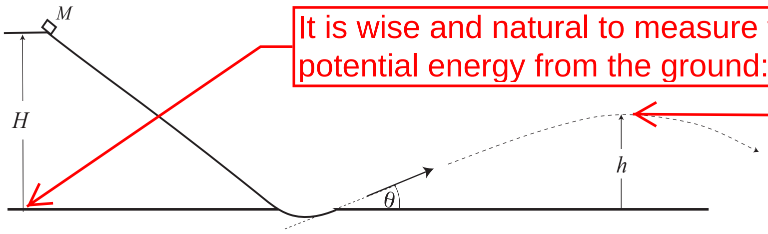
(b) Suppose M is much larger than m (i.e., $M/m \gg 1$), but still the total mechanical energy is conserved. What is a approximately? [5]

```
Block B looks like an unmovable wall, but it is perfectly
reflecting (elastic; no energy is lost), so a = -V, almost.
```

Physics 101 (S11) **Q6D**

Name: ________________ **Section:** __________ **Score:** __________/**20**

1. A block of mass M slides down the frictionless slope from the initial height of H as illustrated below. The initial speed of the block is zero. After sliding down the slope, the block leaves the ground with the velocity making an angle θ with the horizontal.



It is wise and natural to measure the potential energy from the ground: MgH

Mgh, here

(a) Write down the height h of the highest point the block can reach in terms of H and θ. [5]

(1/2)MV^2 = MgH

```
This is a conservation-of-energy question, so the launching speed
is V = \sqrt{2MH}.
At the highest point, the speed of the block is the x-component of
the initial velocity, so it is V cos\theta.
The initial total mechanical energy = MgH.
The kinetic energy at the highest point = (1/2)MV^2 cos^2\theta.
The potential energy at the highest point = Mgh.
Conservation of energy implies
     MgH = (1/2)MV^2 \cos^2\theta + Mgh = MgH\cos^2\theta + Mgh
Hence,
     h = H - H cos^2\theta = H sin^2\theta.
```

(1/2)MV^2 = MgH

(b) Write down the magnitude p of the momentum immediately before the block lands on the ground (after it passes its highest point) in terms of g, H and M. [5]

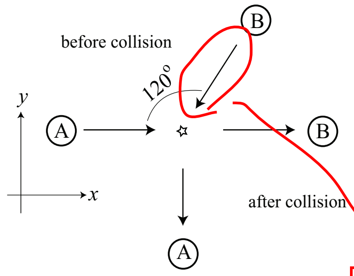
```
Don't be fooled by the apparent complexity of the problem.

The potential energy is zero finally, so MgH must be the total
kinetic energy.
     p^2/2M = MgH, so p = M\sqrt{2gH}.
```

A cleverer way is to use the y-component of the initial velocity:
V sin\theta. Mgh = (1/2)M(V sin\theta)^2.
Using MV^2 = 2MgH, we can immediately obtain the answer.

(**2** on the next page)

2. Two dry ice pucks slide on a horizontal frictionless surface. Puck A of mass 2 kg initially moves along the x-axis with speed 6 m/s, and Puck B of mass 1 kg initially moves with speed 6 m/s in the direction shown in the figure below (it is a top view of the system). After the collision near the star mark in the figure Puck B moves along the x axis and Puck A along the y-axis as illustrated below.



You must know sin 60 = cos 30 = \sqrt{3}/2.

Don't forget the minus signs

(a) What is the speed of Puck B after the collision? [5]

The total momentum is conserved, because there is no external force in the horizontal direction.

The initial velocity of A is (6, 0)(m/s); that of B is (-3, -3\sqrt{3}) (m/s).

Let us write the final velocity of A as (0, v); that of B as (u, 0).
Conservation of momentum implies

    2(6, 0) + 1(-3, -3\sqrt{3}) = 2(0, v) + 1(u, 0) = (u, 2v)

Therefore,

    x-component: 12 - 3 = u
    y-component: -3\sqrt{3} = 2v.

Thus, u = 9 m/s.

(b) What is the loss of the mechanical energy? [5]

The initial total kinetic energy = (1/2)(2 + 1)6^2= 54 J
The final total kinetic energy = (1/2)2 (3\sqrt{3}/2)^2 + (1/2)9^2
                = 27/4 + 81/2 = 47.25 J
Hence, 47.25 - 54 = - 6.75. 6.75 J lost.

Change is always:
Final - Initial