Do not confuse / and \ (backslash). \something usually implies a single symbol. For example, \pi is a single letter `pi.'

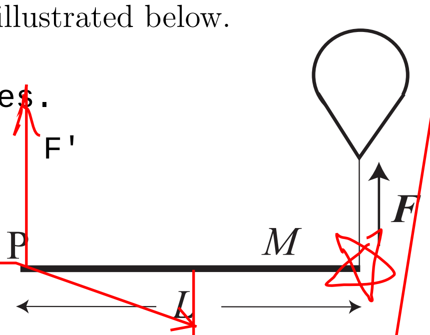
This is true around any point in the universe, but P, CM or the Star are the only practical candidate points in our problem.
P is out of question, since F' disappears.
CM requires two unknown forces F and F', so if you wish to use this, you need one more relation, say, the force balance F' + F - Mg = 0. This is OK, but not the best.

Physics 101 (F11) ... 7A

Name: ______________ S ... 20

1. A uniform stick of mass $M = 2.1$ kg and length $L = 1.2$ m is initially horizontally at rest. Its one end is fixed to a fulcrum P around which the stick can rotate freely. The other end is suspended by a balloon as illustrated below.

```
First, itemize all the forces.
```



You may treat an extended object as a mass concentrated at its center of mass.

(a) What is the magnitude of the force acting on the stick from the fulcrum at P? [5]

```
We may always use \sum F = 0 and \sum \tau = 0, but you must clearly
recognize what is known/unknown and what you want.
     We want F' and we do not know F.  \sum F = 0 is not convenient.
Torque balance around the star mark is very convenient:
     + Mg(L/2) - F' L = 0
Therefore, F' = Mg/2 = 10.29.  You could easily guess it.
```

Counterclockwise +
Clockwise -

(b) The balloon punctures and the force $\boldsymbol{F}$ is gone, so the stick starts to rotate around P. What is the (magnitude of the) initial angular acceleration of the stick? (The moment of inertia of a uniform rod around its end is given by $I = ML^2/3$; you may treat the gravitational force as acting at the center of mass of the stick.) [5]

```
I \a = \tau is the fundamental equation of motion.
\tau is due to gravity, so Mg times (L/2) (clockwise).
The equation of motion reads
     ML^2/3 \a = -MgL/2
Therefore,
      \a = -3g/2L = -12.25 rad/s^2
```

\a = alpha

We use the equation around P.

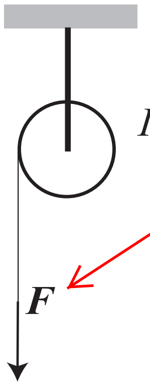
- implies a clockwise angular acceleratioin

The equation of rotational motion is always around a particular point around which you wish to know the rotational motion.
I must be calculated around the point, and
the torque must also be around the same point.

(**2** on the next page)

2. Around a uniform disk of radius $R = 0.3$ m and mass $M = 2.2$ kg is a string, which is pulled down with a constant force $\boldsymbol{F}$.

\Delta E = W,
the work-energy theorem
is the key.
W = FD



This force does work to the disk.

\w = omega

(a) When the string is pulled down over a distance of $D = 0.3$ m, the angular speed of the disk reaches $\omega = 3.1$ rad/s. What is the magnitude F of the constant force $\boldsymbol{F}$? [5]

The final kinetic energy = (1/2)I\w^2, I = (1/2)MR^2.
The initial kinetic energy = 0.
Therefore,
(1/2)[(1/2)MR^2]\w^2 = FD.
or

F = M(R\w)^2/4D = 2.2(0.3 × 3.1)^2/(4 × 0.3) = 1.586 N

Uniform disk

There are two 1/2 factors.

(b) Suppose the magnitude of the constant pulling force is halved, but the distance D is the same. What is the angular speed of the disk in this case? [5]

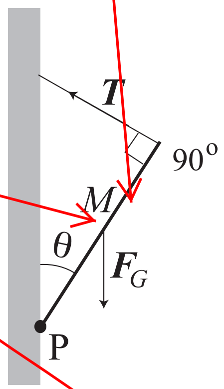
The work is halved, so the kinetic energy is halved. Then,
\w becomes \w/\sqrt{2} = 3.1/1.414 = 2.19 rad/s.

\sum torque = 0 holds around any point in the universe, but P, CM or the tip of the bar are the only practical candidate points in our problem.
We wish to know T, so the tip of the bar is out of question
CM is also out of question, because we do not know the force acting at P.
Therefore, we must choose P to study the torque balance.

Name: ____________________ **Section:** __________ **Score:** __________/**20**

1. One end of a uniform bar of length $L = 1$ m and mass $M = 2.5$ kg is fixed at a fulcrum P. At the other end is attached a massless string, which hangs the bar from the wall as illustrated below.

You may treat an extended object as a mass concentrated at its center of mass.



Counterclockwise +
Clockwise -

(a) The angle $\theta = 30°$. What is the tension T in the string? [5]

```
Since the force acting at P is not easy to guess, the only practical
approach is to study the torques around P.
     Torque due to T: +TL
     Torque due to gravity: Mg sin(\theta)(L/2) = -MgL/4.
Therefore, TL - MgL/4 = 0.
That is,
     T = Mg/4 = 6.125 N.
```

(b) Just after the string snaps, what is the angular acceleration of the rod around P? (The moment of inertia of a uniform rod around its end is given by $I = ML^2/3$; you may treat the gravitational force as acting at the center of mass of the rod.) [5]

```
The equation of motion is
     I\a = \tau
\tau (torque around P) is -MgL/4.
Therefore,
     (ML^2/3)\a = -MgL/4.
That is,
     \a = -3g/4L = -7.35 rad/s^2
```

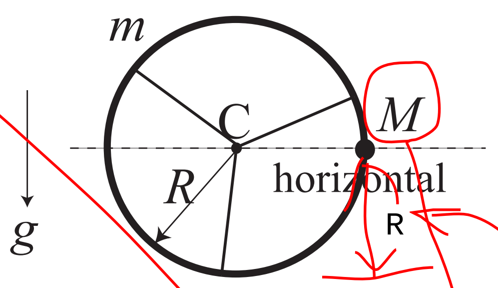
The equation of rotational motion must be used consistently around the same point we are interested (P).

Here, - implies a clockwise acceleration.

(**2** on the next page)

2. A hoop of radius $R = 0.5$ m and mass $m = 4$ kg can rotate freely in a vertical plane around a horizontal axle through the center C (Ignore the masses of the spokes). On the hoop is fixed a small ball of mass $M = 2$ kg, which is initially at the height of the axle as illustrated.

no friction,
so
mechanical energy is
conserved.



(a) What is the angular speed ω of the hoop when the ball reaches its lowest position? Assume that initially the system is at rest. [5]

```
Initial: K = 0, U = MgR (relative to the lowest point)
Final: K = (1/2)I\w^2, where I = mR^2 + MR^2, U = 0.
Therefore,
     MgR = (1/2)[mR^2 + MR^2]ω^2
     MgR = 2×0.5g = 9.8 J.
     I=mR^2+MR^2 =1.5kg·m^2,so ω = (2 × 9.8/1.5)^{1/2} = 3.61 rad/s.
```

I = \sum m r^2

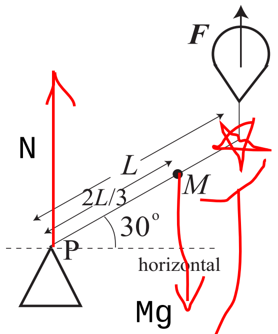
(b) We wish to make the ball complete one rotation around the center. What minimum initial angular speed of the hoop do you need? (Hint: stare at (a) and you will see the answer almost without any calculation, although some justification should be written.) [5]

Since we need at least MgR more energy to reach the highest point, this must be supplied as the initial kinetic energy. Therefore, ω = 3.61 rad/s. Look at the formula you used just above.

Name: ________________ **Section:** __________ **Score:** __________/**20**

1. Around a fulcrum P can freely rotate a massless bar of length $L = 3$ m. At its other end is attached a balloon exerting an upward force of magnitude F. At a distance 2 m from P fixed is a point mass of mass $M = 12$ kg.

What you should do first
is to itemize all the
forces.



As to the choice of the point to apply the torque balance, see A or B.

clockwise

(a) Initially, the system is at rest. What is the magnitude of the force on the fulcrum P? [5]

Since we may assume that the force balance condition \sum F = 0 must hold,
\sum torque= 0 must hold around any point in the universe. In particular
around the star above.

Torque due to N: -NL cos 30

Torque due to gravity: Mg(L/3) cos 30.

Notice that sin 120 = cos 30.

Therefore,

MgL/3 - NL = 0

Or

N = Mg/3 = 4g = 39.2 N.

(b) Just after the string snaps, what is the angular acceleration of the point mass? [5]}

\a = alpha

The equation of rotational motion (around P) is

I \a = torque

I = M(2L/3)^2 = 4ML^2/9

torque = Mg(2L/3)cos 30 = MgL/\sqrt{3}

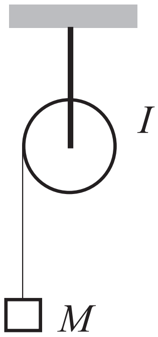
Therefore,

\a = 3\sqrt{3}g/4L = \sqrt{3}g/4 = 3.90 rad/s^2.

(**2** on the next page)

2. Around a disk of radius $R = 0.5$ m and mass $m = 4$ kg is a massless string to which a block of mass $M = 2$ kg is attached. The disk can rotate around its center freely in the vertical plane (corresponding to the sheet of this paper). Initially, the block is going up with a speed $v = 2$ m/s.

It is understood that the disk is also rotating consistently.



Energy is conserved.

To use the conservation law we must clearly recognize the initial and the final situations.

(a) To what height h will the block climb up? [5]

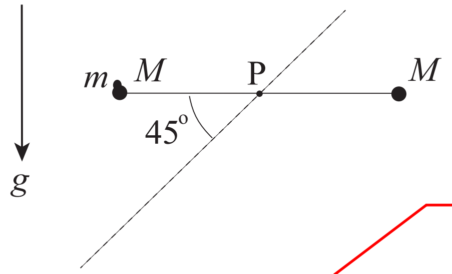
```
Initial: K = (1/2)Mv^2 + (1/2) I\w^2, where R\w = v, and
          I = (1/2)mR^2.  Therefore,
         K = (1/2)Mv^2 + (1/4)mv^2 = 2v^2 = 8 J
         U = 0.
Final: K = 0 at the highest point, no motion.
       U = Mgh = 2gh.
Therefore,
     2gh = 8, or  h = 4/g = 0.408 m.
```

(b) Suppose we double the masses (i.e., $m \to 2m$, $M \to 2M$) but keep R, what happens to h? [5]

Notice that both U and K just scale as masses, so there is no change.

Name: ____________ **Section:** ________ **Score:** ________/**20**

1. Two identical small balls of mass $M = 2$ kg are attached to the ends of a massless stick of length $L = 2$ m as illustrated below. On one of the balls is further attached a small mass of $m = 1$ kg as illustrated below.



must be around P

\a = alpha

(a) Initially, the stick is horizontal and at rest. Then, it is released gently to rotate around the midpoint P of the stick in the vertical plane (the sheet of this paper). What is the angular acceleration immediately after the release? [5]

Counterclockwise +

```
The equation of rotational motion is
     I\a = torque <- this is actually the total torque
     I = M(L/2)^2 + (m+M)(L/2)^2 = 5 kgm^2
     total torque: the torques due to Ms cancel each other.  Thus,
     +mg(L/2) = 9.8 Nm.
Hence,
     \a = g/5 = 1.96 rad/s^2.
Obviously, counterclockwise.
```

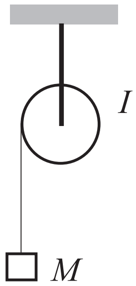
(b) What is the torque around P when the stick makes an angle 45° with the horizontal (the dotted line in the figure)? You must give the correct unit. [5]

```
mg (L/2) cos 45 = g/\sqrt{2}= 6.93 Nm is the torque (still
counterclockwise) due to gravity.
```

(**2** on the next page)

2. A drum of radius 0.5 m with a moment of inertia $I = 12$ kg·m^2 around its horizontal axle is suspended from the ceiling. Around it is wound a massless string which hangs a block of mass $M = 5$ kg as illustrated below. Initially, the (upward) speed of the block and the tangential speed of the outer rim of the drum are identical and 2 m/s.

To apply any conservation law, clearly recognize the initial and final situations.



We assume the mechanical energy is conserved.

(a) What is the height of the highest point of the center of mass of the block measured from its initial height? [5]

```
Initial: K = (1/2)Mv^2 + (1/2)I\w^2, where \w = v/R, so
          K = (5/2)2^2 + 6(2/0.5)^2 = 106 J  U = 0 relative to the
               lowest point.
Final: K = 0 at the highest point.
         U = Mgh = 49h.
Therefore,
     106 = 49h, so h = 2.16 m.
```

(b) When the block reaches its highest point, the string is cut. When the block returns to the initial height, is its speed larger or smaller than its initial speed? You must justify your answer.[5]

When M goes up, the kinetic energy of not only M but also the disk is converted to the potential energy of M. That is, M goes up far more than it could go by itself.
When M falls from the highest point, the potential energy is totally converted into the kinetic energy of M.
Therefore, M is much faster when it comes back to the original height than when it started from the same height.