Homework 4

due Feb 22, 2017 (at 18:00) [in PDF with LaTeX to Tong Wang twang92@illinois.edu]

As I announced, HWs are regarded as substitutes for Discussion, so you may submit your ‘draft’ electronically for my comments/suggestions/hints (by the time reasonably before the deadline). Also you can freely discuss with your friends as long as it facilitates your understanding. I hope all of you will get 100%.

In order to use the results of statistical mechanical calculations effectively, we need sound knowledge of thermodynamics. Thus, let us confirm our practical thermodynamic knowledge.

Thermodynamics Checklist:

Answer 5 problems of your choice but the sum of the problem numbers should be in \([23, 32]\).

(e.g., you can choose 1, 3, 5, 8, 10, but cannot choose 1, 6, 8, 9, 10)

1. Suppose there is no mixing entropy. Show that the second law is violated.

2. The following equations of state cannot be realized, because they violate some general rules of thermodynamics. State explicitly the reason why they are impossible. The symbols are the standard choice as in the notes. \(\alpha\) is a positive constant.
   (1) \(S = \alpha [N E/V]^{1/4}\).
   (2) \(E = \alpha V^{4/N^2} S\).
   (3) \(S = \alpha (N^2 E)^{1/3} e^{-E V/N}\).

3. Choose the signs in the following Maxwell relations:

   (1) \(\left(\frac{\partial S}{\partial M}\right)_T = \pm \left(\frac{\partial B}{\partial T}\right)_M\),
   (2) \(\left(\frac{\partial S}{\partial B}\right)_M = \pm \left(\frac{\partial M}{\partial T}\right)_S\),
   (3) \(\left(\frac{\partial V}{\partial B}\right)_P = \pm \left(\frac{\partial M}{\partial P}\right)_B\),
   (4) \(\left(\frac{\partial T}{\partial B}\right)_S = \pm \left(\frac{\partial M}{\partial S}\right)_B\).

4. Show

   \[ C_B = C_M - T \left( \frac{\partial M}{\partial T} \right)_B \left( \frac{\partial B}{\partial T} \right)_M \quad (0.0.1) \]

   and calculate this when Curie’s law holds: \(M = c B/T\), where \(c\) is constant.

5. Demonstrate

   (1) \(\frac{1}{T} \left( \frac{\partial C_V}{\partial T} \right)_T = \left( \frac{\partial^2 P}{\partial T^2} \right)_V \).

   (2) Assuming that the work coordinates of the system are \(V\) and \(N\) (i.e., the ordinary one component
fluid system), show
\[
\left( \frac{\partial N}{\partial V} \right)_{\mu,T} = \frac{N}{V},
\]  
(0.0.3)

6. Define \( \tilde{E} = E - MB \), where \( E \) is the internal energy. Show:
(1) When there is a change of states under a constant external magnetic field \( B \), the heat absorbed by the system is equal to the increase of \( \tilde{E} \) as long as the initial and the final states are in equilibrium. Assume that the volume does not change.
(2) When pressure \( P \) is kept constant as well in (1) instead of the volume, find the expression for the heat absorbed by the system.

7. There is a box of volume \( V \) with a small hole in its adiabatic wall through which the gas in the box can escape (that is, the pressure \( P \) of the gas is always equal to the ambient pressure). The initial temperature of the gas is \( T_0 \). The gas is slowly heated up to temperature \( T \). The molar specific heat at constant volume of the gas is \( C_V \). Find the necessary heat for the process.

8. There is a magnetic substance with the following relations (under constant \( B \)):
\[
S = S_0 - \frac{a}{2} M^2 - \frac{1}{4!} g M^4, \quad E = E_0 - \frac{b}{2} M^2,
\]  
(0.0.4)

where \( a, b \) and \( g \) are constants. Find \( M \) as a function of \( T \).

9. The internal energy \( u \) per volume of a gas depends only on \( T \), and the equation of state is \( P = u(T) \). Find the functional form of \( u \).

10. At temperature \( T \), \( V = f(P) \) and \( (\partial V/\partial T)_P = g(P) \), where \( f \) and \( g \) are assumed to be given. When we isothermally change the pressure from \( P_0 \) to \( P_1 \), find the necessary heat \( Q \).