**Discussion 1 Jan 22, 2019**

‘Discussions’ will be mainly problem-solving sessions with occasional (perhaps open-ended) discussions moderated by the instructor.

The exercise problems are directly related to the topics in the previous week, and the homework problems are more or less directly related to the Discussion problems.

1. **[Very elementary questions]**
   (1) The unit of pressure is ‘pascal’ Pa. Write this in terms of s, kg and m. [1 atm = 101,325 Pa.]
   (2) What is the total kinetic energy of the gas in this room? You must supply reasonable values for \( P \), \( T \), etc.
      Can you supply an explanation/illustration that makes the number more intuitively understandable?
   (3) Estimate the number of molecules in the air of the discussion room. You may assume the standard ideal gas equation of state \( PV = Nk_B T \) with the accepted value for the Boltzmann constant \( k_B = 1.380662 \times 10^{-23} \) J/K. Clearly recognize that without knowing this constant, you cannot get the number of molecules. Thus, a fundamental question of physics is how to measure this value. This is the topic of Brownian motion.
   (4) Suppose all the air molecules are condensed to a point mass of mass \( M = Nm \) (without any internal structure), where \( m \) is the mass of the gas molecule. What is the speed of the point mass, if it has the same (total) kinetic energy \( K \) as obtained in (2)?
      You cannot answer this question without knowing the mass of air in the room.¹ Let us use the air density = 1.2 kg/m³.
      Now, compare it with the mean square velocity \( \langle v^2 \rangle \) of the molecules. What is your observation?
   (5) It is usually taught that the kinetic theory of gases was a triumph of atomism, but is it really so? Consider the limit \( N \to \infty \), keeping \( M = Nm \) constant. What do you obtain?
      What is the logical conclusion of this observation?
   (6) Suppose you place a sphere of radius 1 m in the room. How many air molecules collide its surface on the average in one second?
      First outline how to obtain the formula for the number \( N_C \) of molecules colliding a one side of a plane with unit area on the average in one second in terms of the quantities appearing in 2.10.
      We may be able to estimate \( N_C \) later more accurately, but here let us assume that we know only (the modern interpretation of) Daniel Bernoulli’s kinetic theory.
      What can you say or is there any way to estimate the number of particles impinging on the sphere approximately?

¹This tells you how important it is that you can make a vacuum. Creating vacua was important only second to heliocentrism, as noted in the lecture notes 2.3.
2 [Does gravity matter?]
The actual room is influenced by the gravitational field of the earth.

(1) What is the potential energy difference of an oxygen molecule between the floor and the ceiling?

(2) Estimate the ratio of this potential energy and the kinetic energy. Is it the order of 1%, 0.1 %, or ...? Is it ignorable? [Think of 0.1% of $T$.]

(3) ['open ended' discussion topic] If a particle climbs up from the floor to the ceiling, its kinetic energy would be converted into its potential energy, so the particle would slow down.

Can you conclude that the temperature at the ceiling must be cooler (to the order estimated in (2)) than on the floor (after the air in the room settles down to time-independent state = equilibrium state)?

This is not a very trivial question, so I wish you to guess the answer, supplying plausible supportive arguments.

Comments
Naively speaking, we may expect that the air is cooler near the ceiling than near the floor. Indeed, there was a very famous scientist who concluded the existence of the temperature difference. Actually, there is no temperature difference due to external field. I wish to prove this later, but it is not very trivial, so here, I wish you to discuss (i) what is the likely situation, $\Delta T = 0$ or not?; (ii) Give justifying or plausibility arguments for your guess.

3 [Law of partial pressure from the kinetic point of view].
Demonstrate Dalton’s law of partial pressure with the kinetic theory of D Bernoulli. If you think this is too trivial, you can skip it.

4 [Ideal gas in $d$-space].
What is Bernoulli’s equation of state in $d$-dimensional space? Again, a trivial question, right?

5 [Equipartition-related] This is just from the notes.

(1) In an equilibrium\(^2\) mixture ideal gas maintained at temperature $T$ are two molecules, 1 and 2, with mass $m$ and $M$, respectively. Suppose $m/M = 0.31$. What is the ratio between the mean square relative velocity of these two molecules and the mean square velocity of molecule 1?

(2) What is $\langle (1/2)m(v_1 - v_2)^2 \rangle$ in the $M/m \to \infty$ limit? The answer should be obvious, so state your answer first with your supporting argument and then confirm it, using the formulas you should have used to answer (1).

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\(^2\)We have not clearly defined what ‘equilibrium’ is, but here you may understand that the system is isolated and left alone for a sufficiently long time. The molecules move in a mutually unrelated manner, and, in particular, the equipartition of energy holds.
Homework 1

1 [How big is \(N_A\)]
Propose a way to show/illustrate how big Avogadro’s constant \(N_A\) (or, more generally, the number of molecules in a macro object) is. My ‘spoon performance’ was an example. One more illustration:
The total number of cells in human bodies on the earth at present is still less than \(N_A\), even if you include your beloved gut microbes.

2 [Otto von Guericke 1654]
The Magdeburg hemispheres\(^3\) has a 50 cm diameter. One of them had a tube connection to attach the pump, with a valve to close it off. When the air was sucked out from inside the hemispheres, and the valve was closed, the hose from the pump could be detached, and the hemisphere were held firmly together by the air pressure of the surrounding atmosphere. Estimate the force required to separate the hemispheres.

3 [Mean square velocity].
There is a gas of mass 19 g in a container of 21 liters. In equilibrium, its pressure is 1.1 atm. What is the root mean-square velocity of the molecules in the gas?

4 [Equipartition related].
In a mixture ideal gas maintained at temperature \(T\) are two molecules, 1 of mass \(m\) and 2 of mass \(M\) (\(> m\)). The ratio between the mean square relative velocity of these two molecules and the mean square velocity of molecule 1 is 1.2. What is the mass ratio \(M/m\)?

\(^3\)Wikipedia https://en.wikipedia.org/wiki/Magdeburg_hemispheres is nice. von Guerick was a successful statesman for his town Magdeburg: “He often would not explain scientifically how his shows worked leading people to believe in his wizardry, promoting his status as a great leader.” (Wiki von Guericke).