Homework 12 due 9 am on April 23 (Tu), 2019.
Submit to compass2g

You may discuss with your friends AFTER you have made due efforts of your own to solve the problems. I trust you. I wish you to fully understand the solutions when you submit your homeworks (and get the full credit).

No solution without your justification will get any credit. You must know how to write proper reports. Writing only formulas is totally unacceptable; your solution must read as a proper English composition.

As to the use of TeX: It was announced that from week 10 use of TeX (of some version) would be strictly imposed. The purpose is that you learn how to write mathematics properly, so if proper math orthography would be met, anything, including extremely neat hand writing, will be accepted. You must use proper aligning of formulas, correct punctuations, and correct fonts,\footnote{Italicized or not in particular; basically, all the formulas are in italic and all the ordinary English sentences are in upright} etc., even with handwriting. Errors in math orthography will be penalized (but at most 30\% of the total score). Except for punctuations most requirements will be automatically satisfied, if you use (La)TeX.\footnote{Although I have no intention to recommend my own macros, if you use something like them, then you need only to be able to ‘read formulas loud.’ That is why I posted all the source files; in most cases you can copy some parts of them with modifications.} With Words you will have to struggle to meet the requirement. Handwriting is strongly discouraged.

You may send me (yoono@illinois.edu) TeX questions like: how to write/program “...

1. [Cosmic background temperature]
At present, the cosmic background radiation is at 3 K. Suppose the universe expands adiabatically (but not necessarily quasistatically). What can you say about the temperature of the cosmic background radiation when the total volume of the universe was one half of the present volume?

2. [Electron-positron-photon equilibrium]
In Discussion 11.6 we discussed the electrons ‘e’ and the positrons ‘p’ in equilibrium
with the photon field (electromagnetic field) and determined their chemical potentials: $\mu_e = \mu_p = 0$.

1. Calculate the total energy $E_p$ of electrons [You need not perform the integrals].
2. Find their $T$ dependence.
3. Let $E_{h\nu}$ be the total electromagnetic wave in this same volume. Which is larger, $E_e$ or $E_{h\nu}$?

3. [Molecular vibration]
The vibrational spectrum of I$_2$ is at (reciprocal wavelength) 213 cm$^{-1}$.
1. What is the occupation number ratio of the ground and the first excited states at room temperature 300 K?
2. What is the contribution in % of vibration to the total $C_V$?