Homework 11 due 9 am on April 18 (Th), 2019.
Submit to compass2g

You may discuss with your friends AFTER you have made due efforts of your own to solve the problems. I trust you. I wish you to fully understand the solutions when you submit your homeworks (and get the full credit).

No solution without your justification will get any credit.

As to the use of TeX: It was announced that from week 10 use of TeX (of some version) would be strictly imposed. The purpose is that you learn how to write mathematics properly, so if proper math orthography would be met, anything, including extremely neat hand writing, will be accepted. You must use proper aligning of formulas, correct punctuations, and correct fonts, etc., even with handwriting. Errors in math orthography will be penalized (but at most 20% of the total score). Except for punctuations most requirements will be automatically satisfied, if you use (La)TeX. With Words (and handwriting) you will have to struggle to meet the requirement.

You may send me (yoono@illinois.edu) TeX questions like: how to write/program “...”?

1 [Finite $T$ correction for fermion chemical potential in $D$-space]. (1) Assuming that the dispersion relation (the momentum-energy relation) is $\epsilon = |\mathbf{p}|^2/2m$, find the relation among $E$, $\mathbf{P}$ and $V$ in $D$-dimensional space of an ideal gas system. [Mimic the derivation of $PV = (2/3)E$.]
(2) What is the ratio $E/\mu(0)N$ at $T = 0$, if all the particles are identical fermions?
(3) For fermions we can conclude (under constant $V$ and $N$) that

$$E = E_0 + \frac{1}{2} \alpha_D N T^2 + o[T^2]$$

(DH11.1)

where $\alpha_D > 0$ is a ($D$-dependent) constant. Using this fact, compute the correction to the Fermi energy $\mu(T)$ to order $T^2$. [Fully use thermodynamics.]

---

1 This is not an error; since the Midterm deadline is 16th, I shift the deadline for two days.
2 italicized or not in particular; basically, all the formulas are in italic and all the ordinary English sentences are in upright
3 Although I have no intention to recommend my own macros, if you use something like them, then you need only to be able to ‘read formulas loud.’ That is why I posted all the source files; in most cases you can copy some parts of them with modifications.
(4) For bosons below $T_c (D \geq 3)$, find the temperature dependence of the non-condensate $N_1$.

2 [Adiabatic free expansion]
There is a cylinder with a piston. It contains $N$ identical particles and is thermally isolated. The volume of the cylinder is suddenly expanded (by pulling the piston out a bit) by 10%.

We wish to know what happens after the system equilibrates. Let $P_i (P_f)$ be the initial (final) pressure and $T_i (T_f)$ be the initial (final) temperature.
Fermion case
(F1) Find $P_f / P_i$.
(F2) Which is larger, $T_i$ or $T_f$? Explain your answer qualitatively in plain terms.
Boson case
(B1) Find $P_f / P_i$.
(B2) What happens, qualitatively, to the BEC (= Bose-Einstein condensation) temperature $T_c$? Explain your answer in plain terms intuitively.
(B3) Suppose $T_i = T_c$ for the initial system. Does the system maintain BEC after expansion?

3. Consider a quantum ideal gas (fermion and boson cases separately, if different). No hand-waving argument will be accepted. [Hint: Use thermodynamics (as much as possible to save your time) and $PV = 2E/3$ in this problem.]
(1) The volume is increased under constant temperature. Does the entropy increase? You must demonstrate your result without any hand-waving argument. Notice that thermodynamics alone cannot answer this question.
(2) You wish to decrease the temperature while keeping the pressure. How do you have to change the system volume?