Homework 10 due 9 am on April 9 (Tu), 2019.
Submit to compass2g

You may discuss with your friends AFTER you have made due efforts of your own to solve the problems. I trust you. I wish you to fully understand the solutions when you submit your homeworks (and get the full credit).

No solution without your justification will get any credit.

As to the use of TeX: It was announced that from week 10 use of TeX (of some version) would be strictly imposed. The purpose is that you learn how to write mathematics properly, so if proper math orthography would be met, anything, including extremely neat hand writing, will be accepted. You must use proper aligning of formulas, correct punctuations, and correct fonts,1 etc., even with handwriting. Errors in math orthography will be penalized (but at most 20% of the total score). Except for punctuations most requirements will be automatically satisfied, if you use (La)TeX.2 With Words (and handwriting) you will have to struggle to meet the requirement.

You may send me (yoono@illinois.edu) TeX questions like: how to write/program “…”?

1. [Easy colligative property questions]
   (1) 12.3 g of an unknown substance (with negligible vapor pressure) is dissolved in 100 g of water. Its vapor pressure is 743 mmHg at 100 °C. What is its molecular weight $M$?
   (2) A 1 l solution containing 25 g of a substance at 0°C exhibits the osmotic pressure 2.34 atm. What is the osmotic pressure of the solution containing 31 g/l of the same substance (with the same solvent) at 30°C?

2. [Steam distillation]

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1Italicized or not in particular; basically, all the formulas are in italic and all the ordinary English sentences are in upright

2Although I have no intention to recommend my own macros, if you use something like them, then you need only to be able to ‘read formulas loud.’ That is why I posted all the source files; in most cases you can copy some parts of them with modifications.
According to the Clapeyron-Clausius equation, the vapor pressure of a liquid obeys

\[ \frac{dP}{dT} = \frac{\Delta H}{T \Delta V}, \]  

(DH10.1)

where \( \Delta H \) denotes the evaporation heat, and \( \Delta V \) is the volume increase by evaporation. Usually, the vapor is approximated as an ideal gas and the liquid volume is ignored, so \( \Delta V = V_{\text{vapor}} = RT/P \) is used. Therefore, the original Clapeyron-Clausius equation may be written as

\[ \frac{dP}{dT} = \frac{P \Delta H}{RT^2}. \]  

(DH10.2)

(1) Show that this implies \( P = ce^{-\Delta H/RT} \), where \( c \) is a positive constant, if we may assume \( \Delta H \) is constant.

(2) There are two immiscible liquids A and B with the vapor pressure (in atm) \( T \) in K

\[ P_A(T) = 942293e^{-5135/T}, \quad P_B(T) = 605380e^{-5364/T}. \]  

(DH10.3)

What is the boiling point of the A, B mixture? [You can use graphic method to solve this. For example, you can use http://dlippman.imathas.com/graphcalc/graphcalc.html]

(3) What is the mole fraction of B in the vapor phase? [In this example, A is actually water.]

3. [Adsorption question]

Consider a surface with \( N \) adsorption centers, each of which can accommodate at most one particle. The energy of the adsorbing site is reduced by \( \epsilon (> 0) \), that is the one particle energy is \(-\epsilon\) when a particle is adsorbed. On the surface there is a conversion reaction between A and B, which is \( \Delta (> 0) \) more stable than A. B cannot detach from the surface. The surface is placed in a large tank filled with gas of A with chemical potential \( \mu \).

(1) Write down the grand canonical partition function for the adsorbing surface.

(2) Find the total coverage fraction \( \theta \) (i.e., the number of sites occupied by A or B divided by \( N \)).

(3) What do you expect to happen to the chemical potential of A, if you change \( \Delta \) while fixing the total coverage \( \theta \)?
4. [Elementary ideal quantum particles]
The one particle state energies are equally spaced (with spacing $\epsilon$) for a particle.

(1) There are $N = 1232$ identical fermions in the system and its ground state has energy $E_0 = 2311$ eV (relative to the one-particle ground state).

(i) What is the spacing $\epsilon$ and the Fermi level $\mu_F$ of the system?
(ii) What is the specific heat of the system at very low temperatures? You may assume $\beta \epsilon \gg 1$.

(2) Now, let us assume these particles are identical bosons and $\epsilon$ is the same.

(i) What is the specific heat of the system at very low temperatures?
(ii) The system is expanded and $\epsilon$ becomes smaller. What do you expect to happen to the specific heat of the system?