27 Lecture 27. Baker's transformation

27.1 Baker's transformtion

The baker's transformation is a one-to-one map $T : M = [0,1)^2 \to [0,1)^2$ defined as follows:

$$T(x,y) = \begin{cases} (2x, y/2) & x \in [0,1/2) \\ (2x-1, (y+1)/2 & x \in [1/2,1) \end{cases} . \tag{27.1}$$

The transformation and its inverse are illustrated in Fig. 27.1

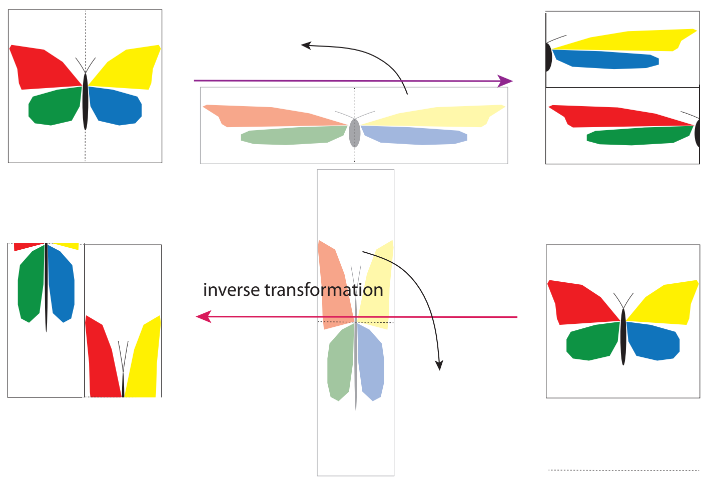

Figure 27.1: Baker's transformation and its inverse

27.2 Stable and unstable manifolds of baker's transformation

M may be decomposed into vertical sets and horizontal sets. The vertical set containing (x,y) is

$$\gamma_-(x,y) = \{(x,y_1) \,|\, y_1 \in [0,1)\} \tag{27.2}$$

and the horizontal set containing (x,y) is

$$\gamma_+(x,y) = \{(x_1,y) \,|\, x_1 \in [0,1)\}. \tag{27.3}$$

Since T expands $\gamma_+(x,y)$, it is an (local) unstable manifold of (x,y); we see $\gamma_-(x,y)$ is a (local) stable manifold of (x,y).[327]

Notice that this decomposition defines an equivalence relation: If $x \in \gamma_\pm(y) \Rightarrow$

[327] 'local' in general, because they may not be continuous.

$\gamma_\pm(x) = \gamma_\pm(y)$.

27.3 Invariant measure of baker's transformation

Obviously the area is preserved, so the usual Lebesgue measure is an invariant measure.

We can show that the measure is a mixing (of course ergodic) measure.

27.4 Symbolic dynamical expression of baker's transformation

Let us define the partition $\mathcal{A} = \{M_0, M_1\}$ (see **32.2** for a precise definition): $M = M_0 \cup M_1$, where $M_0 = [0, 1/2) \times [0, 1)$ and $M_1 = [1/2, 1) \times [0, 1)$.

How this partition is transformed according to T or T^{-1} may be understood easily from the figure 27.2.

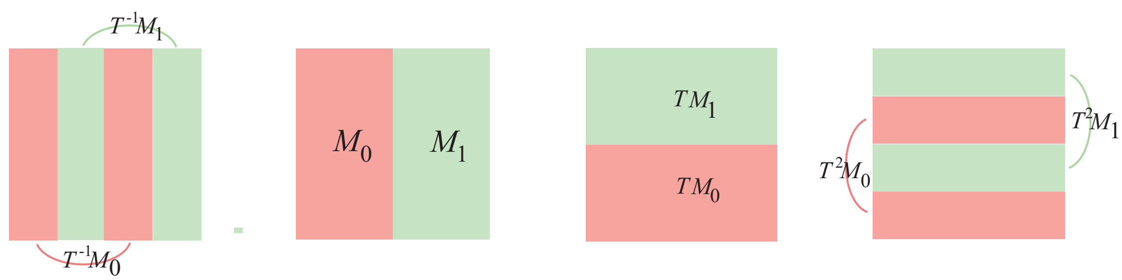

Figure 27.2: The fate of partition $M_0 \vee M_1$

As is clear from the figure $\vee_{n=0}^{\infty} T^{-n}\mathcal{A}$ consists of the totality of γ_+, and $\vee_{n=0}^{\infty} T^{n}\mathcal{A}$ consists of the totality of γ_-. Thus, an element of $\vee_{-\infty}^{\infty} T^{n}\mathcal{A}$ specifies a point in M. This allows us to assign a 01 sequence to each point in M such that TM corresponds to the shift on $\{0, 1\}^{\mathbb{Z}}$. The rule may be more explicitly stated as follows: For $x \in M$ if $T^{-n}x \in M_\alpha$ ($\alpha = 0$ or 1) $\omega(x)_n = \alpha$.

The correspondence is not one to one. $M \to \{0, 1\}^{\mathbb{Z}}$ is injective. However, for binary rational numbers its binary expansion is not unique. However, these points are measure zero, so as a probabilistic system (= measure-theoretical dynamical system) we may totally ignore them and identify baker's transformation (with the Lebesgue measure) and the Bernoulli system $B(1/2, 1/2)$.