

# Index

## A

Abel's formula ... 541  
absolute continuous spectrum ... 473  
absolute value ... 96  
absolutely continuity ... 286  
absolutely convergence ... 526  
accumulating point ... 528  
addition theorem, for spherical harmonics ... 374  
adjoint operator ... 468  
adjoint ... 468  
admittance ... 139  
advanced Green's function ... 503  
advection ... 25  
afterglow effect ... 452, 505  
Airy equation ... 459  
Airy function ... 395  
Airy integral ... 459  
Aleksandrov's theorem ... 411  
algebraic branch point ... 125  
algebraic function ... 88  
aliasing ... 438  
almost everywhere ... 274  
Ampère's law ... 39  
analytic completion ... 118  
analytic continuation ... 117  
analytic function ... 114, 117  
analyticity ... 114  
Anderson localization ... 474  
annular problem ... 378  
anti-difusion ... 403  
area element ... 63  
argument ... 96  
associate Legendre functions ... 353,  
  371  
asymptotic expansion ... 357

asymptotic sequence ... 357

asymptotic series ... 357  
atomic measure ... 10  
Ausstrahlungsbedingung ... 500  
autonomous ... 166  
axial vector ... 41

## B

backward Euler method ... 426  
balance equation ... 25  
Banach space ... 291  
band-limited function ... 437  
Beppo-Levi's theorem ... 277  
Bernoulli equation ... 173  
Bessel function, addition theorem ... 388  
Bessel function, generating function ... 385  
Bessel function, orthonormal relation ... 393  
Bessel function ... 130, 382  
Bessel transform ... 454  
Bessel's equation ... 381  
Bessel's inequality ... 295, 552  
Bessel's integral ... 385  
beta function ... 145  
biholomorphic map ... 154  
Biot-Savart's law ... 39  
bodily forces ... 32  
Bolzano-Weierstrass' theorem ... 529  
Borel measure ... 287  
Borel summable ... 367  
Borel transform ... 367  
bounded operator ... 472  
bounded set ... 528  
bounded variation ... 432

- bra ... 294  
 branch point ... 125  
 branch ... 93  
 Burgers equation ... 4
- C**
- Cantor, biography ... 260
  - Cantor set ... 168
  - Casorati-Weierstrass theorem ... 127
  - Cauchy, biography ... 107
  - Cauchy principal value ... 209
  - Cauchy sequence ... 525
  - [Cauchy-Lipschitz uniqueness theorem ... 168, 556]
  - Cauchy-Peano's theorem ... 167, 556
  - Cauchy-Riemann equation ... 98
  - Cauchy's formula ... 106
  - Cauchy's theorem ... 103
  - Cauchy-Schwartz inequality ... 293
  - causal solution ... 505
  - causality ... 139
  - chain rule ... 51
  - chaos ... 317
  - characteristic curve, for wave equation ... 414
  - characteristic curves ... 191
  - characteristic differential equation ... 191
  - characteristic exponent ... 355
  - characteristic polynomial ... 177
  - Chebychev polynomial ... 316
  - Chebychev's inequality ... 329
  - circline ... 157
  - Clairaut's differential equation ... 165, 180
  - classical polynomials ... 306
  - classical solution ... 401
  - classical special functions ... 335
  - cocircline condition ... 157
  - coherent state ... 437
  - Cole-Hopf transformation ... 4
  - compact operator ... 475
  - comparison theorem ... 411
  - complete ... 247, 251, 291
  - complex conjugate ... 95
  - complex Fourier series ... 552
  - complex function ... 54, 87
  - complex number ... 95
  - complex plane ... 96
  - complexification ... 182
  - conditional convergence ... 527
  - conditional extremum ... 79
  - confluent hypergeometric equation ... 354
  - confluent hypergeometric function ... 354
  - conformal invariance ... 161
  - conformal map ... 153
  - conformal transform ... 224
  - conformality ... 153, 155
  - conjugate harmonic function ... 100
  - conjugate point ... 84
  - conservation law, local ... 26
  - conservation of charge ... 39
  - consistency, in numerical analysis ... 423
  - constitutive relations ... 32
  - continuity equation ... 37
  - continuous spectrum ... 473
  - continuum ... 24
  - contour integration ... 102
  - convection ... 25
  - convergence ... 97
  - convergence circle ... 115
  - convergence coordinate ... 458
  - convergence disk ... 115
  - convergence radius ... 114
  - convex function ... 50
  - convexity ... 535
  - convolution, Fourier transform ... 554
  - convolution, Laplace transform ... 459
  - convolution ... 211
  - cosine-Fourier series ... 257
  - Coulomb gauge ... 41
  - Coulomb's law ... 39
  - countable set ... 527

- Courant-Friedrichs-Lowy condition ... 429  
 Cranck-Nicholson method ... 426  
 cross ratio ... 157  
 curl ... 33, 64, 560  
 curl, curvilinear coordinates ... 75  
 curvilinear coordinates ... 71  
 cylinder function ... 386  
 cylindrical coordinates ... 73
- D**
- d'Alembert, biography ... 61  
 d'Alembertian ... 42  
 d'Alembert's formula ... 57  
 d'Alembert's transformation ... 175  
 Darboux's equation ... 418  
 deBois Raymond's theorem ... 552  
 decomposition of unity ... 297,  
     300, 469  
 deconvolution ... 431  
 degree of ramification ... 125  
 delta function ... 82  
 delta function, definition ... 203  
 delta function, Fourier expansion  
     ... 442  
 delta function, Fourier transform  
     ... 445  
 delta function, Laplace transform  
     ... 461  
 dense ... 528  
 derivative ... 98, 109  
 Descartes' sign rule ... 310  
 diagonal method ... 260  
 differentiable ... 47, 55, 98  
 differential coefficient ... 47  
 differential form ... 69  
 differential operator ... 202  
 diffusion constant ... 4, 26  
 diffusion equation ... 4, 27  
 diffusion equation, Brownian particle ... 28  
 diffusion equation, short time behavior ... 405  
 Dini's condition ... 253
- Dirac ... 294  
 direct method ... 85  
 Dirichlet condition ... 18, 253  
 Dirichlet function ... 258, 275  
 discrete Fourier transform ... 440  
 discrete spectrum ... 473  
 discretization ... 13, 421  
 discriminant ... 17  
 displacement current ... 40  
 displacement vector ... 32  
 divergence ... 63, 559  
 divergence, curvilinear coordinates  
     ... 75  
 domain of dependence ... 415  
 domain, of operator ... 467  
 double exponential decay formula  
     ... 326  
 double layer ... 483  
 double series ... 549  
 drumhead ... 34  
 dual space ... 294
- E**
- Earnshaw's theorem ... 409  
 eigenvalue problem ... 263  
 electrical displacement ... 42  
 elementary functions ... 88  
 elementary transcendental functions ... 88  
 elliptic cylindrical coordinates ... 73  
 elliptic equation ... 16, 17, 21,  
     411  
 elliptic integral ... 160  
 energy integral ... 35, 415  
 entire function ... 88  
 equidimensional equation ... 178  
 essential spectrum ... 473  
 Euclidean space ... 291  
 Euler, biography ... 89  
 Euler-Lagrange equation ... 78  
 Euler-MacLaurin sum formula ... 443  
 Euler's constant ... 147  
 Euler's equation, fluid dynamics  
     ... 37

- Euler's equation (variational calculus) ... 78  
 Euler's formula ... 88  
 Euler's integral ... 142  
 Euler's theorem (variational calculus) ... 78  
 existence of mollifier ... 536  
 exponential function ... 88  
 exponential, of matrix ... 181, 189  
 extension of operator ... 467  
 exterior problem ... 21, 378
- F**
- factorial ... 143  
 Faraday's law ... 39  
 fast Fourier transform ... 440  
 fast inverse Laplace transform ... 463  
 Fejer's theorem ... 254, 552  
 Feynman-Kac formula ... 496  
 Feynman-Kac path integral ... 496  
 FFT ... 440  
 Fick's law ... 26  
 first digit problem ... 287  
 Floque's theorem ... 355  
 flux density ... 25  
 focusing effect ... 419  
 Fokker-Planck equation ... 30  
 formally self-adjoint ... 470  
 Fourier, biography ... 9  
 Fourier cosine transform ... 267, 433  
 Fourier expansion ... 130, 243  
 Fourier series ... 244, 300  
 Fourier sine transform ... 266, 433  
 Fourier transform ... 430, 553  
 Fourier transform, of generalized functions ... 444  
 Fourier-Bessel-Dini expansion ... 394  
 Fourier-Bessel(-Dini) transformation ... 339  
 Fourier's law ... 26  
 Fredholm integral equation ... 475  
 Fredholm operator ... 475  
 Fresnel integral ... 136
- Frobenius' method ... 345  
 Frobenius' theorem (on minimal polynomial) ... 189  
 Fubini's theorem ... 277  
 function element ... 117  
 functional ... 77  
 functional derivative ... 81  
 functional differentiation ... 82  
 functional integral ... 287  
 fundamental matrix ... 342  
 fundamental set ... 284  
 fundamental solution ... 177, 202  
 fundamental system of solutions ... 183, 341
- G**
- Gårding's theorem ... 420  
 Galerkin method ... 422  
 Gamma function ... 142, 364  
 gap theorem ... 119  
 gauge symmetry ... 41  
 gauge transformation ... 41  
 Gauss, biography ... 111  
 Gauss formula, for integration ... 320  
 Gauss formula (numerical integration) ... 320  
 Gauss quadrature ... 321  
 Gaussian integral ... 280  
 Gauss-Stokes-Green's theorem ... 68, 561  
 Gegenbauer-Neumann formula ... 388  
 general solution, ODE ... 164, 555  
 generalized eigenspace ... 189  
 generalized Fourier expansion ... 296  
 generalized function ... 202, 444  
 generalized function, differentiation ... 207  
 generalized homogeneous function ... 196  
 generalized Rodrigues formula ... 48, 306  
 generalized solution ... 5  
 generating function ... 115, 130  
 generating function, for orthog-

- onal polynomials ··· 307  
 genuine singularity ··· 127  
 Gershgorin's theorem ··· 186  
 Gibbs' phenomenon ··· 251  
 Glass pattern ··· 167  
 good function principle ··· 278  
 gradient ··· 53, 62, 558  
 gradient, curvilinear coordinates  
     ··· 74  
 Gram-Schmidt orthonormaliza-  
     tion ··· 297, 304  
 Green, biography ··· 11  
 Green's formula ··· 227, 407  
 Green's function ··· 11, 202, 214,  
     239, 302, 493  
 Green's function, diffusion equa-  
     tion ··· 493  
 Green's function, Sturm-Liouville  
     problem ··· 215  
 Green's theorem ··· 68  
 group ··· 156
- H**
- Hölder continuity ··· 167  
 Hankel function ··· 393  
 Hankel's theorem ··· 453  
 harmonic function ··· 27, 67, 100,  
     407  
 Hartman-Grobman theorem ··· 185  
 Haselgrove method ··· 328  
 Hausdorff's theorem ··· 288  
 heat conductivity ··· 26  
 Heaviside step function ··· 209  
 Heine-Borel covering theorem ··· 529  
 Helmholtz equation ··· 395, 498  
 Helmholtz formula ··· 505  
 Helmholtz' theorem (vector field)  
     ··· 70  
 Helmholtz-Hodge theorem ··· 70  
 Helmholtz-Stokes-Blumenthal the-  
     orem ··· 71  
 Hermite polynomial ··· 313  
 Hilbert, biography ··· 291  
 Hilbert space ··· 290
- Hilbert transform ··· 140, 452  
 Hilbert-Schmidt' theorem ··· 475  
 holomorphic function ··· 99, 155  
 holomorphy ··· 155  
 homogeneous conditions ··· 19  
 homogeneous equation ··· 172  
 homogeneous function ··· 195  
 Hooke's law ··· 33  
 Huygens' principle ··· 418  
 hyperbolic equation ··· 16, 22  
 hyperbolic equation in Gårding's  
     sense ··· 419  
 hyperbolic fixed point ··· 185  
 hyperbolic function ··· 92  
 hyperbolicity in Gårding's sense  
     ··· 419  
 hypergeometric equation ··· 352
- I**
- ideal fluid ··· 37  
 idempotency ··· 297  
 identity theorem (real analytic  
     function) ··· 536  
 image charge ··· 225  
 imaginary axis ··· 96  
 imaginary part ··· 95  
 imaginary unit ··· 95  
 implicit method ··· 426  
 incompressibility ··· 38  
 indentation ··· 138  
 indicial equation ··· 346  
 infinitely many-valued function  
     ··· 93  
 infinite product ··· 544  
 inner product ··· 300  
 integral equation ··· 324  
 integral, of generalized function  
     ··· 211  
 integrating factor ··· 171, 556  
 integration, bilinearity ··· 103  
 interior problem ··· 21, 377  
 inverse trigonometric function ··· 94  
 irregular singular point ··· 345,

- irrotational field ··· 67  
 irrotational flow ··· 38
- J**
- Jensen's inequality ··· 50  
 Jordan's lemma ··· 135  
 Joukowski transformation ··· 158
- K**
- ket ··· 294  
 kinematic viscosity ··· 37  
 Kramers-Kronig relation ··· 139  
 Kummer's equation ··· 354  
 $K$ -vector space ··· 289
- L**
- $L_2$ -convergence ··· 298, 552  
 $L_2$ -norm ··· 298  
 $L_2$ -space ··· 292, 298  
 lacunary series ··· 118  
 Lagrange, biography ··· 79  
 Lagrange interpolation formula ··· 322  
 Lagrange multiplier ··· 79  
 Lagrange's method of variation of constants ··· 177, 558  
 Laplace, biography ··· 456  
 Laplace equation ··· 5, 41, 263, 406  
 Laplace equation, Cartesian coordinates ··· 265  
 Laplace equation, Cylindrical coordinates ··· 336  
 Laplace euqation, fundamental solution ··· 482  
 Laplace transformation ··· 455  
 Laplace's first integral ··· 313  
 Laplacian, curvilinear coordinates ··· 75  
 Laplacian ··· 3, 14, 67  
 Laplacian for vector field ··· 562  
 Laplacian for vector field ··· 67  
 Laurent series ··· 128  
 Laurent's theorem ··· 128
- least square approximation ··· 304  
 Lebesgue extension ··· 286  
 Lebesgue integrable ··· 275  
 Lebesgue integral ··· 275  
 Lebesgue integral, relation to Riemann integral ··· 276  
 Lebesgue's convergence theorem ··· 277  
 Lebesgue's measure problem ··· 288  
 left differentiability ··· 532  
 Legendre equation ··· 351, 370  
 Legendre function ··· 352  
 Legendre polynomial ··· 305, 311, 351  
 Legendre's condition, variational calculus ··· 83  
 liminf ··· 526  
 limit ··· 97  
 limsup ··· 526  
 linear operator ··· 7, 467  
 linear phenomenological law ··· 26  
 linear response ··· 52  
 linearity ··· 7  
 linearly stable ··· 185  
 Liouville's theorem ··· 411  
 Lipschitz condition ··· 167, 556  
 logarithm ··· 93  
 logarithmic branch point ··· 125  
 Lommel's formula ··· 392  
 Lommel's integral ··· 390  
 long-time tail ··· 36  
 Lorentz gauge ··· 41
- M**
- Möbius transformation ··· 156  
 Malgrange-Ehrenpreis' theorem ··· 197  
 Markov property ··· 495  
 maximum principle ··· 408  
 Maxwell-Cattaneo equation ··· 405  
 Maxwell's equations ··· 40  
 Maxwell's equation, in material ··· 43  
 Maxwell's equation, in vacuum

...40  
 mean-value theorem ...407  
 mean-value theorem, converse ...408  
 measurable ...275, 284  
 measurable function ...286  
 measurable set ...284  
 measure ...285  
 measure zero ...274  
 Mellin transformation ...465  
 method of images ...225, 486  
 method of lowering the order ...175  
 method of quadrature ...170  
 method of spherical means ...418  
 metric tensor ...72  
 minimal polynomial ...189  
 minimization sequence ...85  
 minimum algebra ...285  
 Möbius transformation ...156  
 modified Bessel functions ...394  
 modified Radon transform ...450  
 modulus ...96  
 momentum flux density ...35  
 Monte Carlo method ...329  
 Morera's theorem ...111  
 multipole expansion ...375

## N

nabla ...63, 559  
 natural boundary ...118  
 Navier-Stokes equation ...37  
 $n$ -ball, volume of ...150  
 Neumann condition ...19  
 Neumann function ...390, 485  
 Nevanlinna's theorem ...368  
 Newtonian viscosity ...36  
 nodal set ...488  
 Noether's theorem ...84  
 nonautonomous ...166  
 non-standard ...89  
 norm ...78, 291  
 norm, for matrix ...181  
 normal form ...165  
 normal form, ODE ...555  
 normal operator ...469

null set ...284  
 numerical integration ...319

## O

observable ...467  
 Ohm's law ...44  
 optimal truncation, of asymptotic series ...358  
 ordinary differential equation ...164  
 orientation ...104  
 orthogonal curvilinear coordinate system ...72  
 orthogonal projection ...297  
 orthonormal basis ...295  
 outer measure ...284

## P

parabolic boundary ...401  
 parabolic equation ...17  
 parallelogram law ...293  
 Parseval's equality ...295, 552  
 partial derivative ...55  
 partial wave expansion ...397  
 partially differentiable ...55  
 particular solution, ODE ...164, 555  
 partner ...344  
 path integral ...287  
 perfect differential equation ...170  
 perfect fluid ...37  
 perturbation series ...366  
 Plancherel's formula ...435  
 Plancherel's theorem ...435  
 Plemelj formula ...139, 447  
 Poincaré's lemma ...69  
 Poincaré's lemma, converse of ...69  
 point spectrum ...473  
 Poisson equation, fundamental solution ...210  
 Poisson sum formula ...442  
 Poisson's equation, derivation ...6, 41  
 Poisson's formula ...240, 269  
 Poisson's ratio ...33

- Poisson's sum formula ··· 442  
 polar form ··· 96  
 polar vector ··· 41  
 polarization ··· 42  
 pole ··· 127  
 potential ··· 67  
 potential field ··· 67  
 potential flow ··· 38  
 power ··· 93  
 power series ··· 114  
 power spectrum ··· 439  
 preservation of order ··· 404  
 principal part ··· 16, 129  
 principal value of integral ··· 137  
 principal value ··· 93, 96, 138  
 principle of invariance of functional relations ··· 117  
 probability measure ··· 287  
 projection ··· 297  
 propagator ··· 503  
 pure point spectrum ··· 473  
 Pythagoras' theorem ··· 293
- Q**
- quasi Monte Carlo method ··· 327  
 quasilinear first order PDE ··· 190
- R**
- radiation condition ··· 500  
 Radon inversion formula ··· 450  
 Radon transform ··· 449  
 random walk ··· 494  
 rational function, integration ··· 134  
 rational function ··· 88  
 rational integral function ··· 88  
 reaction-diffusion equation ··· 27  
 real axis ··· 96  
 real part ··· 95  
 rectification ··· 166  
 reflection principle ··· 224  
 region ··· 6  
 regular distribution ··· 205  
 regular point ··· 472  
 regular singular point ··· 345
- regular Sturm-Liouville problem ··· 214  
 removable singularity ··· 127  
 residue theorem ··· 131  
 residue ··· 131  
 resolvent ··· 472  
 resolvent equation ··· 472  
 resolvent set ··· 472  
 response function ··· 139  
 retarded Green's function ··· 500  
 Reynolds number ··· 38  
 Riccati's equation ··· 174  
 Riemann, biography ··· 121  
 Riemann geometry ··· 72  
 Riemann mapping theorem ··· 158  
 Riemann surface ··· 119  
 Riemann-Lebesgue lemma ··· 255, 435  
 Riemann-Lebesgue's theorem (convergence of Fourier series) ··· 253  
 right differentiability ··· 532  
 Ritz's method ··· 85  
 Robin condition ··· 20  
 Rodrigues' formula ··· 306
- S**
- sampling function ··· 438  
 Sampling theorem ··· 437  
 Sato hyperfunction ··· 141  
 scalar potential ··· 41  
 scalar product ··· 291, 294  
 scalar product, of functions ··· 300  
 scale invariant equation ··· 179  
 Schläfli's integral ··· 308  
 Schwartz class ··· 445  
 Schwartz-class functions ··· 205  
 Schwarz' theorem ··· 241, 547  
 Schwarz-Christoffel formula ··· 159  
 Schwinger-Feynman parameter formula ··· 149  
 second order PDE, classification ··· 16  
 second variation ··· 83

- self-adjoint operator … 468  
 semibounded operator … 470  
 separable … 295  
 separation constant … 263  
 separation of variables … 262, 332,  
     471  
 separation theorem … 343  
 shear viscosity … 36  
 $\sigma$ -additivity … 285  
 simple function … 275  
 sine-Fourier series … 257  
 singly connected … 64  
 singular continuous spectrum … 473  
 singular point … 345  
 singular solution, ODE … 164,  
     555  
 singularity … 125  
 Smoluchowski equation … 29  
 Sobolev space … 293  
 solenoidal field … 67, 561  
 solution, classical … 5  
 source term … 26  
 special functions … 334  
 special functions, classical … 335  
 spectral decomposition … 469  
 spectral weight … 469  
 spectrum … 472  
 spectrum, absolutely continuous  
     … 473  
 spectrum, discrete … 473  
 spectrum, essential … 473  
 spectrum, point … 473  
 spectrum, pure point … 473  
 spectrum, singular continuous … 473  
 spherical Bessel function … 396  
 spherical Bessel function, orthonormal relation … 397  
 spherical coordinates … 73  
 spherical Hankel function … 396  
 spherical harmonic function … 373  
 spherical harmonics … 372  
 spherical Neumann functions … 396  
 stability, in numerical analysis  
     … 423
- stability, of fixed point … 185  
 steepest descent … 365  
 Stirling’s formula … 150, 364  
 Stokes approximation … 38  
 Stokes equation … 38  
 Stokes line … 360  
 Stokes’ phenomenon … 360  
 strain tensor … 33  
 stress tensor … 31  
 strong derivative … 53  
 strong differentiation … 53  
 strong maximum principle … 409  
 Sturm-Liouville eigenvalue problem … 478  
 Sturm-Liouville problem … 214,  
     307  
 Sturm’s theorem … 309  
 substantial derivative … 38  
 superposition principle … 8  
 supplementary equations … 44  
 symmetric operator … 467  
 system of fundamental solutions  
     … 177

## T

- telegrapher’s equation … 45  
 test function … 204, 445  
 theorem, Carlson’s … 254  
 theorem, Cauchy’s … 103  
 theorem, Dini … 253  
 theorem, Gershgorin’s … 186  
 theorem, Hartman-Grobman … 185  
 theorem, Hilbert-Schmidt’ … 475  
 theorem, indefinite integral … 106  
 theorem, Morera’s … 111  
 theorem, Navanlinna’s … 368  
 theorem of identity … 117  
 theorem, Sturm’s … 309  
 theorem, von Neumann’s … 427  
 theorem, Weyl-Stone-Titchmarsh-Kodaira … 480  
 theorem, Wiener-Khinchin … 439  
 thermal diffusion constant … 27  
 thermodynamics … 172

$\theta$ -method	...426	well-posedness, of wave equation	...417
time correlation	...438	Weyl's equidistribution theorem	...328
time-delay	...460	Weyl-Stone-Titchmarsh-Kodaira theorem	...480
time-dependent Ginzburg-Landau equation	...21	Wick's theorem	...280
total differential	...547	Wiener-Khinchin's theorem	...439
transversality conditions	...80	Wronskian	...342
triangle inequality	...97, 294		
trigonometric function	...91		
<b>U</b>			
uncertainty principle	...436	X, Y, Z	
uncountable set	...527	X-ray tomography	...450
uniform convergence	...542	Young's modulus	...33
uniform measure	...287	Young's theorem	...547
upstream scheme	...429	zeta function	...147
<b>V</b>			
variation	...77	z-transformation	...456
variational calculus	...77		
vector potential	...41, 70		
vector space, complex	...289		
vector space, real	...289		
Veinberg's theorem	...84		
velocity potential	...38		
volume	...283		
volume element	...24, 75		
volume element, curvilinear coordinates	...75		
volume viscosity	...36		
von Neumann's theorem	...427		
<b>W</b>			
Watson's lemma	...362		
wave equation	...3, 34, 419, 452		
wave equation, fundamental solution	...502		
wave equation in $d$ -space	...451		
wave speed	...3		
weak solution	...5		
Weierstrass, biography	...248		
Weierstrass function	...51		
Weierstrass' theorem	...246		
well-posed	...402		